Question 1

(a) $a(1) = \langle x''(1), y''(1) \rangle \quad \langle 19.975, -4.546 \rangle$ Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 11.678$	$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$
(b) Distance $= \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 6.704$ (or 6.703)	$3 : \begin{cases} 2 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $y(1) = 5 + \int_0^1 y'(t)\, dt = 4.057$ (or 4.056)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{10\cos\left(1+\sqrt{t}\right)}{-12\sin\left(2t^2\right)} = \frac{1}{2}$ $t = 1.072$	$2 : \begin{cases} 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

Question 2

(a) Area $= \frac{1}{2}\int_0^{\pi/2} (2 + \sin(4\theta) + \cos(\theta))^2 \, d\theta$

$= 6.194$ (or 6.193)

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $2 + \sin(4\theta) + \cos(\theta) = 2 \Rightarrow \theta = 0.942478$

Let $c = 0.942478$

Area $= \frac{1}{2}\int_c^{\pi/2} \left[2^2 - (2 + \sin(4\theta) + \cos(\theta))^2\right] d\theta$

$= 0.456$

$4 : \begin{cases} 1 : \text{value of } \theta \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $r'(\theta) = 4\cos(4\theta) - \sin(\theta) = 0$

$\Rightarrow \theta = 0.370064, 1.237726$

θ	$r(\theta)$
0	3
0.370064	3.928208
1.237726	1.355256
$\pi/2$	2

$\theta = 0.370$ corresponds to the point on the curve with the greatest distance from the origin.

$3 : \begin{cases} 1 : \text{identifies } \theta = 0.370 \text{ as a candidate} \\ 2 : \text{answer with justification} \end{cases}$

Free Response – Non – Calculator

Question 3

(a) $C'(3.5) \approx \dfrac{C(4)-C(3)}{4-3} = \dfrac{12.8-11.2}{1} = 1.6$ ounces/min	$2: \begin{cases} 1: \text{approximation} \\ 1: \text{units} \end{cases}$
(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval) $\dfrac{C(4)-C(2)}{4-2} = \dfrac{12.8-8.8}{2} = 2$ Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.	$2: \begin{cases} 1: \dfrac{C(4)-C(2)}{4-2} \\ 1: \text{conclusion, using MVT} \end{cases}$
(c) $\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}[2\cdot C(1) + 2\cdot C(3) + 2\cdot C(5)]$ $= \frac{1}{6}(2\cdot 5.3 + 2\cdot 11.2 + 2\cdot 13.8)$ $= \frac{1}{6}(60.6) = 10.1$ ounces $\frac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.	$3: \begin{cases} 1: \text{midpoint sum} \\ 1: \text{approximation} \\ 1: \text{interpretation} \end{cases}$
(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$ $B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min	$2: \begin{cases} 1: B'(t) \\ 1: B'(5) \end{cases}$

Question 4

(a) $g(3) = \int_{-3}^{3} f(t)\,dt = 6 + 4 - 1 = 9$	1 : answer
(b) $g'(x) = f(x)$ The graph of g is increasing and concave down on the intervals $-5 < x < -3$ and $0 < x < 2$ because $g' = f$ is positive and decreasing on these intervals.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$
(c) $h'(x) = \dfrac{5xg'(x) - g(x)5}{(5x)^2} = \dfrac{5xg'(x) - 5g(x)}{25x^2}$ $h'(3) = \dfrac{(5)(3)g'(3) - 5g(3)}{25 \cdot 3^2}$ $= \dfrac{15(-2) - 5(9)}{225} = \dfrac{-75}{225} = -\dfrac{1}{3}$	$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$
(d) $p'(x) = f'\left(x^2 - x\right)(2x - 1)$ $p'(-1) = f'(2)(-3) = (-2)(-3) = 6$	$3 : \begin{cases} 2 : p'(x) \\ 1 : \text{answer} \end{cases}$

Question 5

(a) $f\left(\frac{1}{2}\right) \approx f(1) + \left(\left.\frac{dy}{dx}\right\|_{(1,\,0)}\right) \cdot \Delta x$ $= 0 + 1 \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}$ $f(0) \approx f\left(\frac{1}{2}\right) + \left(\left.\frac{dy}{dx}\right\|_{\left(\frac{1}{2},\,-\frac{1}{2}\right)}\right) \cdot \Delta x$ $\approx -\frac{1}{2} + \frac{3}{2} \cdot \left(-\frac{1}{2}\right) = -\frac{5}{4}$	$2: \begin{cases} 1: \text{Euler's method with two steps} \\ 1: \text{answer} \end{cases}$
(b) Since f is differentiable at $x = 1$, f is continuous at $x = 1$. So, $\lim_{x\to 1} f(x) = 0 = \lim_{x\to 1}(x^3 - 1)$ and we may apply L'Hospital's Rule. $\lim_{x\to 1}\frac{f(x)}{x^3 - 1} = \lim_{x\to 1}\frac{f'(x)}{3x^2} = \frac{\lim_{x\to 1} f'(x)}{\lim_{x\to 1} 3x^2} = \frac{1}{3}$	$2: \begin{cases} 1: \text{use of L'Hospital's Rule} \\ 1: \text{answer} \end{cases}$
(c) $\frac{dy}{dx} = 1 - y$ $\int \frac{1}{1-y}\,dy = \int 1\,dx$ $-\ln\|1 - y\| = x + C$ $-\ln 1 = 1 + C \Rightarrow C = -1$ $\ln\|1 - y\| = 1 - x$ $\|1 - y\| = e^{1-x}$ $f(x) = 1 - e^{1-x}$	$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables

Question 6

(a) $f'(0) = f(0) + 0 + 1 = 3$ An equation for the tangent line is $y = 3x + 2$.	1 : tangent line equation
(b) $f''(x) = f'(x) + 1;\ f''(0) = f'(0) + 1 = 3 + 1 = 4$ $P_2(x) = 2 + 3x + \frac{4}{2!}x^2 = 2 + 3x + 2x^2$	$2 : \begin{cases} 1 : f''(0) \\ 1 : \text{second-degree Taylor polynomial} \end{cases}$
(c) $f'''(x) = f''(x);\ f'''(0) = f''(0) = 4$ $f^{(4)}(x) = f'''(x);\ f^{(4)}(0) = f'''(0) = 4$ $P_4(x) = 2 + 3x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4$ $= 2 + 3x + 2x^2 + \frac{2}{3}x^3 + \frac{1}{6}x^4$	$2 : \begin{cases} 1 : f'''(0) \text{ and } f^{(4)}(0) \\ 1 : \text{fourth-degree Taylor polynomial} \end{cases}$
(d) $f^{(n)}(0) = 4$ for $n \ge 2$ $f(x) = 2 + 3x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4 + \frac{4}{5!}x^5 + \cdots$ $4e^x = 4 + 4x + \frac{4}{2!}x^2 + \frac{4}{3!}x^3 + \frac{4}{4!}x^4 + \frac{4}{5!}x^5 + \cdots$ Therefore, $f(x) - 4e^x = -2 - x$.	$4 : \begin{cases} 1 : f^{(n)}(0) \text{ for } n \ge 2 \\ 1 : \text{Taylor series for } f \\ 1 : \text{Taylor series for } e^x \\ 1 : \text{polynomial expression} \end{cases}$