AB/BC Calculus Exam – Review Sheet – Solutions

A. Precalculus Type problems

	When you see the words …	This is what you think of doing
A1	Find the zeros of $f(x)$.	Set function equal to 0. Factor or use quadratic equation if quadratic. Graph to find zeros on calculator.
A2	Find the intersection of $f(x)$ and $g(x)$.	Set the two functions equal to each other. Find intersection on calculator.
A3	Show that $f(x)$ is even.	Show that $f(-x)=f(x)$. This shows that the graph of f is symmetric to the y-axis.
A4	Show that $f(x)$ is odd.	Show that $f(-x)=-f(x)$. This shows that the graph of f is symmetric to the origin.
A5	Find domain of $f(x)$.	Assume domain is $(-\infty,\infty)$. Restrict domains: denominators $\neq$ 0, square roots of only non-negative numbers, logarithm or natural log of only positive numbers.
A6	Find vertical asymptotes of $f(x)$.	Express $f(x)$ as a fraction, express numerator and denominator in factored form, and do any cancellations. Set denominator equal to 0.
A7	If continuous function $f(x)$ has $f(a)<k$ and $f(b)>k$, explain why there must be a value c such that $a<c<b$ and $f(c)=k$.	This is the Intermediate Value Theorem.

B. Limit Problems

	When you see the words …	This is what you think of doing
B1	Find $\lim\limits_{x\to a} f(x)$.	Step 1: Find $f(a)$. If you get a zero in the denominator, Step 2: Factor numerator and denominator of $f(x)$. Do any cancellations and go back to Step 1. If you still get a zero in the denominator, the answer is either ∞, $-\infty$, or does not exist. Check the signs of $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$ for equality.
B2	Find $\lim\limits_{x\to a} f(x)$ where $f(x)$ is a piecewise function.	Determine if $\lim\limits_{x\to a^-} f(x)=\lim\limits_{x\to a^+} f(x)$ by plugging in a to $f(x), x<a$ and $f(x), x>a$ for equality. If they are not equal, the limit doesn't exist.
B3	Show that $f(x)$ is continuous.	Show that 1) $\lim\limits_{x\to a} f(x)$ exists 2) $f(a)$ exists 3) $\lim\limits_{x\to a} f(x)=f(a)$
B4	Find $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$.	Express $f(x)$ as a fraction. Determine location of the highest power: Denominator: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to-\infty} f(x)=0$ Both Num and Denom: ratio of the highest power coefficients Numerator: $\lim\limits_{x\to\infty} f(x)=\pm\infty$ (plug in large number)
B5	Find horizontal asymptotes of $f(x)$.	$\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$

	When you see the words ...	This is what you think of doing
B6	Find $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}$ if $\lim\limits_{x\to 0} f(x)=0$ and $\lim\limits_{x\to 0} g(x)=0$	Use L'Hospital's Rule: $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}=\lim\limits_{x\to 0}\frac{f'(x)}{g'(x)}$
B7 BC	Find $\lim\limits_{x\to 0} f(x)\cdot g(x)=0(\pm\infty)$	Express $g(x)=\frac{1}{\frac{1}{g(x)}}$ and apply L'Hospital's rule.
B8 BC	Find $\lim\limits_{x\to 0} f(x)-g(x)=\infty-\infty$	Express $f(x)-g(x)$ with a common denominator and use L'Hospital's rule.
B9 BC	Find $\lim\limits_{x\to 0} f(x)^{g(x)}=1^{\infty}$ or 0^0 or ∞^0	Take the natural log of the expression and apply L'Hospital's rule, remembering to take the resulting answer and raise *e* to that power.

C. Derivatives, differentiability, and tangent lines

	When you see the words ...	This is what you think of doing
C1	Find the derivative of a function using the derivative definition.	Find $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ or $\lim\limits_{x\to a}\frac{f(x)-f(a)}{x-a}$
C2	Find the average rate of change of *f* on [*a*, *b*].	Find $\frac{f(b)-f(a)}{b-a}$
C3	Find the instantaneous rate of change of *f* at $x=a$.	Find $f'(a)$
C4	Given a chart of *x* and $f(x)$ and selected values of *x* between *a* and *b*, approximate $f'(c)$ where *c* is a value between *a* and *b*.	Straddle *c*, using a value of $k\geq c$ and a value of $h\leq c$. $f'(c)\approx\frac{f(k)-f(h)}{k-h}$
C5	Find the equation of the tangent line to *f* at (x_1,y_1).	Find slope $m=f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$
C6	Find the equation of the normal line to *f* at (x_1,y_1).	Find slope $m\perp=-1/f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$
C7	Find *x*-values of horizontal tangents to *f*.	Write $f'(x)$ as a fraction. Set numerator of $f'(x)=0$.
C8	Find *x*-values of vertical tangents to *f*.	Write $f'(x)$ as a fraction. Set denominator of $f'(x)=0$.
C9	Approximate the value of $f(x_1+a)$ if you know the function goes through point (x_1,y_1).	Find slope $m=f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$. Evaluate this line for *y* at $x=x_1+a$. Note: The closer *a* is to 0, the better the approximation will be. Also note that using concavity, it can be determine if this value is an over or under-approximation for $f(x_1+a)$.
C10	Find the derivative of $f(g(x))$.	This is the chain rule. You are finding $f'(g(x))\cdot g'(x)$.
C11	The line $y=mx+b$ is tangent to the graph of $f(x)$ at (x_1,y_1).	Two relationships are true: 1) The function *f* and the line share the same slope at x_1: $m=f'(x_1)$ 2) The function *f* and the line share the same *y*-value at x_1.

	When you see the words ...	This is what you think of doing
C12	Find the derivative of the inverse to $f(x)$ at $x = a$.	Follow this procedure: 1) Interchange x and y in $f(x)$. 2) Plug the x-value into this equation and solve for y (you may need a calculator to solve graphically) 3) Using the equation in 1) find $\frac{dy}{dx}$ implicitly. 4) Plug the y-value you found in 2) to $\frac{dy}{dx}$
C13	Given a piecewise function, show it is differentiable at $x = a$ where the function rule splits.	First, be sure that $f(x)$ is continuous at $x = a$. Then take the derivative of each piece and show that $\lim_{x\to a^-} f'(x) = \lim_{x\to a^+} f'(x)$.

D. Applications of Derivatives

	When you see the words ...	This is what you think of doing
D1	Find critical values of $f(x)$.	Find and express $f'(x)$ as a fraction. Set both numerator and denominator equal to zero and solve.
D2	Find the interval(s) where $f(x)$ is increasing/decreasing.	Find critical values of $f'(x)$. Make a sign chart to find sign of $f'(x)$ in the intervals bounded by critical values. Positive means increasing, negative means decreasing.
D3	Find points of relative extrema of $f(x)$.	Make a sign chart of $f'(x)$. At $x = c$ where the derivative switches from negative to positive, there is a relative minimum. When the derivative switches from positive to negative, there is a relative maximum. To actually find the point, evaluate $f(c)$. OR if $f'(c)=0$, then if $f''(c)>0$, there is a relative minimum at $x = c$. If $f''(c)<0$, there is a relative maximum at $x = c$. (2nd Derivative test).
D4	Find inflection points of $f(x)$.	Find and express $f''(x)$ as a fraction. Set both numerator and denominator equal to zero and solve. Make a sign chart of $f''(x)$. Inflection points occur when $f''(x)$ switches from positive to negative or negative to positive.
D5	Find the absolute maximum or minimum of $f(x)$ on $[a, b]$.	Use relative extrema techniques to find relative max/mins. Evaluate f at these values. Then examine $f(a)$ and $f(b)$. The largest of these is the absolute maximum and the smallest of these is the absolute minimum
D6	Find range of $f(x)$ on $(-\infty,\infty)$.	Use relative extrema techniques to find relative max/mins. Evaluate f at these values. Then examine $f(a)$ and $f(b)$. Then examine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
D7	Find range of $f(x)$ on $[a, b]$	Use relative extrema techniques to find relative max/mins. Evaluate f at these values. Then examine $f(a)$ and $f(b)$. Then examine $f(a)$ and $f(b)$.
D8	Show that Rolle's Theorem holds for $f(x)$ on $[a, b]$.	Show that f is continuous and differentiable on $[a, b]$. If $f(a)=f(b)$, then find some c on $[a, b]$ such that $f'(c)=0$.

D9	Show that the Mean Value Theorem holds for $f(x)$ on [*a*, *b*].	Show that *f* is continuous and differentiable on [*a*, *b*]. If $f(a)=f(b)$, then find some *c* on [*a*, *b*] such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$
D10	Given a graph of $f'(x)$, determine intervals where $f(x)$ is increasing/decreasing.	Make a sign chart of $f'(x)$ and determine the intervals where $f'(x)$ is positive and negative.
D11	Determine whether the linear approximation for $f(x_1+a)$ over-estimates or under-estimates $f(x_1+a)$.	Find slope $m=f'(x_i)$. Then use point slope equation: $y-y_1=m(x-x_1)$. Evaluate this line for *y* at $x=x_1+a$. If $f''(x_1)>0$, *f* is concave up at x_1 and the linear approximation is an underestimation for $f(x_1+a)$. $f''(x_1)<0$, *f* is concave down at x_1 and the linear approximation is an overestimation for $f(x_1+a)$.
D12	Find intervals where the slope of $f(x)$ is increasing.	Find the derivative of $f'(x)$ which is $f''(x)$. Find critical values of $f''(x)$ and make a sign chart of $f''(x)$ looking for positive intervals.
D13	Find the minimum slope of $f(x)$ on [*a*, *b*].	Find the derivative of $f'(x)$ which is $f''(x)$. Find critical values of $f''(x)$ and make a sign chart of $f''(x)$. Values of *x* where $f''(x)$ switches from negative to positive are potential locations for the minimum slope. Evaluate $f'(x)$ at those values and also $f'(a)$ and $f'(b)$ and choose the least of these values.

E. Integral Calculus

	When you see the words …	This is what you think of doing
E1	Approximate $\int_a^b f(x)\,dx$ using left Riemann sums with *n* rectangles.	$A=\left(\dfrac{b-a}{n}\right)\left[f(x_0)+f(x_1)+f(x_2)+\ldots+f(x_{n-1})\right]$
E2	Approximate $\int_a^b f(x)\,dx$ using right Riemann sums with *n* rectangles.	$A=\left(\dfrac{b-a}{n}\right)\left[f(x_1)+f(x_2)+f(x_3)+\ldots+f(x_n)\right]$
E3	Approximate $\int_a^b f(x)\,dx$ using midpoint Riemann sums.	Typically done with a table of points. Be sure to use only values that are given. If you are given 7 points, you can only calculate 3 midpoint rectangles.
E4	Approximate $\int_a^b f(x)\,dx$ using trapezoidal summation.	$A=\left(\dfrac{b-a}{2n}\right)\left[f(x_0)+2f(x_1)+2f(x_2)+\ldots+2f(x_{n-1})+f(x_n)\right]$ This formula only works when the base of each trapezoid is the same. If not, calculate the areas of individual trapezoids.

E5	Find $\int_b^a f(x)\,dx$ where $a<b$.	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

E6	Meaning of $\int_a^x f(t)\,dt$.	The accumulation function – accumulated area under function f starting at some constant a and ending at some variable x.
E7	Given $\int_a^b f(x)\,dx$, find $\int_a^b \left[f(x)+k\right]dx$.	$\int_a^b \left[f(x)+k\right]dx = \int_a^b f(x)\,dx + \int_a^b k\,dx$
E8	Given the value of $F(a)$ where the antiderivative of f is F, find $F(b)$.	Use the fact that $\int_a^b f(x)\,dx = F(b)-F(a)$ so $F(b)=F(a)+\int_a^b f(x)\,dx$. Use the calculator to find the definite integral.
E9	Find $\frac{d}{dx}\int_a^x f(t)\,dt$.	$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. The 2nd Fundamental Theorem.
E10	Find $\frac{d}{dx}\int_a^{g(x)} f(t)\,dt$.	$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$. The 2nd Fundamental Theorem.
E11 BC	Find $\int_0^\infty f(x)\,dx$.	$\int_0^\infty f(x)\,dx = \lim_{h\to\infty}\int_0^h f(x)\,dx = \lim_{h\to\infty} F(h)-F(0)$.
E12 BC	Find $\int f(x)\cdot g(x)\,dx$	If u-substitution doesn't work, try integration by parts: $\int u\cdot dv = uv - \int v\cdot du$

F. Applications of Integral Calculus

	When you see the words ...	This is what you think of doing
F1	Find the area under the curve $f(x)$ on the interval $[a, b]$.	$\int_a^b f(x)\,dx$
F2	Find the area between $f(x)$ and $g(x)$.	Find the intersections, a and b of $f(x)$ and $g(x)$. If $f(x)\ge g(x)$ on [a,b], then area $A=\int_a^b \left[f(x)-g(x)\right]dx$.
F3	Find the line $x=c$ that divides the area under $f(x)$ on $[a, b]$ into two equal areas.	$\int_a^c f(x)\,dx = \int_c^b f(x)\,dx$ or $\int_a^b f(x)\,dx = 2\int_a^c f(x)\,dx$

F4	Find the volume when the area under $f(x)$ is rotated about the x-axis on the interval $[a, b]$.	Disks: Radius = $f(x)$: $V=\pi\int_a^b [f(x)]^2 \, dx$
F5	Find the volume when the area between $f(x)$ and $g(x)$ is rotated about the x-axis.	Washers: Outside radius = $f(x)$. Inside radius = $g(x)$. Establish the interval where $f(x)\geq g(x)$ and the values of a and b where $f(x)=g(x)$. $V=\pi\int_a^b \left([f(x)]^2-[g(x)]^2\right) dx$

F6	Given a base bounded by $f(x)$ and $g(x)$ on $[a, b]$ the cross sections of the solid perpendicular to the x-axis are squares. Find the volume.	Base = $f(x)-g(x)$. Area = $\text{base}^2=[f(x)-g(x)]^2$. Volume = $\int_a^b [f(x)-g(x)]^2 dx$
F7	Solve the differential equation $\frac{dy}{dx}=f(x)g(y)$.	Separate the variables: x on one side, y on the other with the dx and dy in the numerators. Then integrate both sides, remembering the $+C$, usually on the x-side.
F8	Find the average value of $f(x)$ on $[a, b]$.	$F_{avg}=\frac{\int_a^b f(x)\,dx}{b-a}$
F9	Find the average rate of change of $F'(x)$ on $[t_1,t_2]$.	$\frac{\frac{d}{dt}\int_{t_1}^{t_2} F'(x)\,dx}{t_2-t_1}=\frac{F'(t_2)-F'(t_1)}{t_2-t_1}$
F10	y is increasing proportionally to y.	$\frac{dy}{dt}=ky$ which translates to $y=Ce^{kt}$
F11	Given $\frac{dy}{dx}$, draw a slope field.	Use the given points and plug them into $\frac{dy}{dx}$, drawing little lines with the calculated slopes at the point.
F12 BC	Find $\int \frac{dx}{ax^2+bx+c}$	Factor ax^2+bx+c into non-repeating factors to get $\int \frac{dx}{(mx+n)(px+q)}$ and use Heaviside method to create partial fractions and integrate each fraction.
F13 BC	Use Euler's method to approximate $f(1.2)$ given a formula for $\frac{dy}{dx}$, (x_0,y_0) and $\Delta x=0.1$	$dy=\frac{dy}{dx}(\Delta x)$, $y_{new}=y_{old}+dy$
F14 BC	Is the Euler's approximation an over- or under-approximation?	Look at sign of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the interval. This gives increasing/decreasing and concavity information. Draw a picture to ascertain the answer.
F15 BC	A population P is increasing logistically.	$\frac{dP}{dt}=kP(C-P)$.
F16 BC	Find the carrying capacity of a population growing logistically.	$\frac{dP}{dt}=kP(C-P)=0\Rightarrow C=P$.

F17 BC	Find the value of P when a population growing logistically is growing the fastest.	$\frac{dP}{dt} = kP(C-P) \Rightarrow$ Set $\frac{d^2P}{dt^2} = 0$
F18 BC	Given continuous $f(x)$, find the arc length on $[a, b]$	$L = \int_a^b \sqrt{1+[f'(x)]^2}\,dx$

G. Particle Motion and Rates of Change

	When you see the words …	This is what you think of doing
G1	Given the position function $s(t)$ of a particle moving along a straight line, find the velocity and acceleration.	$v(t) = s'(t) \quad a(t) = v'(t) = s''(t)$
G2	Given the velocity function $v(t)$ and $s(0)$, find $s(t)$.	$s(t) = \int v(t)\; dt + C$. Plug in $s(0)$ to find C.
G3	Given the acceleration function $a(t)$ of a particle at rest and $s(0)$, find $s(t)$.	$v(t) = \int a(t)\; dt + C_1$. Plug in $v(0) = 0$ to find C_1. $s(t) = \int v(t)\; dt + C_2$. Plug in $s(0)$ to find C_2.
G4	Given the velocity function $v(t)$, determine if a particle is speeding up or slowing down at $t = k$.	Find $v(k)$ and $a(k)$. If both have the same sign, the particle is speeding up. If they have different signs, the particle is slowing down.
G5	Given the position function $s(t)$, find the average velocity on $[t_1, t_2]$.	Avg. vel. $= \frac{s(t_2) - s(t_1)}{t_2 - t_1}$
G6	Given the position function $s(t)$, find the instantaneous velocity at $t = k$.	Inst. vel. $= s'(k)$.
G7	Given the velocity function $v(t)$ on $[t_1, t_2]$, find the minimum acceleration of a particle.	Find $a(t)$ and set $a'(t) = 0$. Set up a sign chart and find critical values. Evaluate the acceleration at critical values and also t_1 and t_2 to find the minimum.
G8	Given the velocity function $v(t)$, find the average velocity on $[t_1, t_2]$.	Avg. vel. $= \frac{\int_{t_1}^{t_2} v(t)\; dt}{t_2 - t_1}$
G9	Given the velocity function $v(t)$, determine the difference of position of a particle on $[t_1, t_2]$.	Displacement $= \int_{t_1}^{t_2} v(t)\; dt$
G10	Given the velocity function $v(t)$, determine the distance a particle travels on $[t_1, t_2]$.	Distance $= \int_{t_1}^{t_2} \lvert v(t) \rvert\; dt$

G11	Calculate $\int_{t_1}^{t_2} \lvert v(t) \rvert \, dt$ without a calculator.	Set $v(t)=0$ and make a sign charge of $v(t)=0$ on $[t_1,t_2]$. On intervals $[a, b]$ where $v(t)>0$, $\int_a^b \lvert v(t) \rvert \, dt = \int_a^b v(t) \, dt$ On intervals $[a, b]$ where $v(t)<0$, $\int_a^b \lvert v(t) \rvert \, dt = \int_b^a v(t) \, dt$
G12	Given the velocity function $v(t)$ and $s(0)$, find the greatest distance of the particle from the starting position on $[0,t_1]$.	Generate a sign chart of $v(t)$ to find turning points. $s(t)=\int v(t) \, dt + C$. Plug in $s(0)$ to find C. Evaluate $s(t)$ at all turning points and find which one gives the maximum distance from $s(0)$.

G13	The volume of a solid is changing at the rate of …	$\frac{dV}{dt} = \ldots$
G14	The meaning of $\int_a^b R'(t) \, dt$.	This gives the accumulated change of $R(t)$ on $[a, b]$. $\int_a^b R'(t) \, dt = R(b)-R(a)$ or $R(b)=R(a)+\int_a^b R'(t) \, dt$
G15	Given a water tank with g gallons initially, filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1,t_2]$ a) The amount of water in the tank at $t = m$ minutes. b) the rate the water amount is changing at $t = m$ minutes and c) the time t when the water in the tank is at a minimum or maximum.	a) $g+\int_0^m [F(t)-E(t)] \, dt$ b) $\frac{d}{dt}\int_0^m [F(t)-E(t)] \, dt = F(m)-E(m)$ c) set $F(m)-E(m)=0$, solve for m, and evaluate $g+\int_0^m [F(t)-E(t)] \, dt$ at values of m and also the endpoints.

H. Parametric and Polar Equations - BC

	When you see the words …	**This is what you think of doing**
H1	Given $x=f(t), y=g(t)$, find $\frac{dy}{dx}$	$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$
H2	Given $x=f(t), y=g(t)$, find $\frac{d^2y}{dx^2}$	$x=f(t), y=g(t)$, find $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$
H3	Given $x=f(t), y=g(t)$, find arc length on $[t_1,t_2]$	$L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} \, dt$
H4	Express a polar equation in the form of $r=f(\theta)$ in parametric form.	$x=r\cos\theta=f(\theta)\cos\theta \quad y=r\sin\theta=f(\theta)\sin\theta$

H5	Find the slope of the tangent line to $r = f(\theta)$	$x = r\cos\theta \quad y = r\sin\theta \Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$
H5	Find horizontal tangents to a polar curve $r = f(\theta)$	$x = r\cos\theta \quad y = r\sin\theta$ Find where $r\sin\theta = 0$ when $r\cos\theta \neq 0$
H6	Find vertical tangents to a polar curve $r = f(\theta)$	$x = r\cos\theta \quad y = r\sin\theta$ Find where $r\cos\theta = 0$ when $r\sin\theta \neq 0$
H7	Find the area bounded by the polar curve $r = f(\theta)$ on $[\theta_1, \theta_2]$	$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta = \frac{1}{2}\int_{\theta_1}^{\theta_2} [f(\theta)]^2\,d\theta$
H8	Find the arc length of the polar curve $r = f(\theta)$ on $[\theta_1, \theta_2]$	$s = \int_{\theta_1}^{\theta_2} \sqrt{[f(\theta)]^2 + [f'(\theta)]^2}\,d\theta = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$

I. Vectors and Vector-valued functions - BC

	When you see the words ...	This is what you think of doing
I1	Find the magnitude of vector $v\langle v_1, v_2 \rangle$	$\lVert v \rVert = \sqrt{v_1^2 + v_2^2}$
I2	Find the dot product: $\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle$	$\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle = u_1 v_1 + u_2 v_2$
I3	The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find a) the velocity vector and b) the acceleration vector.	a) $v(t) = \langle x'(t), y'(t) \rangle$ b) $a(t) = \langle x''(t), y''(t) \rangle$
I4	The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the speed of the particle at time *t*.	Speed $= \lVert v(t) \rVert = \sqrt{[x'(t)]^2 + [y'(t)]^2}$ - a scalar
I5	Given the velocity vector $v(t) = \langle x(t), y(t) \rangle$ and position at time $t = 0$, find the position vector.	$s(t) = \int x(t)\,dt + \int y(t)\,dt + C$ Use $s(0)$ to find *C*, remembering that it is a vector.
I6	Given the velocity vector $v(t) = \langle x(t), y(t) \rangle$, when does the particle stop?	$v(t) = 0 \Rightarrow x(t) = 0$ AND $y(t) = 0$
I7	The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$. Find the distance the particle travels from t_1 to t_2.	Distance $= \int_{t_1}^{t_2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$

J. Taylor Polynomial Approximations - BC

	When you see the words …	This is what you think of doing
J1	Find the nth degree Maclaurin polynomial to $f(x)$	$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$
J2	Find the nth degree Taylor polynomial to $f(x)$ centered at $x = c$	$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!} + \ldots + \frac{f^{(n)}(c)(x-c)^n}{n!}$
J3	Use the first-degree Taylor polynomial to $f(x)$ centered at $x = c$ to approximate $f(k)$ and determine whether the approximation is greater than or less than $f(k)$	Write the first-degree TP and find $f(k)$. Use the signs of $f'(c)$ and $f''(c)$ to determine increasing/decreasing and concavity and draw your line (1^{st} degree TP) to determine whether the line is under the curve (under-approximation) or over the curve (over-approximation).

J4	Given an nth degree Taylor polynomial for f about $x = c$, find $f(c), f'(c), f''(c), \ldots, f^{(n)}(c)$	$f(c)$ will be the constant term in your Taylor polynomial (TP) $f'(c)$ will be the coefficient of the x term in the TP. $\frac{f''(c)}{2!}$ will be the coefficient of the x^2 term in the TP. $\frac{f^{(n)}(c)}{n!}$ will be the coefficient of the x^n term in the TP.
J5	Given a Taylor polynomial centered at c, determine if there is enough information to determine if there is a relative maximum or minimum at $x = c$.	If there is no first-degree x-term in the TP, then the value of c about which the function is centered is a critical value. Thus the coefficient of the x^2 term is the second derivative divided by 2! Using the second derivative test, we can tell whether there is a relative maximum, minimum, or neither at $x = c$.
J6	Given an nth degree Taylor polynomial for f about $x = c$, find the Lagrange error bound (remainder).	$R_n(x) = \left\lvert \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1} \right\rvert$. The value of z is some number between x and c. $f^{(n+1)}(z)$ represents the $(n+1)^{st}$ derivative of z. This usually is given to you.
J7	Given an nth degree Maclaurin polynomial P for f, find the $\lvert f(k) - P(k) \rvert$	This is looking for the Lagrange error – the difference between the value of the function at $x = k$ and the value of the TP at $x = k$.

K. Infinite Series - BC

	When you see the words …	This is what you think of doing
K1	Given a_n, determine whether the sequence a_n converges.	a_n converges if $\lim_{n\to\infty} a_n$ exists.
K2	Given a_n, determine whether the series a_n could converge.	If $\lim_{n\to\infty} a_n = 0$, the series could converge. If $\lim_{n\to\infty} a_n \neq 0$, the series cannot converge. (*n*th term test).
K3	Tests to determine where a series converges.	Examine the *n*th term of the series. Assuming it passes the *n*th term test, the most widely used series forms and their rule of convergence are: Geometric: $\sum_{n=0}^{\infty} ar^n$ - converges if $\lvert r\rvert < 1$ *p*-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ - converges if $p > 1$ Alternating: $\sum_{n=1}^{\infty} (-1)^n a_n$ - converges if $0 < a_{n+1} < a_n$ Limit Comparison: If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ where L is finite and positive, a_n and b_n either both converge or both diverge. Ratio: $\sum_{n=0}^{\infty} a_n$ - converges if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$
K4	Determine whether a series converges absolutely or conditionally.	Absolute: If $\sum a_n$ converges and $\sum \lvert a_n\rvert$ converges Conditional: If $\sum a_n$ converges and $\sum \lvert a_n\rvert$ diverges
K5	Find the sum of a geometric series.	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$
K6	Find the interval of convergence of a series.	If not given, you will have to generate the *n*th term formula. Use a test (usually the ratio test) to find the interval of convergence and then check out the endpoints.

K7	$1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$	The harmonic series – divergent.
K8	$f(x)=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots$	$f(x)=e^x$
K9	$f(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$	$f(x)=\sin x$
K10	$f(x)=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$	$f(x)=\cos x$
K11	$f(x)=1+x+x^2+x^3+\ldots+x^n+\ldots$	$f(x)=\frac{1}{1-x}$ Convergent: $(-1, 1)$

K12	Given a formula for the *n*th derivative of $f(x)$. Write the first four terms and the general term for the power series for $f(x)$ centered at $x = c$.	$f(x)=f(c)+f'(c)(x-c)+\frac{f''(c)(x-c)^2}{2!}+\frac{f'''(c)(x-c)^3}{3!}+...+\frac{f^{(n)}(c)(x-c)^n}{n!}+...$
K13	Let S_4 be the sum of the first 4 terms of an alternating series for $f(x)$. Approximate $\lvert f(x)-S_4\rvert$.	This is the error for the 4th term of an alternating series which is simply the 5th term. It will be positive since you are looking for an absolute value.
K14	Write a series for expressions like e^{x^2}.	Rather than go through generating a Taylor polynomial, use the fact that if $f(x)=e^x$, then $f(x^2)=e^{x^2}$. So $f(x)=e^x=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\frac{x^4}{4!}+...+\frac{x^n}{n!}+...$ and $f(x^2)=e^{x^2}=1+x^2+\frac{x^4}{2}+\frac{x^6}{3!}+\frac{x^8}{4!}+...+\frac{x^{2n}}{n!}+...$