Calculus
Test #2 Review – Differentiation

Evaluate each limit.

1.
$$\lim_{h \to 0} \frac{\sqrt[3]{5+h} - \sqrt[3]{5}}{h}$$

2.
$$\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{4} + h\right) - \cos \frac{3\pi}{4}}{h}$$
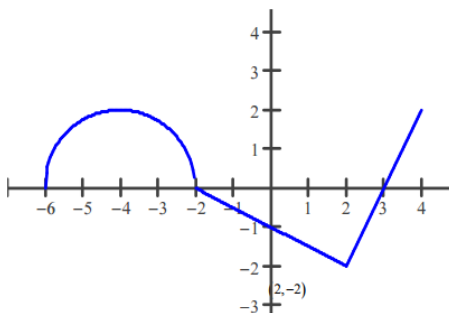
3.
$$\lim_{h \to 0} \frac{\ln(3+h) - \ln 3}{h}$$

For each function, f(x), show work to determine whether the functions is continuous and/or differentiable where the piecewise function meet.

4.
$$f(x) = \begin{cases} 4 - x^2, x < 1 \\ 2x + 2, x \geq 1 \end{cases}$$

5.
$$f(x) = \begin{cases} 3 + (x+2)^{1/3}, x \geq -2 \\ 3 - (x+2)^{2/3}, x < -2 \end{cases}$$
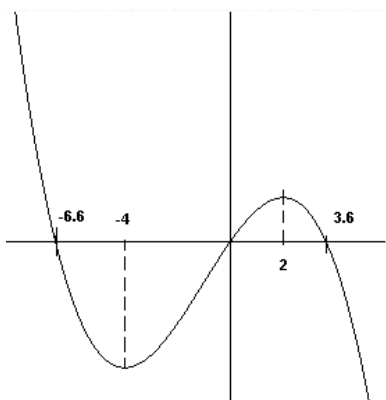
6.
$$f(x) = \begin{cases} \sqrt{x} - 3, x > 1 \\ \frac{1}{2}x - \frac{5}{2}, x \leq 1 \end{cases}$$

Use the graph of the following function to answer each question.

5. What intervals is the derivative positive?

6. What intervals is the derivative negative?

7. What values does the derivative equal zero?

8. Where is the derivative undefined?

Use the graph of the derivative to answer each question.

9. Where is the function increasing?

10. Where is the function decreasing?

11. Where does the function have horizontal tangents?

Find the derivative of the following functions. Answers should be simplified.

12.
$$g(x) = \ln\left[\frac{\sqrt{x^2 + 4}}{(6x - 5)^2}\right]$$

13.
$$f(x) = \sqrt{\frac{2x + 5}{7x - 9}}$$

14.
$$f(x) = \ln(xe^{7x})$$

15. $f(x) = \sec^2 x \cdot \tan x$

16. $f(x) = \ln(5x^2 + 9)^3$

17.
$$y = \frac{x}{\sqrt{x^2 - 1}}$$

18. $h(x) = \dfrac{1 - \cos}{\sin x}$

19. $f(x) = [(x^2 - 1)^5 - x]^3$

20 $f(x) = \arcsin(x^3 + 1)$

21. $g(x) = x^3 \sec^4(2x)$

22. $j(x) = \left[\dfrac{3x+2}{x-9}\right]^5$

23. $y = \dfrac{\sec^3(2x)}{x^2}$

24. $f(x) = (2x+3)e^{x^2}$

25. $y = \dfrac{1}{4}\arctan(\dfrac{x}{4})$

26. $g(x) = \sin\left(\dfrac{2x+1}{x-3}\right)$

Calculate the equation of the tangent line and the normal line at the given value.

19. $p(x) = \dfrac{x}{\sqrt{x^2+1}}$ at x = 0

20. $g(x) = x^2 \cos x$ at x = π

21. $y = x^2 + \ln(4x-7)$ at (2,4)

Use a linear approximation to approximate the given values on the functions.

22. Approximate $f(1.9)$ on the function $f(x) = x^3 - 2x + 3$ at the point where $x = 2$.

23. Approximate $f(1.11)$ on the function $f(x) = x + \dfrac{1}{x}$ at the point where $x = 1$.

24. Approximate $f(3.1)$ on the function $f(x) = \tan x$ at the point where $x = \pi$.

Find the second derivative of the following functions.

25. $f(x) = \dfrac{x^4}{12} + \dfrac{x^3}{6} - 3x^2 - 2x + 4.$

26. $y = x^2 \ln x$

27. $f(x) = e^x \sin x$

Find $\dfrac{dy}{dx}$ for the following relations.

28. $2x^3 = (3xy+1)^2$

29. $\sin 2x^2y^3 = 3x^3 + 1$

30. $3x^2 + 3 = \ln 5xy^2$

Evalute the following limits.

31. $\lim\limits_{x \to 1} \dfrac{1-1/x}{1-1/x^2}$

32. $\lim\limits_{x \to 0}\left(\csc x - \dfrac{1}{x}\right)$

33. $\lim\limits_{x \to \infty}\left(1+\dfrac{1}{x}\right)^{3x}$