Question 1

(a) $\sqrt{(x'(2))^2 + (y'(2))^2} = 3.272461$ The speed of the particle at time $t = 2$ seconds is 3.272 meters per second.	$2 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{answer with units} \end{cases}$
(b) $s(t) = \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{(2\cos(2t))^2 + (2t-1)^2}$ $s'(4) = 2.16265$ Since $s'(4) > 0$, the speed of the particle is increasing at time $t = 4$.	$2 : \begin{cases} 1 : \text{considers } s'(4) \\ 1 : \text{answer with reason} \end{cases}$
(c) $\int_0^5 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 22.381767$ The total distance the particle travels over the time interval $0 \le t \le 5$ seconds is 22.382 (or 22.381) meters.	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(d) $x(10) = x(8) + x'(8) \cdot 2 = \sin 16 + x'(8) \cdot 2 = -4.118541$ $y(10) = y(8) + y'(8) \cdot 2 = \left(8^2 - 8\right) + y'(8) \cdot 2 = 86$ The position of the particle at time $t = 10$ seconds is $(-4.119, 86)$ (or $(-4.118, 86)$).	$3 : \begin{cases} 1 : \text{uses position at } t = 8 \\ 1 : \text{uses velocity at } t = 8 \\ 1 : \text{position at } t = 10 \end{cases}$

Question 2

(a) $R'(45) = \dfrac{30-0}{35-55} = -\dfrac{3}{2}$

The rate at which water is being pumped into the tank is decreasing at $\dfrac{3}{2}$ liters/min^2 at $t = 45$ minutes.

$$2 : \begin{cases} 1 : R'(45) \\ 1 : \text{explanation} \end{cases}$$

(b) $\displaystyle\int_0^{55} R(t)\,dt = 20 \cdot \frac{10+30}{2} + 15 \cdot 30 + \frac{1}{2} \cdot 20 \cdot 30$

$= 400 + 450 + 300 = 1150$

$$2 : \begin{cases} 1 : \text{sum of areas} \\ 1 : \text{answer} \end{cases}$$

(c) $\displaystyle \text{Amt} = 100 + 1150 - \int_{10}^{55} 10e^{(\sin t)/10}\,dt$

$= 1250 - 450.275371 = 799.725 \text{ (or } 799.724)$

$$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for water in the tank} \\ 1 : \text{answer} \end{cases}$$

(d) $R(45) = 15$
$D(45) = 10.88815$

At time $t = 45$ minutes, the rate of water pumped into the tank is greater than the rate of water draining from the tank. Therefore, the amount of water in the tank is increasing at time $t = 45$ minutes.

2 : answer with justification

Question 3

(a) $\frac{dP}{dt} = \frac{1}{4}(220 - P)$ $\int \frac{dP}{220 - P} = \int \frac{1}{4}\,dt$ $-\ln\lvert 220 - P\rvert = \frac{1}{4}t + C$ Because $P(0) = 20$, $P < 220$, so $\lvert 220 - P\rvert = 220 - P$. $-\ln(220 - 20) = \frac{1}{4}(0) + C \Rightarrow C = -\ln 200$ $220 - P = 200e^{-t/4}$ $P = 220 - 200e^{-t/4}$, $t \ge 0$	$5: \begin{cases} 1: \text{separation of variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } P \end{cases}$ Note: max $2/5$ [1-1-0-0-0] if no constant of integration Note: $0/5$ if no separation of variables
(b) Q satisfies a logistic differential equation with carrying capacity 220. Q grows most rapidly when $Q = \frac{220}{2} = 110$. $\left.\frac{dQ}{dt}\right\rvert_{Q=110} = \frac{110^2}{500} = \frac{121}{5}$	$2: \begin{cases} 1: Q = 110 \\ 1: \text{answer} \end{cases}$
(c) $Q(0) = 20$ $Q'(0) = \frac{1}{500}(20)(200) = 8$ $Q(5) \approx 20 + (8)(5) = 60$ $Q'(5) \approx \frac{1}{500}(60)(220 - 60) = \frac{96}{5}$ $Q(10) \approx 60 + \left(\frac{96}{5}\right)(5) = 156$	$2: \begin{cases} 1: \text{Euler's method with two steps} \\ 1: \text{answer} \end{cases}$

Question 4

(a) $g(0) = 2 \cdot 0 + \int_{-2}^{0} f(t)\,dt = 3$ $g(-5) = 2 \cdot (-5) + \int_{-2}^{-5} f(t)\,dt = -10 + 3 = -7$	$2 : \begin{cases} 1 : g(0) \\ 1 : g(-5) \end{cases}$
(b) $g'(x) = 2 + f(x)$ $g''(x) = f'(x)$ $g''(4) = f'(4) = -1$ $g''(-2) = f'(-2)$ does not exist.	$3 : \begin{cases} 1 : g'(x) \\ 1 : g''(4) \\ 1 : g''(-2) \text{ does not exist} \end{cases}$
(c) The graph of g is concave down on the intervals $(-2, 0)$ and $(2, 8)$ since $g'(x) = 2 + f(x)$ decreases on those intervals.	1 : intervals and reason
(d) $h'(x) = g'\left(x^3 + 1\right) \cdot 3x^2$ $h'(1) = g'(2) \cdot 3 = (2 + f(2)) \cdot 3$ $= (2 + 3) \cdot 3 = 15$	$3 : \begin{cases} 2 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

Question 5

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.	$2: \begin{cases} 1: \text{estimate} \\ 1: \text{conclusion with reason} \end{cases}$
(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$ $\left.\frac{dV}{dt}\right\rvert_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$	$3: \begin{cases} 2: \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$ $= 19.3$ ft $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.	$2: \begin{cases} 1: \text{approximation} \\ 1: \text{explanation} \end{cases}$
(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.	1 : conclusion with reason
Units of ft^3/min in part (b) and ft in part (c)	1 : units in (b) and (c)

Question 6

(a) Since g is continuous,

$$g(0) = \lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{\cos(2x) - 1}{x^2}$$
$$= \lim_{x\to 0} \frac{-2\sin(2x)}{2x}$$
$$= \lim_{x\to 0} \frac{-4\cos(2x)}{2} = -2$$

$2 : \begin{cases} 1 : \text{uses L'Hospital's Rule correctly at least once} \\ 1 : \text{answer} \end{cases}$

(b) $$\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots + (-1)^n \frac{(2x)^{2n}}{(2n)!} + \cdots$$
$$= 1 - \frac{4}{2!}x^2 + \frac{16}{4!}x^4 - \frac{64}{6!}x^6 + \cdots + (-1)^n \frac{2^{2n}}{(2n)!}x^{2n} + \cdots$$

$3 : \begin{cases} 1 : \text{first two terms} \\ 1 : \text{next two terms} \\ 1 : \text{general term} \end{cases}$

(c) $$g(x) = -\frac{4}{2!} + \frac{16}{4!}x^2 - \frac{64}{6!}x^4 + \cdots + (-1)^n \frac{2^{2n}}{(2n)!}x^{2n-2} + \cdots$$

$2 : \begin{cases} 1 : \text{first three terms} \\ 1 : \text{general term} \end{cases}$

(d) $g'(x) = \frac{2 \cdot 16}{4!}x - \frac{4 \cdot 64}{6!}x^3 + \cdots$, so $g'(0) = 0$.

$g''(x) = \frac{2 \cdot 16}{4!} - \frac{3 \cdot 4 \cdot 64}{6!}x^2 + \cdots$, so $g''(0) > 0$.

Therefore, g has a relative minimum at $x = 0$ by the Second Derivative Test.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$