(a) \( \sqrt{(x'(2))^2 + (y'(2))^2} = 3.272461 \)

The speed of the particle at time \( t = 2 \) seconds is 3.272 meters per second.

(b) \( s(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(2 \cos(2t))^2 + (2t - 1)^2} \)

\( s'(4) = 2.16265 \)

Since \( s'(4) > 0 \), the speed of the particle is increasing at time \( t = 4 \).

(c) \( \int_0^3 \sqrt{(x'(t))^2 + (y'(t))^2} \, dt = 22.381767 \)

The total distance the particle travels over the time interval \( 0 \leq t \leq 5 \) seconds is 22.382 (or 22.381) meters.

(d) \( x(10) = x(8) + x'(8) \cdot 2 = \sin 16 + x'(8) \cdot 2 = -4.118541 \)

\( y(10) = y(8) + y'(8) \cdot 2 = (8^2 - 8) + y'(8) \cdot 2 = 86 \)

The position of the particle at time \( t = 10 \) seconds is \((-4.119, 86)\) (or \((-4.118, 86)\)).
(a) \( R'(45) = \frac{30 - 0}{35 - 55} = -\frac{3}{2} \)

The rate at which water is being pumped into the tank is decreasing at \( \frac{3}{2} \) liters/min² at \( t = 45 \) minutes.

(b) \[
\int_0^{55} R(t) \, dt = 20 \cdot \frac{10 + 30}{2} + 15 \cdot 30 + \frac{1}{2} \cdot 20 \cdot 30 \\
= 400 + 450 + 300 = 1150
\]

(c) \[
\text{Amt} = 100 + 1150 - \int_{10}^{55} 10e^{\left(\sin t\right)/10} \, dt \\
= 1250 - 450.275371 = 799.725 \text{ (or 799.724)}
\]

(d) \( R(45) = 15 \)
\( D(45) = 10.88815 \)

At time \( t = 45 \) minutes, the rate of water pumped into the tank is greater than the rate of water draining from the tank. Therefore, the amount of water in the tank is increasing at time \( t = 45 \) minutes.
Question 3

(a) \[
\frac{dP}{dt} = \frac{1}{4}(220 - P)
\]
\[
\int \frac{dP}{220 - P} = \int \frac{1}{4} \, dt
\]
\[-\ln|220 - P| = \frac{1}{4}t + C
\]
Because \(P(0) = 20, \) \(P < 220,\) so \(|220 - P| = 220 - P\).
\[-\ln(220 - 20) = \frac{1}{4}(0) + C \Rightarrow C = -\ln 200
\]
\[220 - P = 200e^{-t/4}
\]
\[P = 220 - 200e^{-t/4}, \quad t \geq 0
\]

(b) \(Q\) satisfies a logistic differential equation with carrying capacity 220. \(Q\) grows most rapidly when \(Q = \frac{220}{2} = 110.\)
\[
\frac{dQ}{dt} \bigg|_{Q=110} = \frac{110^2}{500} = \frac{121}{5}
\]

(c) \(Q(0) = 20\)
\[
Q'(0) = \frac{1}{500}(20)(200) = 8
\]
\[Q(5) \approx 20 + (8)(5) = 60
\]
\[
Q'(5) \approx \frac{1}{500}(60)(220 - 60) = \frac{96}{5}
\]
\[Q(10) \approx 60 + \left(\frac{96}{5}\right)(5) = 156
\]
Question 4

(a) \( g(0) = 2 \cdot 0 + \int_{-2}^{0} f(t) \, dt = 3 \)
\[ g(-5) = 2 \cdot (-5) + \int_{-2}^{-5} f(t) \, dt = -10 + 3 = -7 \]

(b) \( g'(x) = 2 + f(x) \)
\( g''(x) = f'(x) \)
\[ g''(4) = f'(4) = -1 \]
\( g''(-2) = f'(-2) \) does not exist.

(c) The graph of \( g \) is concave down on the intervals \((-2, 0)\) and \((2, 8)\) since \( g'(x) = 2 + f(x) \) decreases on those intervals.

(d) \( h'(x) = g'(x^2 + 1) \cdot 3x^2 \)
\[ h'(1) = g'(2) \cdot 3 = (2 + f(2)) \cdot 3 \]
\[ = (2 + 3) \cdot 3 = 15 \]
Question 5

(a) \( r(5.4) = r(5) + r'(5) \Delta t = 30 + 2(0.4) = 30.8 \text{ ft} \)
    Since the graph of \( r \) is concave down on the interval \( 5 < t < 5.4 \), this estimate is greater than \( r(5.4) \).

(b) \[
\frac{dV}{dt} = 3 \left( \frac{4}{3} \right) \pi r^2 \frac{dr}{dt}
\]

\[
\left. \frac{dV}{dt} \right|_{r=5} = 4\pi (30)^2 \cdot 2 = 7200\pi \text{ ft}^3/\text{min}
\]

(c) \[
\int_0^{12} r'(t) \, dt = 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)
\]
    \[= 19.3 \text{ ft} \]

\( \int_0^{12} r'(t) \, dt \) is the change in the radius, in feet, from \( t = 0 \) to \( t = 12 \) minutes.

(d) Since \( r \) is concave down, \( r' \) is decreasing on \( 0 < t < 12 \). Therefore, this approximation, 19.3 ft, is less than \( \int_0^{12} r'(t) \, dt \).

Units of \( \text{ft}^3/\text{min} \) in part (b) and ft in part (c)
(a) Since \( g \) is continuous,
\[
g(0) = \lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{\cos(2x) - 1}{x^2}
\]
\[
= \lim_{x \to 0} -\frac{2 \sin(2x)}{2x}
\]
\[
= \lim_{x \to 0} -\frac{4 \cos(2x)}{2} = -2
\]

(b) \( \cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots + (-1)^n \frac{(2x)^{2n}}{(2n)!} + \cdots \)
\[
= 1 - \frac{4}{2!} x^2 + \frac{16}{4!} x^4 - \frac{64}{6!} x^6 + \cdots + (-1)^n \frac{2^{2n}}{(2n)!} x^{2n} + \cdots
\]

(c) \( g(x) = -\frac{4}{2!} + \frac{16}{4!} x^2 - \frac{64}{6!} x^4 + \cdots + (-1)^n \frac{2^{2n}}{(2n)!} x^{2n-2} + \cdots \)

(d) \( g'(x) = \frac{2 \cdot 16}{4!} x - \frac{4 \cdot 64}{6!} x^3 + \cdots \), so \( g'(0) = 0 \).

\[
g''(x) = \frac{2 \cdot 16}{4!} - \frac{3 \cdot 4 \cdot 64}{6!} x^2 + \cdots \), so \( g''(0) > 0 \).

Therefore, \( g \) has a relative minimum at \( x = 0 \) by the Second Derivative Test.