Calculus Name ___________________________

Riemann Sums and Trapezoidal Rule

Approximate the area under the given function using the specified Riemann Sum or Trapezoidal Rule.

1. $f(x) = 16 - x^2$ on $[2, 5]$ Midpoint with 6 equal subintervals.

2. $f(x) = 2x^2 - x + 2$ on $[1, 4]$ Left Endpoint with 6 equal subintervals.

3. $f(x) = \frac{1}{x^2}$ on $[1, 3]$ Midpoint with 4 equal subintervals.

4. $f(x) = e^x$ on $[0,2]$ Trapezoidal with 4 equal subintervals.

For each problem, approximate the area under the given function using the specified number of rectangles/trapezoids to fill in the chart.

5.

Function	Interval	Number	Left Rectangles	Right Rectangles	Midpoint Rectangles	Trapezoids
$f(x) = x^2 - 3x + 4$	$[1,4]$	6				
$f(x) = \sqrt{x}$	$[2,6]$	8				
$f(x) = 2^x$	$[0,1]$	5				
$f(x) = \sin x$	$[0,\pi]$	8				

6. Use a left-hand Riemann sum to approximate the integral based off of the values in the table.

x	0	27	35	44	45
$f(x)$	3	4	6	4	1

7. Use a right-hand Riemann sum to approximate the integral based off of the values in the table.

x	0	1	3	7	12	17	20
$f(x)$	1	2	0	−1	1	0	−2

8. Use a midpoint Riemann sum to approximate the integral based off of the values in the table.

x	0	0.5	1	1.5	2	2.5	3
f(x)	0	12	18	25	20	14	20

9. Use the trapezoidal rule to approximate the integral based off of the values in the table.

x	0	1	2	4	6	10
$f(x)$	5	3	2	3	5	7

Riemann Sums and Trapezoidal Rule

1. $$\text{AREA} \approx \frac{1}{2}\left[f(2.25)+f(2.75)+f(3.25)+f(3.75)+f(4.25)+f(4.75)\right]$$

$\text{AREA} \approx 9.063$

2. $$\text{AREA} \approx \frac{1}{2}\left[f(1)+f(1.5)+f(2)+f(2.5)+f(3)+f(3.5)\right]$$

$\text{AREA} \approx 34$

3. $$\text{AREA} \approx \frac{1}{2}\left[f(1.25)+f(1.75)+f(2.25)+f(2.75)\right]$$

$\text{AREA} \approx 0.648$

4. $$\text{AREA} \approx \frac{1}{2}\left[\frac{f(0)+f(0.5)}{2}+\frac{f(0.5)+f(1)}{2}+\frac{f(1)+f(1.5)}{2}+\frac{f(1.5)+f(2)}{2}\right]$$

$\text{AREA} \approx 6.522$

5. $f(x)=x^2-3x+4$
$[1,4]$

LEFT: $\frac{1}{2}\left[f(1)+f(1.5)+f(2)+f(2.5)+f(3)+f(3.5)\right] \approx 9.125$

RIGHT: $\frac{1}{2}\left[f(1.5)+f(2)+f(2.5)+f(3)+f(3.5)+f(4)\right] \approx 12.125$

MIDPOINT: $\frac{1}{2}\left[f(1.25)+f(1.75)+f(2.25)+f(2.75)+f(3.25)+f(3.75)\right] \approx 10.438$

TRAPEZOID: $$\frac{1}{2}\left[\frac{f(1)+f(1.5)}{2}+\frac{f(1.5)+f(2)}{2}+\frac{f(2)+f(2.5)}{2}+\frac{f(2.5)+f(3)}{2}+\frac{f(3)+f(3.5)}{2}+\frac{f(3.5)+f(4)}{2}\right]$$
≈ 10.625

$f(x)=\sqrt{x}$
$[2,6]$

LEFT: $\frac{1}{2}\left[f(2)+f(2.5)+f(3)+f(3.5)+f(4)+f(4.5)+f(5)+f(5.5)\right] \approx 7.650$

RIGHT: $\frac{1}{2}\left[f(2.5)+f(3)+f(3.5)+f(4)+f(4.5)+f(5)+f(5.5)+f(6)\right] \approx 8.168$

MIDPOINT: $\frac{1}{2}\left[f(2.25)+f(2.75)+f(3.25)+f(3.75)+f(4.25)+f(4.75)+f(5.25)+f(5.75)\right] \approx 7.914$

TRAPEZOIDAL: $$\frac{1}{2}\left[\frac{f(2)+f(2.5)}{2}+\frac{f(2.5)+f(3)}{2}+\frac{f(3)+f(3.5)}{2}+\frac{f(3.5)+f(4)}{2}+\frac{f(4)+f(4.5)}{2}+\frac{f(4.5)+f(5)}{2}+\frac{f(5)+f(5.5)}{2}+\frac{f(5.5)+f(6)}{2}\right]$$
≈ 7.909

$f(x) = 2^x$
$[0, 1]$

LEFT: $\frac{1}{5}\left[f(0) + f(\tfrac{1}{5}) + f(\tfrac{2}{5}) + f(\tfrac{3}{5}) + f(\tfrac{4}{5})\right] \approx 1.345$

RIGHT: $\frac{1}{5}\left[f(\tfrac{1}{5}) + f(\tfrac{2}{5}) + f(\tfrac{3}{5}) + f(\tfrac{4}{5}) + f(1)\right] \approx 1.545$

MIDPOINT: $\frac{1}{5}\left[f(\tfrac{1}{10}) + f(\tfrac{3}{10}) + f(\tfrac{5}{10}) + f(\tfrac{7}{10}) + f(\tfrac{9}{10})\right] \approx 1.442$

TRAPEZOIDAL: $\frac{1}{5}\left[\frac{f(0)+f(\frac{1}{5})}{2} + \frac{f(\frac{1}{5})+f(\frac{2}{5})}{2} + \frac{f(\frac{2}{5})+f(\frac{3}{5})}{2} + \frac{f(\frac{3}{5})+f(\frac{4}{5})}{2} + \frac{f(\frac{4}{5})+f(1)}{2}\right] \approx 1.445$

$f(x) = \sin x$
$[0, \pi]$

LEFT: $\frac{\pi}{8}\left[f(0) + f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8}) + f(\tfrac{\pi}{2}) + f(\tfrac{5\pi}{8}) + f(\tfrac{3\pi}{4}) + f(\tfrac{7\pi}{8})\right] \approx 1.974$

RIGHT: $\frac{\pi}{8}\left[f(\tfrac{\pi}{8}) + f(\tfrac{\pi}{4}) + f(\tfrac{3\pi}{8}) + f(\tfrac{\pi}{2}) + f(\tfrac{5\pi}{8}) + f(\tfrac{3\pi}{4}) + f(\tfrac{7\pi}{8}) + f(\pi)\right] \approx 1.974$

MIDPOINT: $\frac{\pi}{8}\left[f(\tfrac{\pi}{16}) + f(\tfrac{3\pi}{16}) + f(\tfrac{5\pi}{16}) + f(\tfrac{7\pi}{16}) + f(\tfrac{9\pi}{16}) + f(\tfrac{11\pi}{16}) + f(\tfrac{15\pi}{16}) + f(\tfrac{15\pi}{16})\right] \approx 2.013$

TRAPEZOID: $\frac{\pi}{8}\left[\frac{f(0)+f(\frac{\pi}{8})}{2} + \frac{f(\frac{\pi}{8})+f(\frac{\pi}{4})}{2} + \frac{f(\frac{\pi}{4})+f(\frac{3\pi}{8})}{2} + \frac{f(\frac{3\pi}{8})+f(\frac{\pi}{2})}{2} + \frac{f(\frac{\pi}{2})+f(\frac{5\pi}{8})}{2} + \frac{f(\frac{5\pi}{8})+f(\frac{3\pi}{4})}{2} + \frac{f(\frac{3\pi}{4})+f(\frac{7\pi}{8})}{2} + \frac{f(\frac{7\pi}{8})+f(\pi)}{2}\right]$

≈ 1.974

6.

AREA $\approx 27(f(0)) + 8(f(27)) + 9(f(35)) + 1(f(44))$

FROM THE TABLE, AT MOST 4 SUBINTERVALS ON $[0,45]$.

$\approx 81 + 32 + 54 + 4 \approx 171$

7. AREA $\approx 1(f(1)) + 2(f(3)) + 4(f(7)) + 5(f(12)) + 5(f(17)) + 3(f(20))$

FROM TABLE, AT MOST 6 SUBINTERVALS ON $[0,20]$

$\approx 2 + 0 + -4 + 5 + 0 + -6 = -3$

8. AREA $\approx 1(f(.5)) + 1(f(1.5)) + 1(f(2.5))$

FROM TABLE, AT MOST 3 SUBINTERVALS ON $[0,3]$

$\approx 12 + 25 + 14 \approx 51$

9. AREA $\approx 1\left(\frac{f(0)+f(1)}{2}\right) + 1\left(\frac{f(1)+f(2)}{2}\right) + 2\left(\frac{f(2)+f(4)}{2}\right) + 2\left(\frac{f(4)+f(6)}{2}\right) + 4\left(\frac{f(6)+f(10)}{2}\right)$

FROM TABLE, AT MOST 5 SUBINTERVALS ON $[0,10]$

$\approx 1(4) + 1(2.5) + 2(2.5) + 2(4) + 4(6) \approx 43.5$