

CALCULUS BC  
WORKSHEET ON PARAMETRICS AND CALCULUS

On problems 1 – 5, find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .

1.  $x = t^2$ ,  $y = t^2 + 6t + 5$

2.  $x = t^2 + 1$ ,  $y = 2t^3 - t^2$

3.  $x = \sqrt{t}$ ,  $y = 3t^2 + 2t$

4.  $x = \ln t$ ,  $y = t^2 + t$

5.  $x = 3\sin t + 2$ ,  $y = 4\cos t - 1$

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6. A curve  $C$  is defined by the parametric equations  $x = t^2 + t - 1$ ,  $y = t^3 - t^2$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $t$ .

(b) Find an equation of the tangent line to  $C$  at the point where  $t = 2$ .

7. A curve  $C$  is defined by the parametric equations  $x = 2\cos t$ ,  $y = 3\sin t$ .

(a) Find  $\frac{dy}{dx}$  in terms of  $t$ .

(b) Find an equation of the tangent line to  $C$  at the point where  $t = \frac{\pi}{4}$ .

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On problems 8 – 10, find:

(a)  $\frac{dy}{dx}$  in terms of  $t$ .

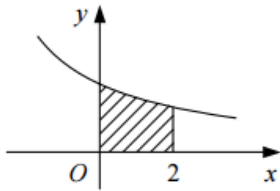
(b) all points of horizontal and vertical tangency

8.  $x = t + 5$ ,  $y = t^2 - 4t$

9.  $x = t^2 - t + 1$ ,  $y = t^3 - 3t$

10.  $x = 3 + 2\cos t$ ,  $y = -1 + 4\sin t$

11.



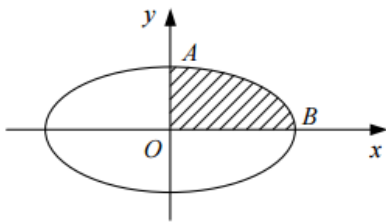
The diagram shows part of the curve with parametric equations

$$x = 2t - 4, \quad y = \frac{1}{t}.$$

The shaded region is bounded by the curve, the coordinate axes and the line  $x = 2$ .

- a Find the value of the parameter  $t$  when  $x = 0$  and when  $x = 2$ .
- b Show that the area of the shaded region is given by  $\int_2^3 \frac{2}{t} dt$ .
- c Hence, find the area of the shaded region.
- d Verify your answer to part c by first finding a cartesian equation for the curve.

12.



The diagram shows the ellipse with parametric equations

$$x = 4 \cos \theta, \quad y = 2 \sin \theta, \quad 0 \leq \theta < 2\pi,$$

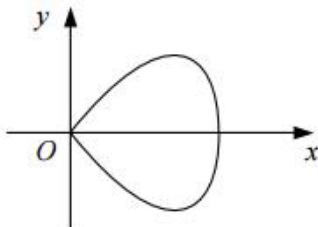
which meets the positive coordinate axes at the points  $A$  and  $B$ .

- a Find the value of the parameter  $\theta$  at the points  $A$  and  $B$ .
- b Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by

$$\int_0^{\frac{\pi}{2}} 8 \sin^2 \theta \, d\theta$$

- c Hence, show that the area of the region enclosed by the ellipse is  $8\pi$ .

13.



The diagram shows the curve with parametric equations

$$x = 2 \sin t, \quad y = 5 \sin 2t, \quad 0 \leq t < \pi.$$

- a Show that the area of the region enclosed by the curve is given by  $\int_0^{\frac{\pi}{2}} 20 \sin 2t \cos t \, dt$ .
- b Evaluate this integral.