AP Calculus

Infinite Sequences and Series

Sequences and Series

1. $\sum_{n=1}^{\infty} \frac{(3)^{n+1}}{5^n} =$

(A) $\frac{3}{5}$ (B) $\frac{5}{2}$ (C) $\frac{9}{2}$ (D) The series diverges

2. If $f(x) = \sum_{n=1}^{\infty} (\tan x)^n$, then $f(1) =$

(A) -2.794 (B) -0.61 (C) 0.177 (D) The series diverges

3. $\sum_{n=2}^{\infty} \frac{2}{n^2 - 1} =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$

4. The sum of the geometric series $\frac{2}{21} + \frac{4}{63} + \frac{8}{189} + \ldots$ is

(A) $\frac{5}{21}$ (B) $\frac{2}{7}$ (C) $\frac{4}{7}$ (D) The series diverges

5. If $S_n = \left(\frac{3^{n-1}}{(4+n)^{20}}\right)\left(\frac{(7+n)^{20}}{3^n}\right)$, to what number does the sequence $\{S_n\}$ converge?

(A) $\frac{1}{3}$ (B) $\frac{7}{4}$ (C) $\left(\frac{7}{4}\right)^{20}$ (D) Diverges

6. Which of the following sequences converge?

I. $\left\{\frac{\cos^2 n}{(1.1)^n}\right\}$ II. $\left\{\frac{e^n - 3}{3^n}\right\}$ III. $\left\{\frac{n}{9+\sqrt{n}}\right\}$

(A) I only (B) II only (C) III only (D) I and II only

7. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{10(n+1)}$ II. $\sum_{n=1}^{\infty} \arctan n$ III. $\sum_{n=1}^{\infty} \frac{-6}{(-5)^n}$

(A) I only (B) II only (C) III only (D) II and III only

Free Response Questions

8. Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{3}{n(n+3)} + \frac{1}{7^n}\right)$.

Integral Test and P-Series

1. If $\int_1^{\infty} \frac{dx}{x^2+1} = \frac{\pi}{4}$, then which of the following must be true?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ diverges.

II. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.

III. $\sum_{n=1}^{\infty} \frac{1}{n^2+1} = \frac{\pi}{4}$

(A) none (B) I only (C) II only (D) II and III only

2. What are all values of p for which $\int_1^{\infty} \frac{1}{\sqrt[3]{x^p}}$ converges?

(A) $P < -3$
(B) $P < -1$
(C) $P > 1$
(D) $P > 3$

3. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{2n^2+1}$ II. $\sum_{n=1}^{\infty} ne^{-n^2}$ III. $\sum_{n=2}^{\infty} \frac{1}{x \ln x}$

(A) I only (B) II only (C) III only (D) I and II only

4. What are all values of p for which $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^p+1}$ converges?

(A) $p > 0$ (B) $p > \frac{1}{2}$ (C) $p > 1$ (D) $p > \frac{3}{2}$

5. What are all values of k for which the series $1+(\sqrt{2})^k+(\sqrt{3})^k+(\sqrt{4})^k+\cdots+(\sqrt{n})^k+\cdots$ converges?

(A) $k<-2$ (B) $k<-1$ (C) $k>1$ (D) $k>2$

Free Response Questions

6. Determine whether the following series converge or diverge.

(a) $1+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\cdots$

(b) $1+\frac{1}{(\sqrt[3]{2})^2}+\frac{1}{(\sqrt[3]{3})^2}+\frac{1}{(\sqrt[3]{4})^2}+\cdots$

Comparison Tests

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2+n+3}$ II. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+2}$ III. $\sum_{n=1}^{\infty} \frac{1+4^n}{3^n}$

(A) I only (B) II only (C) III only (D) I and II only

2. Which of the following series diverge?

I. $\sum_{n=1}^{\infty} \frac{1}{n!}$ II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2}$ III. $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$

(A) I only (B) II only (C) II and III only (D) I, II, and III

3. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n^{3/2}}{3n^3+7}$ II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4+1}}$ III. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(A) I only (B) I and II only (C) I and III only (D) I, II, and III

4. Which of the following series cannot be shown to converge using the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{2^n}$?

(A) $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$

(B) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

(C) $\sum_{n=1}^{\infty} \frac{2n}{2^{n+1}\sqrt{n^2+1}}$

(D) $\sum_{n=1}^{\infty} \frac{2n^2-3n}{2^n(n^2+n-100)}$

Free Response Questions

5. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{\cos(2n)}{1+(1.6)^n}$

(b) $\sum_{n=1}^{\infty} \frac{4^n}{2^n + 3^n}$

Alternating Series and Error Bound

Multiple Choice Questions

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n}$ II. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ III. $\sum_{n=1}^{\infty} \cos(n\pi)$

(A) I only (B) II only (C) III only (D) I and II only

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^n \cos(\frac{\pi}{n})$ II. $\sum_{n=1}^{\infty} \sin(\frac{2n-1}{2})\pi$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2n}{n^2+1}$

(A) I only (B) II only (C) III only (D) I and II only

3. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \frac{n^2\sqrt{n}}{n^k+1}$ converge?

(A) 3 (B) 4 (C) 5 (D) 6

4. Let $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ and s_n be the sum of the first n terms of the series. If $|s - s_n| < \frac{1}{500}$ what is the smallest value of n?

(A) 6 (B) 7 (C) 8 (D) 9

5. Which of the following series converge?

I. $\sum_{n=2}^{\infty} (-1)^n \sqrt[n]{3}$

II. $\sum_{n=1}^{\infty} \frac{3^{n+1}}{\pi^n}$

III. $\sum_{n=1}^{\infty} (\tan^{-1}(n+1) - \tan^{-1}(n))$

(A) I only (B) II only (C) III only (D) II and III only

6. Which of the following statements about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+3)}{n^2}$ is true?

(A) The series converges conditionally.

(B) The series converges absolutely.

(C) The series converges but neither conditionally nor absolutely.

(D) The series diverges.

7. Which of the following series is absolutely convergent?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt{n}}{n}$

(B) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{n^2\sqrt{n}}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}n}{n^2-\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(n^2+1)}{n^3}$

8. An alternating series is given by $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+3}$. Let S_3 be the sum of the first three terms of the given alternating series. Of the following, which is the smallest number M for which the alternating series error bound guarantees that $|S - S_3| \leq M$?

(A) $\frac{1}{4}$ (B) $\frac{1}{7}$ (C) $\frac{1}{19}$ (D) $\frac{1}{28}$

Free Response Questions

9. Let $f(x) = 1 - \frac{3x}{2!} + \frac{9x^2}{4!} - \frac{27x^3}{6!} + \cdots + \frac{(-1)^n (3x)^n}{(2n)!} + \cdots$.

   Use the alternating series error bound to show that $1 - \frac{3}{2!} + \frac{9}{4!}$ approximates $f(1)$ with an error less than $\frac{1}{20}$.

Ratio Test

1. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ II. $\sum_{n=1}^{\infty} \frac{n}{3^n}$ III. $\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$

(A) I only (B) II only (C) II and III only (D) I, II, and III

2. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ II. $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ III. $\sum_{n=1}^{\infty} \frac{n^9}{9^n}$

(A) I only (B) II only (C) I and II only (D) I, II, and III

Free Response Questions

3. Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{n!}{n\ 2^n}$

(b) $\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$

(c) $\sum_{k=1}^{\infty} \frac{3^k k!}{(k+3)!}$

Power Series Convergence

1. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n^3}$ converges?

(A) $-1 < x < 1$ (B) $-1 \le x \le 1$ (C) $-1 < x \le 1$ (D) $-1 \le x < 1$

2. What are all values of x for which the series $\sum_{n=0}^{\infty} \frac{n(x-2)^n}{3^n}$ converges?

(A) $-1 < x < 5$ (B) $-1 < x \le 5$ (C) $-2 \le x < 4$ (D) $-2 < x \le 4$

3. What are all values of x for which the series $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$ converges?

(A) $0 < x < 2$ (B) $0 \le x < 2$ (C) $-1 < x \le 2$ (D) All real x

4. What are all values of x for which the series $\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{2^n \sqrt{n}}$ converges?

(A) $-2 < x < 2$ (B) $-2 \le x < 2$ (C) $-2 < x \le 2$ (D) All real x

5. What are all values of x for which the series $\sum_{n=1}^{\infty} n!(3x-2)^n$ converges?

(A) No values of x (B) $(-\infty, \frac{2}{3}]$ (C) $x = \frac{2}{3}$ (D) $[\frac{2}{3}, \infty)$

Free Response Questions

6. Find the radius of convergence and the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)} x^{n+1}.$$

Representations of Functions as Power Series

1. The power series expansion for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{1}{1+x^3}$?

(A) $1+x^2+x^4+x^6+\cdots$

(B) $1-x^3+x^6-x^9+\cdots$

(C) $1+\frac{x^3}{3}+\frac{x^6}{6}+\frac{x^9}{9}+\cdots$

(D) $1-\frac{x^3}{3}+\frac{x^6}{6}-\frac{x^9}{9}+\cdots$

2. The power series expansion for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{1}{2-x}$?

(A) $1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}+\cdots$

(B) $1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\cdots$

(C) $\frac{1}{2}+\frac{x}{4}+\frac{x^2}{8}+\frac{x^3}{16}+\cdots$

(D) $\frac{1}{2}-\frac{x}{4}+\frac{x^2}{8}-\frac{x^3}{16}+\cdots$

3. If $f(x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-2)^n}{n!}=(x-2)-\frac{(x-2)^2}{2!}+\frac{(x-2)^3}{3!}-\frac{(x-2)^4}{4!}+\cdots$, which of the following represents $f'(x)$?

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-2)^{n-1}}{n!}$

(B) $\sum_{n=1}^{\infty}(-1)^{n}\frac{(x-2)^{n-1}}{(n+1)!}$

(C) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{(x-2)^{n-1}}{n!}$

(D) $\sum_{n=0}^{\infty}(-1)^{n}\frac{(x-2)^{n}}{n!}$

Free Response Questions

4. A power series expansion for $f(x)=\frac{1}{1-x}$ can be obtained from the sum of the geometric series $\sum_{n=0}^{\infty}ar^n=\frac{a}{1-r}$, if you let $a=1$ and $r=x$.. Let $g(x)$ be defined as $g(x)=\frac{1}{1+x}$.

(a) Write the first four terms and the general term of the power series expansion of $g(x)$.

(b) Write the first four terms and the general term of the power series expansion of $g(x^2)$.

(c) Write the first four terms and the general term of the power series expansion of h, where $h(x)=\int g(x^2)\,dx$ and $h(0)=0$.

(d) Find the value of $h(1)$.

5. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2n+1)x^{2n}}{(2n)!} = 1 - \frac{3x^2}{2!} + \frac{5x^4}{4!} - \frac{7x^6}{6!} + \cdots + (-1)^n \frac{(2n+1)x^{2n}}{(2n)!} + \cdots$$

for all real numbers x.

(a) Find $f'(0)$ and $f''(0)$. Determine whether f has a local maximum, a local minimum, or neither at $x = 0$. Give a reason for your answer.

(b) Show that $1 - \frac{3}{2!} + \frac{5}{4!}$ approximates $f(1)$ with an error less than $\frac{1}{100}$.

(c) Let g be the function given by $g(x) = \int_0^x f(t)\,dt$. Write the first four terms and the general term of the power series expansion of $\frac{g(x)}{x}$.

Taylor Polynomial and Lagrange Error Bound

1. Let $P(x) = \frac{1}{3} - \frac{2}{3}x + \frac{2}{3}x^2 - \frac{4}{9}x^3 + \frac{2}{9}x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?

(A) $-\frac{32}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{8}{9}$ (D) $\frac{16}{3}$

2. Let $P(x) = 4 - 3x^2 + \frac{13}{12}x^4 - \frac{121}{360}x^6$ be the sixth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

(A) $-\frac{121}{15}$ (B) $-\frac{3}{2}$ (C) 0 (D) $\frac{121}{15}$

3. Let f be a function that has derivatives of all orders for all real numbers. If $f(1) = 2$, $f'(1) = -3$, $f''(1) = 4$, and $f'''(1) = -9$, which of the following is the third-degree Taylor polynomial for f about $x = 1$?

(A) $P(x) = 2 - 3(x-1) + 2(x-1)^2 - \frac{3}{2}(x-1)^3$

(B) $P(x) = 2 - 3(x+1) + 2(x+1)^2 - \frac{3}{2}(x+1)^3$

(C) $P(x) = 2 - 3(x-1) + 4(x-1)^2 - 9(x-1)^3$

(D) $P(x) = 2 - 3(x+1) + 2(x+1)^2 - 3(x+1)^3$

4. The third-degree Taylor polynomial of xe^x about $x=0$ is

(A) $P_3(x)=x-\frac{1}{2}x^2+\frac{1}{6}x^3$

(B) $P_3(x)=x+x^2+\frac{1}{2}x^3$

(C) $P_3(x)=x+x^2-\frac{1}{3}x^3$

(D) $P_3(x)=1-x+\frac{1}{2}x^2-\frac{1}{6}x^3$

5. The second-degree Taylor polynomial of $\sec x$ about $x=\frac{\pi}{4}$ is

(A) $P_2(x)=1+\sqrt{2}(x-\frac{\pi}{4})+\sqrt{2}(x-\frac{\pi}{4})^2$

(B) $P_2(x)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}(x-\frac{\pi}{4})+\frac{3\sqrt{2}}{3!}(x-\frac{\pi}{4})^2$

(C) $P_2(x)=\sqrt{2}+\sqrt{2}(x-\frac{\pi}{4})+\frac{3\sqrt{2}}{2!}(x-\frac{\pi}{4})^2$

(D) $P_2(x)=1+\sqrt{2}(x-\frac{\pi}{4})+\frac{3\sqrt{2}}{3!}(x-\frac{\pi}{4})^2$

6. A function f has derivatives of all orders at $x=0$. Let P_n denote the nth- degree Taylor polynomial for f about $x=0$. It is known that $f(0)=\frac{1}{3}$ and $f''(0)=\frac{4}{3}$. If $P_2(\frac{1}{2})=\frac{1}{8}$, what is the value of $f'(0)$?

(A) $-\frac{3}{8}$ (B) $-\frac{3}{4}$ (C) $-\frac{5}{4}$ (D) $-\frac{3}{2}$

Free Response Questions

7. Let $P(x) = 3 - 2(x-2) + 5(x-2)^2 - 12(x-2)^3 + 3(x-2)^4$ be the fourth-degree Taylor polynomial for the function f about $x = 2$. Assume f has derivatives of all orders for all real numbers.

   (a) Find $f(2)$ and $f'''(2)$.

   (b) Write the third-degree Taylor polynomial for f' about 2 and use it to approximate $f'(2.1)$.

   (c) Write the fourth-degree Taylor polynomial for $g(x) = \int_2^x f(t)\,dt$ about 2.

   (d) Can $f(1)$ be determined from the information given? Justify your answer.

8. Let f be the function given by $f(x) = \sin(2x) + \cos(2x)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

   (a) Find $P(x)$.

   (b) Find the coefficient of x^{19} in the Taylor series for f about $x = 0$.

   (c) Use the Lagrange error bound to show that $\left| f(\frac{1}{5}) - P(\frac{1}{5}) \right| < \frac{1}{100}$

   (d) Let h be the function given by $h(x) = \int_0^x f(t)\,dt$. Write the third-degree Taylor polynomial for h about $x = 0$.

Taylor and Maclaurin Series

1. A series expansion of $\dfrac{\arctan x}{x}$ is

(A) $1-\dfrac{x}{3}+\dfrac{x^3}{5}-\dfrac{x^5}{7}+\cdots$

(B) $1-\dfrac{x^2}{3}+\dfrac{x^4}{5}-\dfrac{x^6}{7}+\cdots$

(C) $1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\cdots$

(D) $x-\dfrac{x^3}{3}+\dfrac{x^4}{5}-\dfrac{x^6}{7}+\cdots$

2. The coefficient of x^3 in the Taylor series for e^{-2x} about $x=0$ is

(A) $-\dfrac{4}{3}$ (B) $-\dfrac{2}{3}$ (C) $-\dfrac{1}{3}$ (D) $\dfrac{4}{3}$

3. A function f has a Maclaurin series given by $-\dfrac{x^4}{3!}+\dfrac{x^6}{5!}-\dfrac{x^8}{7!}+\cdots+\dfrac{(-1)^n x^{2n+2}}{(2n+1)!}+\cdots$.

Which of the following is an expression for $f(x)$?

(A) $x^3e^x-x^2$

(B) $x\ln x-x^2$

(C) $\tan^{-1}x-x$

(D) $x\sin x-x^2$

4. A series expansion of $\dfrac{x-\sin x}{x^2}$ is

(A) $\dfrac{1}{2!}-\dfrac{x^2}{4!}+\dfrac{x^4}{6!}+\cdots+\dfrac{(-1)^{n+1}x^{2n-2}}{(2n)!}+\cdots$

(B) $\dfrac{x}{2!}-\dfrac{x^3}{4!}+\dfrac{x^5}{6!}+\cdots+\dfrac{(-1)^{n+1}x^{2n+1}}{(2n)!}+\cdots$

(C) $\dfrac{x}{3!}-\dfrac{x^3}{5!}+\dfrac{x^5}{7!}+\cdots+\dfrac{(-1)^{n+1}x^{2n-1}}{(2n+1)!}+\cdots$

(D) $\dfrac{x^2}{3!}-\dfrac{x^4}{5!}+\dfrac{x^6}{7!}+\cdots+\dfrac{(-1)^{n+1}x^{2n}}{(2n+1)!}+\cdots$

5. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+2}}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $x\sin x$ (B) $x\cos x$ (C) x^2e^{-x} (D) $x\ln(x+1)$

6. The graph of the function represented by the Maclaurin series $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+\cdots+\dfrac{(-1)^n x^{2n+1}}{2n+1}+\cdots$ intersects the graph of $y=e^{-x}$ at $x=$

(A) 0.495 (B) 0.607 (C) 1.372 (D) 2.166

7. What is the coefficient of x^4 in the Taylor series for $\cos^2 x$ about $x=0$?

(A) $\dfrac{1}{12}$ (B) $\dfrac{1}{8}$ (C) $\dfrac{1}{6}$ (D) $\dfrac{1}{3}$

8. The fifth-degree Taylor polynomial for $\tan x$ about $x=0$ is $x+\dfrac{1}{3}x^3+\dfrac{2}{15}x^5$. If f is a function such that $f'(x)=\tan(x^2)$, then the coefficient of x^7 for $f(x)$ about $x=0$ is

(A) $\dfrac{1}{21}$ (B) $\dfrac{3}{42}$ (C) 0 (D) $\dfrac{1}{7}$

9. The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty}\frac{(-2)^{n-1}}{n+1}x^n = \frac{1}{2}x - \frac{2}{3}x^2 + x^3 - \cdots + \frac{(-2)^{n-1}}{n+1}x^n + \cdots$.
Which of the following is the third-degree Taylor polynomial for $g(x) = \cos x \cdot f(x)$ about $x = 0$?

(A) $x - \frac{1}{2}x^2 - \frac{2}{3}x^3$

(B) $1 - \frac{1}{2}x^2 + \frac{2}{3}x^3$

(C) $\frac{1}{2}x - \frac{2}{3}x^2 + \frac{3}{4}x^3$

(D) $\frac{1}{2}x - \frac{11}{12}x^2 + x^3$

10. The Maclaurin series for the function f is given by
$f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)!} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots + \frac{(-1)^n x^{2n}}{(2n+1)!} + \cdots$ on its interval of convergence.
Which of the following statements about f must be true?

(A) f has a relative minimum at $x = 0$.

(B) f has a relative maximum at $x = 0$.

(C) f does not have a relative maximum or a relative minimum at $x = 0$.

(D) f has a point of inflection at $x = 0$.

Free Response Questions

11. Let f be the function given by $f(x) = e^{-x}$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 0$.

(b) Use the result from part (a) to write the first four nonzero terms and the general term of the series expansion about $x = 0$ for $g(x) = \frac{1 - x - f(x)}{x}$.

(c) For the function g in part (b), find $g'(-1)$ and use it to show that $\sum_{n=1}^{\infty}\frac{n}{(n+1)!} = 1$.

12. The Maclaurin series for $f(x)$ is given by $f(x) = \frac{x}{2!} - \frac{x^3}{4!} + \frac{x^5}{6!} + \cdots + \frac{(-1)^{n+1}x^{2n-1}}{(2n)!} + \cdots$.

The Maclaurin series for $g(x)$ is given by $g(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \cdots + \frac{(-1)^n x^n}{n+1} + \cdots$.

(a) Find $f'''(0)$ and $f^{(15)}(0)$.

(b) Find the interval of convergence of the Maclaurin series for $g(x)$.

(c) The graph of $y = f(x) + g(x)$ passes through the point $(0,1)$. Find $y'(0)$ and $y''(0)$ and determine whether y has a relative minimum, a relative maximum, or neither at $x = 0$. Give a reason for your answer.