Calculus
Test #4 Review

Name ______________________________

Approximate the given integrals using the specified technique.

1. Left Endpoint Riemann Sum

a. $\int_{-2}^{1} (x^2 + 2)\, dx$ with 6 subintervals.

b. $\int_{0}^{6} \sqrt{x+1}\, dx$ with 3 subintervals.

2. Right Endpoint Riemann Sum

a. $\int_{0}^{3} (x^3 - 3)\, dx$ with 3 subintervals.

b. $\int_{3}^{5} (-x^2 + 10x - 20)\, dx$ with 4 subintervals.

3. Midpiont Riemann Sum

a. $\int_{0}^{\pi} \sin(2x)\, dx$ with 3 subintervals.

b. $\int_{-2}^{4} (16 - x^2)\, dx$ with 6 subintervals.

4. Trapezoidal Rule

a. $\int_{0}^{\pi} x \sin x\, dx$ with 4 subintervals.

b. $\int_{-2}^{6} e^x - 2\, dx$ with 4 subintervals.

Evaluate each of the following integrals.

5. $\int (x^3 - 2)^2\, dx$

6. $\int \frac{x^2 + 2}{x^2}\, dx$

7. $\int x\sqrt{x^2 - 2}\, dx$

8. $\int x^2 \sec(x^3) \tan(x^3)\, dx$

9. $\int \frac{x^2 + 1}{x^3 + 3x + 7}\, dx$

10. $\int \frac{2x^2 + 7x - 3}{x - 2}\, dx$

11. $\int \frac{2}{\sqrt{1 - x^2}}\, dx$

12. $\int \frac{1 + \cos^2\theta}{\cos^2\theta}\, d\theta$

13. $\int e^{3x+2}\, dx$

14. $\int \frac{e^{2/x}}{x^2}\, dx$

Evaluate each definite integral using the First Fundamental Theorem.

15. $\int_{0}^{3} (3x^2 + x - 2)\, dx$

16. $\int_{4}^{9} \frac{1 - \sqrt{u}}{\sqrt{u}}\, du$

17. $\int_{-2}^{-1} \frac{x^2 - 1}{x^2}\, dx$

18. $\int_{-1}^{1} \frac{1}{\sqrt{1 - x^2}}\, dx$

19. $\int_1^2 (x-1)\sqrt{2-x}\,dx$

20. $\int_0^{\pi/2} \sin(\frac{2x}{3})\,dx$

21. $\int_0^3 |x^2-4|\,dx$

22. $\int_0^{\sqrt{2}} xe^{-x^2/2}\,dx$

Evaluate the derivative of each of the following using the Second Fundamental Theorem.

23. $\frac{d}{dx}\left[\int_1^x \frac{1}{t}\,dt\right]$

24. $\frac{d}{dx}\left[\int_\pi^{x^2} \cos t\,dt\right]$

25. $\frac{d}{dx}\left[\int_x^{x^2} (-2t-2)\,dt\right]$

26. $\frac{d}{dx}\left[\int_x^{x^2} 2\sqrt{t+3}\,dt\right]$

Calculate the average value of the functions on the given intervals.

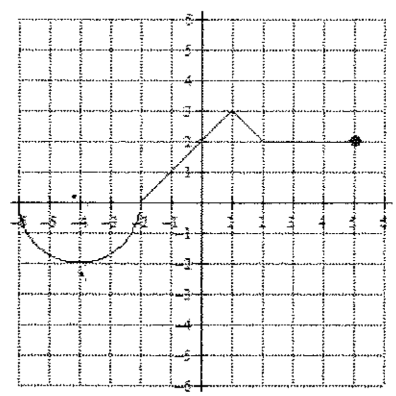
27. $f(x)=\sqrt{x+3}$ [-2, 13]

28. $f(x)=\sin 5x$ $[0,\pi/2]$

29. $f(x)=\frac{2}{4+x^2}$ [-1, $\sqrt{3}$]

30. $f(x)=\frac{x}{x+1}$ [1, e]

Pictured to the right is the graph of *f(t)*. If $F(x)=\int_{-6}^{2x} f(t)dt$



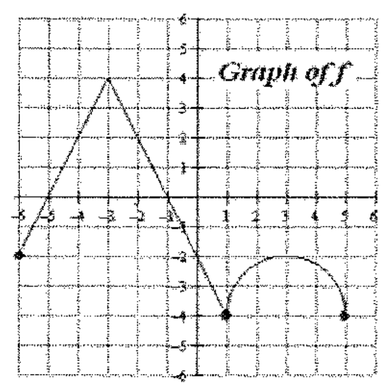
31. Find the value of *F(0)*.

32. Find the value of *F(-1/2)*.

33. Find the value of *F'(-2)*.

34. Find the value of *F'(2.5)*.

Pictured the right is the graph of *f*. If $G(x)=\int_{-2}^{x} f(t)dt$



35. Find the value of *G(3)*.

36. Find the value of *G(-4)*.

37. Find the value of *G'(-2)*.

Test #4 Review Answers

① a. $\Delta x = \frac{1 - -2}{6} = \frac{1}{2}$

$$\int_{-2}^{1} (x^2+2)\,dx \approx \frac{1}{2}\left[f(-2)+f(\tfrac{-3}{2})+f(-1)+f(-\tfrac{1}{2})+f(0)+f(\tfrac{1}{2})\right]$$

$$\approx \frac{1}{2}\left[6+\frac{17}{4}+3+\frac{9}{4}+2+\frac{9}{4}\right] = \frac{79}{8} \approx 9.875$$

b. $\Delta x = \frac{6-0}{3} = 2$

$$\int_0^6 \sqrt{x+1}\,dx \approx 2\left[f(0)+f(2)+f(4)\right]$$

$$\approx 2\left[1+\sqrt{3}+\sqrt{5}\right] \approx 9.936$$

② a. $\Delta x = \frac{3-0}{3} = 1$

$$\int_0^3 (x^3-3)\,dx \approx 1\left[f(1)+f(2)+f(3)\right]$$

$$\approx \left[-2+5+24\right] \approx 27$$

b. $\Delta x = \frac{5-3}{4} = \frac{1}{2}$

$$\int_3^5 (-x^2+10x-20)\,dx \approx \frac{1}{2}\left[f(\tfrac{7}{2})+f(4)+f(\tfrac{9}{2})+f(5)\right]$$

$$\approx \frac{1}{2}\left[\frac{11}{4}+4+\frac{19}{4}+5\right] \approx \frac{33}{4} \approx 8.25$$

③ a. $\Delta x = \frac{\pi-0}{3} = \frac{\pi}{3}$

$$\int_0^{\pi} \sin(2x)\,dx \approx \frac{\pi}{3}\left[f(\tfrac{\pi}{6})+f(\tfrac{\pi}{2})+f(\tfrac{5\pi}{6})\right]$$

$$\approx \frac{\pi}{3}\left[\frac{\sqrt{3}}{2}+0+\frac{-\sqrt{3}}{2}\right] = 0$$

b. $\Delta x = \frac{4--2}{6} = 1$

$$\int_{-2}^{4} (16-x^2)\,dx \approx 1\left[f(\tfrac{-3}{2})+f(\tfrac{-1}{2})+f(\tfrac{1}{2})+f(\tfrac{3}{2})+f(\tfrac{5}{2})+f(\tfrac{7}{2})\right]$$

$$\approx \left[\frac{55}{4}+\frac{63}{4}+\frac{63}{4}+\frac{55}{4}+\frac{39}{4}+\frac{15}{4}\right] \approx \frac{145}{2} \approx 72.5$$

④ a. $\Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$

$$\int_0^{\pi} x \sin x \, dx \approx \left(\frac{\pi}{4}\right)\left(\frac{1}{2}\right)\left[f(0) + 2f\left(\frac{\pi}{4}\right) + 2f\left(\frac{\pi}{2}\right) + 2f\left(\frac{3\pi}{4}\right) + f(\pi)\right]$$

$$\approx \frac{\pi}{8}\left[0 + \frac{\pi\sqrt{2}}{4} + \pi + \frac{3\pi\sqrt{2}}{4} + 0\right] \approx \frac{\pi^2\sqrt{2} + \pi^2}{8} \approx 2.978$$

b. $\Delta x = \frac{6 - -2}{4} = 2$

$$\int_{-2}^{6} (e^x - 2)\, dx \approx 2\left(\frac{1}{2}\right)\left[f(-2) + 2f(0) + 2f(2) + 2f(4) + f(6)\right]$$

$$\approx \left[e^{-2} - 2 + -2 + 2e^2 - 4 + 2e^4 - 4 + e^6 - 2\right] \approx 513.539$$

⑤ $\int (x^3 - 2)^2 dx = \int (x^6 - 4x^3 + 4)\, dx = \frac{1}{7}x^7 - x^4 + 4x + C$

⑥ $\int \frac{x^2 + 2}{x^2} dx = \int (1 + 2x^{-2})\, dx = x - 2x^{-1} + C = x - \frac{2}{x} + C$

⑦ $\int x\sqrt{x^2 - 2}\, dx = \frac{1}{2}\int \sqrt{u}\, du = \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right] = \frac{1}{3}(x^2 - 2)^{3/2} + C$

$u = x^2 - 2$

$du = 2x\, dx$

$\frac{1}{2} du = x\,dx$

⑧ $\int x^2 \sec(x^3) \tan(x^3)\, dx = \frac{1}{3}\int \sec(u)\tan(u)\, du = \frac{1}{3}\left[\sec(u)\right]$

$= \frac{1}{3}\sec(x^3) + C$

$u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

⑨ $\int \frac{x^2 + 1}{x^3 + 3x + 7} dx = \frac{1}{3}\int \frac{1}{u} du = \frac{1}{3}\ln|u|$

$= \frac{1}{3}\ln|x^3 + 3x + 7| + C$

$u = x^3 + 3x + 7$

$du = (3x^2 + 3)\, dx$

$du = 3(x^2 + 1)\, dx$

$\frac{1}{3} du = (x^2 + 1)\, dx$

(10) $\int \frac{2x^2+7x-3}{x-2}\,dx$

$$\begin{array}{r} 2x+11 \\ x-2 \;\overline{)\;2x^2+7x-3} \\ -(2x^2-4x) \\ \hline 11x-3 \\ -(11x-22) \\ \hline 19 \end{array}$$

$\int \left(2x+11+\frac{19}{x-2}\right)dx$

$= x^2+11x+19\ln|x-2|+C$

(11) $\int \frac{2}{\sqrt{1-x^2}}\,dx = 2\int \frac{1}{\sqrt{1^2-x^2}}\,dx = 2\arcsin x + C$

(12) $\int \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int\left(\frac{1}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}\right)d\theta = \int(\sec^2\theta+1)\,d\theta$

$= \tan\theta+\theta+C$

(13) $\int e^{3x+2}\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u$

$u = 3x+2$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}e^{3x+2}+C$

(14) $\int \frac{e^{2/x}}{x^2}\,dx = \int x^{-2}\cdot e^{2x^{-1}}\,dx = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u$

$u = 2x^{-1}$

$du = -2x^{-2}\,dx$

$-\frac{1}{2}du = x^{-2}dx$

$= -\frac{1}{2}e^{2/x}+C$

(15) $\int_0^3 (3x^2+x-2)\,dx = \left(x^3+\frac{1}{2}x^2-2x\right)\Big|_0^3$

$= \left[3^3+\frac{1}{2}(3)^2-2(3)\right]-\left[0^3+\frac{1}{2}(0)^2-2(0)\right]$

$= \frac{51}{2} = 25.5$

(16) $\int_4^9 \frac{1-\sqrt{u}}{\sqrt{u}}\,du = \int_4^9 (u^{-1/2} - 1)\,du = (2u^{1/2} - u)\Big|_4^9$

$= (2\sqrt{9} - 9) - (2\sqrt{4} - 4)$

$= -3$

(17) $\int_{-2}^{-1} \frac{x^2-1}{x^2}\,dx = \int_{-2}^{-1} (1 - x^{-2})\,dx = \left(x + \frac{1}{x}\right)\Big|_{-2}^{-1}$

$= \left(-1 + \frac{1}{-1}\right) - \left(-2 + \frac{1}{-2}\right)$

$= \frac{1}{2}$

(18) $\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)\Big|_{-1}^{1} = \arcsin(1) - \arcsin(-1) = \frac{\pi}{2} - \frac{-\pi}{2} = \pi$

(19) $\int_1^2 (x-1)\sqrt{2-x}\,dx = -\int_1^0 [(2-u-1)\sqrt{u}]\,du = \int_0^1 [(1-u)\sqrt{u}]\,du$

$u = 2-x \rightarrow 2-u = x$

$du = -dx$

$-du = dx$

$u = 1$

$u = 0$

$= \int_0^1 (u^{1/2} - u^{3/2})\,du$

$= \left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^1$

$= \left(\frac{2}{3} - \frac{2}{5}\right) - (0 - 0) = \frac{4}{15}$

(20) $\int_0^{\pi/2} \sin\left(\frac{2x}{3}\right)dx = \frac{3}{2}\int_{-}^{-} \sin(u)\,du = \frac{3}{2}\left(-\cos(u)\right)\Big|_{-}^{-}$

$u = \frac{2x}{3}$

$du = \frac{2}{3}\,dx$

$\frac{3}{2}\,du = dx$

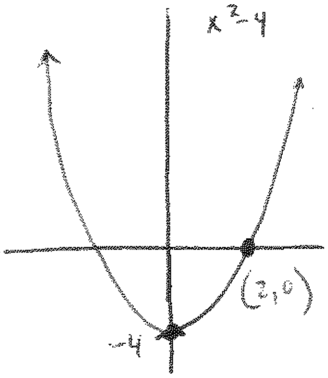
$= -\frac{3}{2}\cos\left(\frac{2x}{3}\right)\Big|_0^{\pi/2}$

$= -\frac{3}{2}\left[\cos\left(\frac{\pi}{3}\right) - \cos(0)\right]$

$= -\frac{3}{2}\left[\frac{1}{2} - 1\right]$

$= \frac{3}{4}$

(21) $\int_0^3 |x^2-4|\,dx = \int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$



$= -\left(\frac{1}{3}x^3-4x\right)\Big|_0^2 + \left(\frac{1}{3}x^3-4x\right)\Big|_2^3$

$= -\left[\left(\frac{8}{3}-8\right)-(0-0)\right] + \left[(9-12)-\left(\frac{8}{3}-8\right)\right]$

$= \frac{16}{3} + \frac{7}{3} = \frac{23}{3}$

(22) $\int_0^{\sqrt{2}} x e^{-x^2/2}\,dx = -\int_0^{-1} e^u\,du = \int_{-1}^0 e^u\,du = e^u\Big|_{-1}^0$

$u = -\frac{1}{2}x^2$ $\quad u=0$

$du = -x\,dx$ $\quad u=-1$

$-du = x\,dx$

$= e^0 - e^{-1}$

$= 1 - \frac{1}{e} \approx 0.632$

(23) $\frac{d}{dx}\left[\int_1^x \frac{1}{t}\,dt\right] = \frac{1}{x}$

(24) $\frac{d}{dx}\left[\int_\pi^{x^2} \cos(t)\,dt\right] = \cos(x^2)\cdot 2x$

(25) $\frac{d}{dx}\left[\int_x^{x^2} (-2t-2)\,dt\right] = (-2(x^2)-2)\cdot 2x - (-2x-2)$

$= -4x^3 - 2x + 2$

(26) $\frac{d}{dx}\left[\int_x^{x^2} 2\sqrt{t+3}\,dt\right] = \left(2\sqrt{x^2+3}\right)\cdot 2x - \left(2\sqrt{x+3}\right)$

$= 4x\sqrt{x^2+3} - 2\sqrt{x+3}$

(27) $\text{Avg} = \frac{1}{13-2}\int_{-2}^{13} \sqrt{x+3}\,dx = \frac{1}{15}\left(\frac{2}{3}(x+3)^{3/2}\right)\Big|_{-2}^{13}$

$= \frac{2}{45}\left(16^{3/2} - 1^{3/2}\right)$

$= \frac{2}{45}(63) = \frac{14}{5}$

(28) $$\text{Avg} = \frac{1}{\pi/2 - 0}\int_0^{\pi/2} \sin(5x)\,dx$$

$u = 5x$ $\quad u = 0$
$du = 5\,dx$ $\quad u = 5\pi/2$
$\frac{1}{5}du = dx$

$$= \frac{2}{\pi}\cdot\frac{1}{5}\int_0^{5\pi/2} \sin(u)\,du$$

$$= \frac{2}{5\pi}\left[-\cos u\right]\Big|_0^{5\pi/2} = -\frac{2}{5\pi}\Big(\cos(5\pi/2) - \cos(0)\Big) = \frac{2}{5\pi}$$

(29) $$\text{Avg} = \frac{1}{\sqrt{3} - -1}\int_{-1}^{\sqrt{3}} \frac{2}{4+x^2}\,dx$$

$$= \frac{2}{\sqrt{3}+1}\int_{-1}^{\sqrt{3}} \frac{1}{(2)^2+(x)^2}\,dx = \frac{2}{\sqrt{3}+1}\left(\frac{1}{2}\arctan\left(\frac{x}{2}\right)\right)\Bigg|_{-1}^{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}+1}\left(\arctan\left(\frac{\sqrt{3}}{2}\right) - \arctan\left(-\frac{1}{2}\right)\right) \approx 0.431$$

(30) $$\text{Avg} = \frac{1}{e-1}\int_1^e \frac{x}{x+1}\,dx = \frac{1}{e-1}\int_{-}^{-} \frac{u-1}{u}\,du = \frac{1}{e-1}\int_{-}^{-}\left(1 - \frac{1}{u}\right)du$$

$u = x+1 \rightarrow u-1 = x$
$du = dx$

$$= \frac{1}{e-1}\Big(u - \ln|u|\Big)\Big|_{-}^{-}$$

$$= \frac{1}{e-1}\Big(x+1 - \ln|x+1|\Big)\Big|_1^e$$

$$= \frac{1}{e-1}\Big[\big(e+1-\ln|e+1|\big) - \big(1+1-\ln|1+1|\big)\Big]$$

$$= 0.639$$

(31) $$F(0) = \int_{-6}^{0} f(t)\,dt = -\tfrac{1}{2}\pi(2)^2 + \tfrac{1}{2}(2)(2) = -2\pi + 2$$

(32) $$F(-1/2) = \int_{-6}^{-1} f(t)\,dt = -\tfrac{1}{2}\pi(2)^2 + \tfrac{1}{2}(1)(1) = -2\pi + \tfrac{1}{2}$$

$$F'(x) = f(2x)\cdot 2$$

(33) $$F'(-2) = f(-4)\cdot 2 = -4$$

(34) $$F'(2.5) = f(5)\cdot 2 = 4$$

(35) $G(3) = \int_{-2}^{3} f(t)\,dt = \frac{1}{2}(1)(2) - \frac{1}{2}(2)(4) - \left[2(4) - \frac{1}{4}\pi(2)^2\right]$

$= 1 - 4 - [8 - \pi]$

$= -11 + \pi$

(36) $G(-4) = \int_{-2}^{-4} f(t)\,dt = -\int_{-4}^{-2} f(t)\,dt = -\left[2 \cdot 2 + \frac{1}{2}(2)(2)\right]$

$= -6$

$G'(x) = f(x)$

(37) $G'(-2) = f(-2) = 2$