## Question 1

(a) \( \frac{dy}{dx} \bigg|_{t=2} = \frac{2}{e^2} \)

Because \( \frac{dx}{dt} \bigg|_{t=2} > 0 \), the particle is moving to the right at time \( t = 2 \).

\( \frac{dy}{dx} \bigg|_{t=2} = \frac{dy/dt}{dx/dt} \bigg|_{t=2} = 3.055 \) (or 3.054)

(b) \( x(4) = 1 + \int_2^4 \frac{x + 2}{e^t} \, dt = 1.253 \) (or 1.252)

(c) Speed = \( \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575 \) (or 0.574)

\[ \text{Acceleration} = \langle x''(4), y''(4) \rangle = \langle -0.041, 0.989 \rangle \]

(d) Distance = \( \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2} \, dt \)

\( \approx 0.651 \) (or 0.650)

## Question 2

Point of intersection
\( e^{-3x} = \sqrt{x} \) at \((7, S) = (0.238734, 0.488604)\)

(a) Area = \( \int_T^{10} (\sqrt{x} - e^{-3x}) \, dx \)

\( = 0.442 \) or 0.443

(b) Volume = \( \pi \int_T^{10} (1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \, dx \)

\( = 0.453 \pi \) or 1.423 or 1.424

(c) Length = \( \sqrt{e^x - e^{-3x}} \)

Height = \( 5(\sqrt{e^x} - e^{-3x}) \)

Volume = \( \int_T^{10} 5(\sqrt{e^x} - e^{-3x})^2 \, dx = 1.554 \)
### Part B

#### Question 3

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<tbody>
<tr>
<td>(a)</td>
<td>Average acceleration of rocket A is</td>
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|      | \[
|      | \frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2
|      | 1: answer |
| (b)  | Since the velocity is positive, \( \int_{10}^{70} v(t) \, dt \) represents the |
|      | distance, in feet, traveled by rocket A from \( t = 10 \) seconds to \( t = 70 \) seconds. |
|      | A midpoint Riemann sum is |
|      | \[
|      | 20[\frac{v(20) + v(40) + v(60)}{3}] = 20[\frac{22 + 35 + 44}{3}] = 2020 \text{ ft}
|      | 1: uses \( v(20), v(40), v(60) \) |
|      | 1: value |
| (c)  | Let \( v_B(t) \) be the velocity of rocket B at time \( t \). |
|      | \[
|      | v_B(t) = \int_0^t \frac{3}{\sqrt{t+1}} \, dt = 6\sqrt{t+1} + C
|      | 2 = v_B(0) = 6 + C
|      | v_B(t) = 6\sqrt{t+1} - 4
|      | v_B(80) = 50 > 49 = v(80)
|      | Rocket B is traveling faster at time \( t = 80 \) seconds. |
|      | Units of \( \text{ft/sec}^2 \) in (a) and \( \text{ft} \) in (b) |
|      | 1: units in (a) and (b) |

#### Question 4

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| (a)  | \[
|      | \frac{d^2 y}{dx^2} = 3 + 2 \frac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5
|      | 1: answer |
| (b)  | If \( y = mx + b + e^{rx} \) is a solution, then |
|      | \[
|      | m + re^{rx} = 3x + 2(mx + b + e^{rx}) + 1.
|      | If \( r = 0 \): \( m = 2b + 1, \quad r = 2, \quad 0 = 3 + 2m, \)
|      | so \( m = -\frac{3}{2}, \quad r = 2, \quad b = -\frac{5}{4} \).
|      | OR |
|      | If \( r = 0 \): \( m = 2b + 3, \quad r = 0, \quad 0 = 3 + 2m, \)
|      | so \( m = -\frac{3}{2}, \quad r = 0, \quad b = -\frac{9}{4} \).
|      | 1: value for \( r \) |
|      | 1: values for \( m \) and \( b \) |
| (c)  | \[
|      | f \left( \frac{1}{2} \right) \approx f(0) + f'(0) \cdot \frac{1}{2} = -2 + (-3) \cdot \frac{1}{2} = -\frac{7}{2}
|      | f \left( \frac{1}{2} \right) \approx 3 \cdot \left( \frac{1}{2} \right) + 2 \left( \frac{7}{2} \right) + 1 = -\frac{9}{2}
|      | f(1) \approx f \left( \frac{1}{2} \right) + f' \left( \frac{1}{2} \right) \cdot \frac{1}{2} = -\frac{7}{2} + \left( -\frac{9}{2} \right) \cdot \frac{1}{2} = -\frac{23}{4}
|      | 1: Euler's method with 2 steps |
|      | 1: Euler's approximation for \( f(1) \) |
| (d)  | \[
|      | g(0) = 3 \cdot 0 + 2, \quad k + 1 = 2k + 1
|      | g(1) \approx g(0) + g'(0) \cdot 1 = k + (2k + 1) = 3k + 1 = 0
|      | k = \frac{1}{3}
|      | 1: value of \( k \) |
Question 5

(a) \( g(-3) = 2(-3) + \int_{0}^{-3} f(t) \, dt = -6 - \frac{9\pi}{4} \)

\[ g'(x) = 2 + f'(x) \]

\[ g'(-3) = 2 + f'(-3) - 2 \]

(b) \( g'(x) = 0 \) when \( f(x) = -2 \). This occurs at \( x = \frac{5}{2} \).

\[ g'(x) > 0 \text{ for } -4 < x < \frac{5}{2} \text{ and } g'(x) < 0 \text{ for } \frac{5}{2} < x < 3. \]

Therefore, \( g \) has an absolute maximum at \( x = \frac{5}{2} \).

(c) \( g''(x) = f'(x) \) changes sign only at \( x = 0 \). Thus the graph of \( g \) has a point of inflection at \( x = 0 \).

(d) The average rate of change of \( f \) on the interval \(-4 \leq x \leq 3\) is

\[ \frac{f(3) - f(-4)}{3 - (-4)} = \frac{2}{7}. \]

To apply the Mean Value Theorem, \( f \) must be differentiable at each point in the interval \(-4 < x < 3\). However, \( f \) is not differentiable at \( x = -3 \) and \( x = 0 \).

Question 6

(a) \( 1 + (x - 1)^2 + \frac{(x - 1)^4}{2} + \frac{(x - 1)^6}{6} + \ldots + \frac{(x - 1)^{2n}}{n!} + \ldots \)

(b) \( 1 + \frac{(x - 1)^2}{2} + \frac{(x - 1)^4}{6} + \frac{(x - 1)^6}{24} + \ldots + \frac{(x - 1)^{2n}}{(n + 1)!} + \ldots \)

(c) \( \lim_{n \to \infty} \frac{(x - 1)^{2n+2}}{(n + 2)!} = \lim_{n \to \infty} \frac{(n + 1)!}{(x - 1)^{2n}} = \lim_{n \to \infty} \frac{(x - 1)^2}{n + 2} = 0 \)

Therefore, the interval of convergence is \((-\infty, \infty)\).

(d) \( f''(x) = 1 + 4 \cdot \frac{3}{6} (x - 1)^2 + 6 \cdot \frac{5}{24} (x - 1)^4 + \ldots \)

\[ + \frac{2n(2n-1)}{(n + 1)!} (x - 1)^{2n-2} + \ldots \]

Since every term of this series is nonnegative, \( f''(x) \geq 0 \) for all \( x \).

Therefore, the graph of \( f \) has no points of inflection.