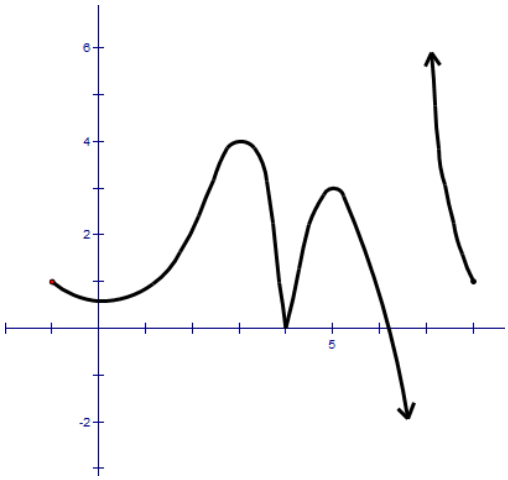
Calculus Name _____

Test #3 – Applications of Derivative Review

1. Given the graph of f(x), find the following:



a. $f'(x) = 0$ at x = _____

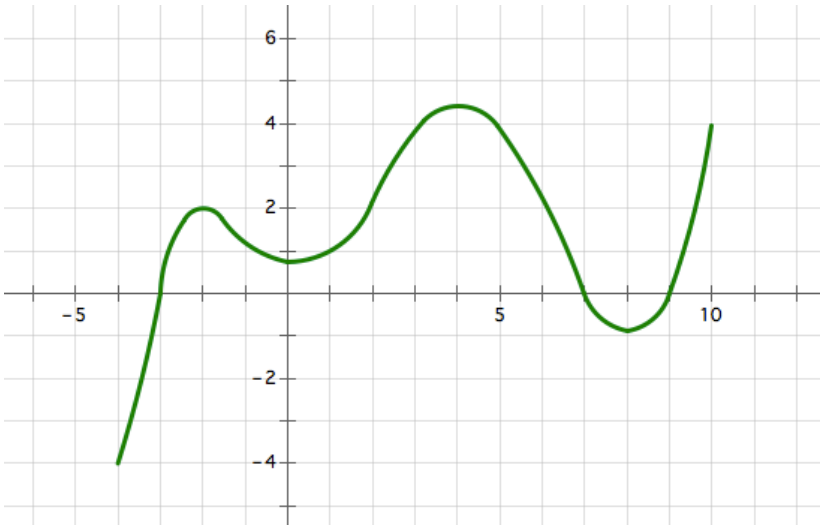
b. Where is $f'(x)$ positive? Negative?

c. Where is $f''(x)$ positive? Negative?

d. $f'(x)$ DNE at x = _____

e. Sketch a graph of $f'(x)$.

2. Given the graph of $f'(x)$, find the following:



a. Where is $f(x)$ increasing? Decreasing?

b. Where are the relative extrema of $f(x)$?

c. Where is $f(x)$concave up? Concave down?

d. Where are the points of inflection?

e. Sketch a possible graph of$f(x)$.

3. Determine if the following function meet the conditions for Rolle's Theorem. If the function does not, explain why. If the function does, find the points that satisfy the theorem.

a. $f(x) = x\sqrt{6-x}$ on $[0, 6]$

b. $y = -\sin 2x$ on $[-\pi, \pi]$

c. $y = 2\sec x$ on $[-\pi, \pi]$

d. $f(x) = \frac{x^2-2x-3}{x+2}$ on $[-1,3]$

e. $f(x) = (1-x^2)^{\frac{1}{3}}$ on $[-3, 3]$

4. Determine if the following function meet the conditions for the Mean Value Theorem. If the function does not, explain why. If the function does, find the values of x that satisfy the theorem.

a. $f(x) = \ln(x-1)$ on $[2, 4]$

b. $y = (3x+15)^{\frac{2}{3}}$ on $[-7, 4]$

c. $y = \frac{x+1}{x}$ on $\left[\frac{1}{2}, 2\right]$

d. $f(x) = \sin x - \cos x$ on $[0,2\pi]$

e. $f(x) = \begin{cases} \sin x, & -\infty < x < 1 \\ \frac{1}{2}x, & 1 \le x \le \infty \end{cases}$ on $[-1, 3]$

5. Find the relative extrema of each function using the first derivative test.

a. $f(x) = (x-2)^2(x+3)$

b. $f(x) = \sec\left(x - \frac{\pi}{2}\right)$ on $[0, 2\pi]$

c. $f(x) = (x-1)^2\sqrt{x+1}$

d. $f(x) = 4xe^{-x^2}$

6. Find the relative extrema of each function using the second derivative test.

a. $f(x) = x^4 - 8x^3 + 2$

b. $f(x) = 2\cos x + \sin 2x$ on $[-\pi, \pi]$

c. $f(x) = xe^x$

d. $f(x) = -4x^2 \ln(\frac{x}{2})$

7. Solve the following related rates problems.

a. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 2 feet per second, how fast is the angle between the top of the ladder and the wall changing when the angle is $\pi/4$ radians?

b. A paper cup has the shape of a cone with height 10 cm and radius 3 cm. If water is poured into the cup at a rate of 2 cubic cm per second, how fast is the water level rising when the water is 5 cm deep?

c. If a snowball melts so that its surface area decreases at a rate of 1 square cm per minute, find the rate at which the radius decreases when the radius is 5 cm.

d. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 2:00 p.m.

8. Solve the following optimization problems.

a. Four pens will be built side-by-side bordering the side of a barn. If there is a total of 150 feet of fencing, what dimensions will maximize the area of the pens?

b. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with a maximum volume?

c. A cylindrical container has a volume of 300 cubic inches. The top and bottom parts of the cylinder cost \$2 per square inch of material and the material wrapping around the sides of the cylinder cost \$6 per square inch. What are the dimensions of the cylinder that minimize the cost of the materials to make the cylinder?

d. A cylinder is inscribed in a sphere with radius 3 inches. What are the dimensions of the cylinder that will maximize the volume of the cylinder? What is the maximum volume?

Solve the following particle motion problems.

9. A particle moves along a path with its postion $s(t) = 2t^3 - 9t^2 + 12t - 4$. Use the position equation to answer the following questions**.**

a. Find an equation representing the velocity and acceleration of the particle. Find the position, velocty and acceleration at t =4.

b.For what values of t is the distance of the particle decreasing? What values of t is the distance increasing? When is the particle at rest?

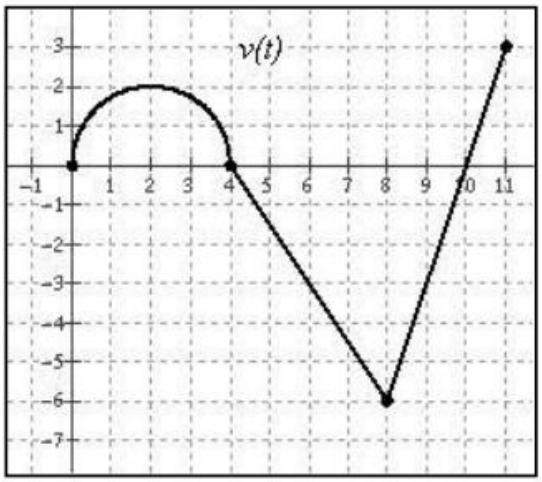
c. On what intervals is the particle speeding up? Slowing down?

d. When is the velocity equal to 12?

e. What is the displacement of the particle on $0 \leq t \leq 8$? What is the total distance traveled on $0 \leq t \leq 8$?

10. The graph to the right shows the velocity of a partcile moving along the x-axis for $0 \leq t \leq 11$. It consists of a semicircle and two line segments. Use the graph to answer the following questions.

a. At what time on $0 \leq t \leq 11$ is the speed of the particle the greatest? Explain.



b. Over what intervals is the particle moving to the left? Moving to the right? When is the particle at rest?

c. Over what intervals is the speed of the particle decreasing? Increasing? Explain.