Calculus BC – Practice Exam #1 FRQ Scoring

Part A:

Question 1

(a) $\left.\frac{dx}{dt}\right\|_{t=2} = \frac{2}{e^2}$ Because $\left.\frac{dx}{dt}\right\|_{t=2} > 0$, the particle is moving to the right at time $t = 2$. $\left.\frac{dy}{dx}\right\|_{t=2} = \frac{dy/dt\|_{t=2}}{dx/dt\|_{t=2}} = 3.055$ (or 3.054)	$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$
(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574) Acceleration $= \langle x''(4), y''(4) \rangle = \langle -0.041, 0.989 \rangle$	$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$
(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 0.651$ (or 0.650)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Question 2

Point of intersection $e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$	1: Correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx = 0.442$ or 0.443	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2 \right) dx = 0.453\pi$ or 1.423 or 1.424	$3 : \begin{cases} 2 : \text{integrand} \\ \quad <-1> \text{ reversal} \\ \quad <-1> \text{ error with constant} \\ \quad <-1> \text{ omits 1 in one radius} \\ \quad <-2> \text{ other errors} \\ 1 : \text{answer} \end{cases}$
(c) Length $= \sqrt{x} - e^{-3x}$ Height $= 5\left(\sqrt{x} - e^{-3x}\right)$ Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$	$3 : \begin{cases} 2 : \text{integrand} \\ \quad <-1> \text{ incorrect but has } \sqrt{x} - e^{-3x} \text{ as a factor} \\ 1 : \text{answer} \end{cases}$

Part B

Question 3

(a) Average acceleration of rocket A is $\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} = \frac{11}{20}$ ft/sec^2	1 : answer
(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is $20[v(20)+v(40)+v(60)]$ $= 20[22+35+44] = 2020$ ft	3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{uses } v(20),\ v(40),\ v(60) \\ 1 : \text{value} \end{cases}$
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$ $2 = v_B(0) = 6 + C$ $v_B(t) = 6\sqrt{t+1} - 4$ $v_B(80) = 50 > 49 = v(80)$ Rocket B is traveling faster at time $t = 80$ seconds.	4 : $\begin{cases} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80)\text{, compares to } v(80)\text{,} \\ \quad \text{and draws a conclusion} \end{cases}$
Units of ft/sec^2 in (a) and ft in (b)	1 : units in (a) and (b)

Question 4

(a) $\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx} = 3 + 2(3x+2y+1) = 6x + 4y + 5$	2 : $\begin{cases} 1 : 3 + 2\frac{dy}{dx} \\ 1 : \text{answer} \end{cases}$
(b) If $y = mx + b + e^{rx}$ is a solution, then $m + re^{rx} = 3x + 2\left(mx + b + e^{rx}\right) + 1.$ If $r \neq 0$: $m = 2b + 1,\ r = 2,\ 0 = 3 + 2m,$ so $m = -\frac{3}{2},\ r = 2,$ and $b = -\frac{5}{4}.$ OR If $r = 0$: $m = 2b + 3,\ r = 0,\ 0 = 3 + 2m,$ so $m = -\frac{3}{2},\ r = 0,\ b = -\frac{9}{4}.$	3 : $\begin{cases} 1 : \frac{dy}{dx} = m + re^{rx} \\ 1 : \text{value for } r \\ 1 : \text{values for } m \text{ and } b \end{cases}$
(c) $f\left(\frac{1}{2}\right) \approx f(0) + f'(0)\cdot\frac{1}{2} = -2 + (-3)\cdot\frac{1}{2} = -\frac{7}{2}$ $f'\left(\frac{1}{2}\right) \approx 3\left(\frac{1}{2}\right) + 2\left(-\frac{7}{2}\right) + 1 = -\frac{9}{2}$ $f(1) \approx f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right)\cdot\frac{1}{2} = -\frac{7}{2} + \left(-\frac{9}{2}\right)\cdot\frac{1}{2} = -\frac{23}{4}$	2 : $\begin{cases} 1 : \text{Euler's method with 2 steps} \\ 1 : \text{Euler's approximation for } f(1) \end{cases}$
(d) $g'(0) = 3\cdot 0 + 2\cdot k + 1 = 2k + 1$ $g(1) \approx g(0) + g'(0)\cdot 1 = k + (2k+1) = 3k + 1 = 0$ $k = -\frac{1}{3}$	2 : $\begin{cases} 1 : g(0) + g'(0)\cdot 1 \\ 1 : \text{value of } k \end{cases}$

Question 5

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$ $g'(x) = 2 + f(x)$ $g'(-3) = 2 + f(-3) = 2$	$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$
(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$. $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$. Therefore g has an absolute maximum at $x = \frac{5}{2}$.	$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$
(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.	1 : answer with reason
(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$. To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.	$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

Question 6

(a) $1 + (x-1)^2 + \frac{(x-1)^4}{2} + \frac{(x-1)^6}{6} + \cdots + \frac{(x-1)^{2n}}{n!} + \cdots$	$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$
(b) $1 + \frac{(x-1)^2}{2} + \frac{(x-1)^4}{6} + \frac{(x-1)^6}{24} + \cdots + \frac{(x-1)^{2n}}{(n+1)!} + \cdots$	$2 : \begin{cases} 1 : \text{first four terms} \\ 1 : \text{general term} \end{cases}$
(c) $\lim_{n\to\infty} \left\lvert \dfrac{\frac{(x-1)^{2n+2}}{(n+2)!}}{\frac{(x-1)^{2n}}{(n+1)!}} \right\rvert = \lim_{n\to\infty} \frac{(n+1)!}{(n+2)!}(x-1)^2 = \lim_{n\to\infty} \frac{(x-1)^2}{n+2} = 0$ Therefore, the interval of convergence is $(-\infty, \infty)$.	$3 : \begin{cases} 1 : \text{sets up ratio} \\ 1 : \text{computes limit of ratio} \\ 1 : \text{answer} \end{cases}$
(d) $f''(x) = 1 + \frac{4\cdot 3}{6}(x-1)^2 + \frac{6\cdot 5}{24}(x-1)^4 + \cdots + \frac{2n(2n-1)}{(n+1)!}(x-1)^{2n-2} + \cdots$ Since every term of this series is nonnegative, $f''(x) \ge 0$ for all x. Therefore, the graph of f has no points of inflection.	$2 : \begin{cases} 1 : f''(x) \\ 1 : \text{answer} \end{cases}$