Parametric Derivatives :

1. $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{1+e^t}{te^t + e^t}$ $\boxed{D}$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{2}{1} = 2$$

2. $\dfrac{dy}{dx} = \dfrac{\cos t}{\sec^2 t} = \cos^3 t$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[dy/dx\right]}{dx/dt} = \frac{3\cos^2 t \cdot -\sin t}{\sec^2 t} = -3\cos^4 t \cdot \sin t$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\pi/6} = -3\left(\frac{\sqrt{3}}{2}\right)^4\left(\frac{1}{2}\right) = \frac{-27}{32}$$ $\boxed{B}$

3. $\dfrac{dy}{dx} = \dfrac{4t}{3t^2} = \dfrac{4}{3t}$ $\quad \left.\dfrac{dy}{dx}\right|_{(5,8)} = \dfrac{4}{6} = \dfrac{2}{3}$ $\boxed{C}$

$$\left.\begin{matrix} 5 = t^3 - 3 \\ 8 = 2t^2 \end{matrix}\right] t = 2$$

$$y - 8 = \frac{2}{3}(x-5)$$

4. $\dfrac{dy}{dx} = \dfrac{4\sin t \cdot \cos t}{-\sin t} = -4\cos t$ $\quad \left.\dfrac{dy}{dx}\right|_{t=1} = -4\cos(1)$ $\boxed{B}$

5. Horizontal Tangent $\Rightarrow$ $\dfrac{dy}{dt} = 0$ and $\dfrac{dx}{dt} \neq 0$

$$\frac{dx}{dt} = \frac{(1+t^3)(5) - 5t(3t^2)}{(1+t^3)^2} = \frac{1-10t^3}{(1+t^3)^2}$$

$$\frac{dy}{dt} = \frac{(1+t^3)(4t) - 2t^2(3t^2)}{(1+t^3)^2} = \frac{4t - 2t^4}{(1+t^3)^2}$$

$$\frac{dy}{dt} = 0 \Rightarrow 4t - 2t^4 = 0$$

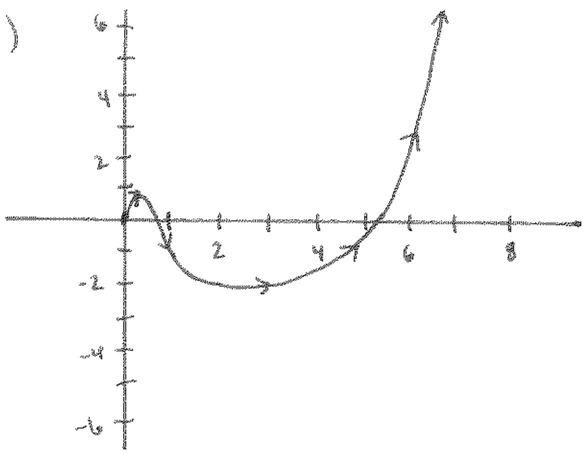
$$2t(2 - t^3) = 0$$

$$t = 0 \quad t = \sqrt[3]{2}$$ $\boxed{D}$

6. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow \frac{3}{4} = \frac{dy/dt}{-2/5} \Rightarrow dy/dt = \frac{-15}{8}$ $\boxed{A}$

7. $x(6) = x(3) + \int_3^6 2 - \sin(t^2)\,dt \Rightarrow x(6) = 8.135$ $\boxed{A}$

8. (a)



t	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$x(t)$	0	$\frac{\pi}{4}-\frac{\sqrt{2}}{2}$	$\frac{\pi-2}{2}$	$\frac{3\pi}{4}-\frac{\sqrt{2}}{2}$	π	$\frac{5\pi}{4}+\frac{\sqrt{2}}{2}$	$\frac{3\pi+2}{2}$	$\frac{7\pi}{4}+\frac{\sqrt{2}}{2}$	2π
$y(t)$	0	$\frac{\pi\sqrt{2}}{8}$	0	$\frac{-3\pi\sqrt{2}}{8}$	$-\pi$	$\frac{-5\pi\sqrt{2}}{8}$	0	$\frac{7\pi\sqrt{2}}{8}$	2π

(b) $y'(t) = t \cdot -\sin t + \cos t = \cos t - t\sin t$

$y'(t) = 0 \Rightarrow t = 3.426$

$x(3.426) = 3.706$

$y(3.426) = -3.288$

(c) $\frac{dy}{dx} = \frac{\cos t - t\sin t}{1-\cos t}$ $\qquad \left.\frac{dy}{dx}\right|_{t=\pi} = -\frac{1}{2}$

$x(\pi) = \pi$

$y(\pi) = -\pi$

$y + \pi = -\frac{1}{2}(x - \pi)$

9. (a) $x(t) = \int \frac{t}{\sqrt{t^2+9}}\,dt = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \sqrt{u} + c = \sqrt{t^2+9} + c$

$u = t^2+9$

$du = 2t\,dt$

$1 = \sqrt{9} + c$

$-2 = c$

$x(t) = \sqrt{t^2+9} - 2$

(b) $\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt} - 8x \cdot \frac{dx}{dt} = 3\left[\sqrt{t^2+9} - 2\right]^2 \cdot \frac{t}{\sqrt{t^2+9}} - 8\left[\sqrt{t^2+9} - 2\right] \cdot \frac{t}{\sqrt{t^2+9}}$

(c) $x(4) = \sqrt{16+9} - 2 = 3$

$y(4) = (3)^3 - 4(3)^2 + 4 = -5$

Position: $\langle 3, -5 \rangle$ @ $t = 4$

(d) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\left[\sqrt{t^2+9} - 2\right]^2 \cdot \frac{t}{\sqrt{t^2+9}} - 8\left[\sqrt{t^2+9} - 2\right] \cdot \frac{t}{\sqrt{t^2+9}}}{\frac{t}{\sqrt{t^2+9}}}$

$= 3\left[\sqrt{t^2+9} - 2\right]^2 - 8\left[\sqrt{t^2+9} - 2\right]$

$\left.\frac{dy}{dx}\right|_{t=4} = 3(3)^2 - 8(3) = 3$

$y + 5 = 3(x - 3)$

Arc Length in Parametric Form:

1. $$\int_1^3 \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt = \int_1^3 \sqrt{(1-2t)^2 + (2t^{1/2})^2}\, dt$$

$$\boxed{B}$$

$$= \int_1^3 \sqrt{1 - 4t + 4t^2 + 4t}\, dt$$

$$= \int_1^3 \sqrt{1 + 4t^2}\, dt = 8.268$$

2. $$\int_0^{\pi} \sqrt{(at\cos t)^2 + (at\sin t)^2}\, dt = \int_0^{\pi} \sqrt{a^2t^2(\cos^2 t + \sin^2 t)}\, dt = \int_0^{\pi} \sqrt{(at)^2}\, dt$$

$$\boxed{C}$$

$$x'(t) = a(-\sin t + t\cos t + \sin t)$$
$$= at\cos t$$
$$y'(t) = a(\cos t + t\sin t - \cos t)$$
$$= at\sin t$$

$$= \int_0^{\pi} at\, dt$$
$$= \left[\frac{1}{2}at^2\right]_0^{\pi}$$
$$= \frac{1}{2}a\pi^2 - \frac{1}{2}a(0)^2$$

3. $$x'(t) = \cos t + \frac{1}{\cos t}\cdot \sin t \qquad y'(t) = -\sin t$$

$$= \cos t - \tan t$$

$$\boxed{D}$$

$$[x'(t)]^2 + [y'(t)]^2 = (\cos t - \tan t)^2 + (-\sin t)^2$$
$$= \cos^2 t - 2\cos t\tan t + \tan^2 t + \sin^2 t$$
$$= \cos^2 t + \sin^2 t - 2\cos t\left(\frac{\sin t}{\cos t}\right) + \sec^2 x - 1$$
$$= 1 - 2\sin t + \sec^2 x - 1$$
$$= \sec^2 x - 2\sin t$$

4. Displacement $= \sqrt{\left[\int_0^2 e^t - t\,dt\right]^2 + \left[\int_0^2 t\sin t\,dt\right]^2} = 4.722$ $\boxed{A}$

5. (a) $\left.\dfrac{dy}{dx}\right|_{(3,2)} = \dfrac{\cos(1)}{2\sin(1)} = 0.321$ $\qquad y - 2 = 0.321(x-3)$

(b) $\displaystyle\int_1^3 \sqrt{\left[2\sin(t^2)\right]^2 + \left[\cos(t^3)\right]^2}\;dt = 3.166$

(c) $x(3) = x(1) + \int_1^3 2\sin(t^2)\,dt = 3.927$ $\qquad (3.927, 1.877)$

$y(3) = y(1) + \int_1^3 \cos(t^3)\,dt = 1.877$

(d) $$\sqrt{\left[\int_1^3 2\sin(t^2)\,dt\right]^2 + \left[\int_1^3 \cos(t^3)\,dt\right]^2} = 0.935$$

Vectors:

1. $v(t) = \left\langle 3t^2 , \frac{1}{\sqrt{t^2+1}} \cdot \frac{1}{2}(t^2+1)^{-1/2} \cdot 2t \right\rangle$ $\boxed{C}$

$= \left\langle 3t^2 , \frac{t}{t^2+1} \right\rangle$

$v(1) = \left\langle 3 , \frac{1}{2} \right\rangle$

2. $v(t) = \left\langle 3t^2 - 2t , 1 + \frac{1}{t} \right\rangle$ $\boxed{D}$

$a(t) = \left\langle 6t - 2 , -\frac{1}{t^2} \right\rangle$ $a(2) = \left\langle 10 , -\frac{1}{4} \right\rangle$

3. Speed $= \sqrt{[x'(t)]^2 + [y'(t)]^2}$

$x'(t) = e^t \cdot -\sin t + e^t \cdot \cos t = e^t(\cos t - \sin t)$

$y'(t) = e^t \cdot \cos t + e^t \sin t = e^t(\cos t + \sin t)$

$\text{Speed}\Big|_{t=2} = \sqrt{\left[e^2(\cos 2 - \sin 2)\right]^2 + \left[e^2(\cos 2 + \sin 2)\right]^2} =$ $\boxed{B}$

$= \sqrt{e^4(\cos 2 - \sin 2)^2 + e^4(\cos 2 + \sin 2)^2}$

$= e^2 \sqrt{\cos^2 2 - 2\cos 2 \sin 2 + \sin^2 2 + \cos^2 2 + 2\cos 2 \sin 2 + \sin^2 2} = \sqrt{2} \cdot e^2$

4. $f'(t) = \left\langle \frac{1}{\sin t} \cdot \cos t , 2t - e^{-t} \right\rangle = \left\langle \cot t , 2t - e^{-t} \right\rangle$ $\boxed{A}$

$f''(t) = \left\langle -\csc^2 x , 2 + e^{-t} \right\rangle$

5. $x\left(\frac{\pi}{2}\right) = x(0) + \int_0^{\pi/2} \cos t \, dt$ $\qquad y(2) = 2+\sqrt{2}$ $\qquad \boxed{D}$

$x\left(\frac{\pi}{2}\right) = 1 + \left[\sin t\right]_0^{\pi/2} = 1 + 1 = 2$

6. $\frac{dy}{dx} = 8x^3 - 1$ $\qquad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 8x^3-1 \Rightarrow$ when $x=1$, $\frac{dy/dt}{dx/dt} = 7 \Rightarrow \frac{dy}{dt} = 7 \cdot \frac{dx}{dt}$

$\left.\frac{dy}{dx}\right|_{x=1} = 7$

$$\text{Speed} = \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2} = \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[7 \cdot \frac{dx}{dt}\right]^2} = 20 \qquad \boxed{C}$$

$\Downarrow$

$$\frac{dx}{dt}\sqrt{50} = 20 \Rightarrow \frac{dx}{dt} = 2\sqrt{2}$$

7. (a) $\sqrt{\left[1+\cos(e^t)\right]^2 + \left[e^{2-t^2}\right]^2} = 3 \Rightarrow t = 0.950$

(b) $\frac{d^2x}{dt^2} = -\sin(e^t) \cdot e^t \Rightarrow$ when $t=2$, $\frac{d^2x}{dt^2} = -e^2\sin(e^2)$

$\frac{d^2y}{dt^2} = e^{(2-t^2)} \cdot -2t \Rightarrow$ when $t=2$, $\frac{d^2y}{dt^2} = -4e^{-2}$ $\qquad a(t) = \left\langle -e^2\sin(e^2), -4e^{-2} \right\rangle$

(c) $\int_1^4 \sqrt{\left[1+\cos(e^t)\right]^2 + \left[e^{2-t^2}\right]^2} \, dt = 3.544$

(d) $\sqrt{\left[\int_1^4 1+\cos(e^t)\, dt\right]^2 + \left[\int_1^4 e^{2-t^2} dt\right]^2} = 2.954$

8. (a) $x(5) = x(1) + \int_1^5 t - \sin(e^t)\,dt = 13.245$

(b) $(1,4)$ $\quad \left.\frac{dy}{dx}\right|_{t=1} = \frac{3}{1-\sin(e)} = 5.091$ $\qquad y-4 = 5.091(x-1)$

(c) $\text{Speed} = \sqrt{(1-\sin(e))^2 + (3)^2} = 3.057$

(d) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = t-2$ for $t \geq 0$

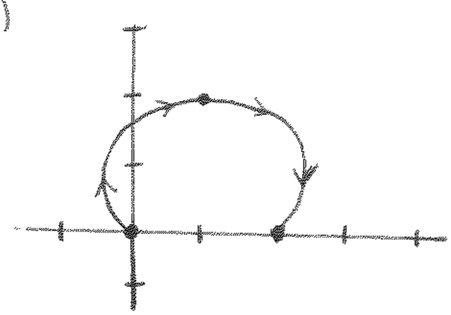
$\frac{dy}{dt} = (t-2)\cdot\frac{dx}{dt}$

$\frac{d^2x}{dt^2} = 1 - \cos(e^t)\cdot e^t$ $\qquad \frac{d^2y}{dt^2} = (t-2)\cdot\frac{d^2x}{dt^2} + \frac{dx}{dt}$

$\left.\frac{d^2x}{dt^2}\right|_{t=3} = -5.6$ $\qquad \left.\frac{d^2y}{dt^2}\right|_{t=3} = (1)\cdot(-5.6) + 3 - \sin(e^3) = -3.544$

$a(3) = \langle -5.6, -3.544 \rangle$

9. (a)



(b) $x(1) = 1 - \sin(\pi) = 1$ $\qquad (1,2)$

$y(1) = 1 - \cos(\pi) = 2$

9. (c) $x'(t) = 1 - \cos(\pi t) \cdot \pi$

$y'(t) = \sin(\pi t) \cdot \pi$

$v(t) = \langle 1 - \pi\cos(\pi t), \pi\sin(\pi t) \rangle$

(d) $\displaystyle\int_0^2 \sqrt{[1 - \pi\cos(\pi t)]^2 + [\pi\sin(\pi t)]^2}\, dt = 6.443$

10. (a) Maximum Height $\Rightarrow$ Horizontal Tangent $\Rightarrow \frac{dy}{dt} = 0$

$4.2 - 9.8t = 0 \Rightarrow t = .429$

(b) Maximum Vertical Distance $\Rightarrow$ Maximum Height

$y(.429) = 10 + \displaystyle\int_0^{.429} 4.2 - 9.8t\, dt = 10.900$ meters

(c) Hit the ground $\Rightarrow y(t) = 0$

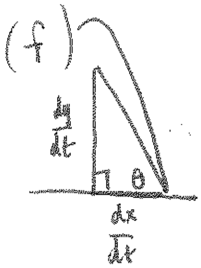
$y(t) = 10 + \displaystyle\int_0^t 4.2 - 9.8y\, dy = 0$

$\left[4.2y - \frac{9.8y^2}{2}\right]_0^t = -10 \Rightarrow t = 1.920$ seconds

(d) $\displaystyle\int_0^{1.920} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 12.384$ meters

(e) $\sqrt{\left(\displaystyle\int_0^{1.92} \frac{dx}{dt}\, dt\right)^2 + \left(\displaystyle\int_0^{1.92} \frac{dy}{dt}\, dt\right)^2} = 10.354$

(f)



$\tan\theta = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{dy}{dx}$

$\left.\dfrac{dy}{dx}\right|_{t=1.920} = -10.44$

$\tan\theta = -10.44$

$\theta = \arctan(10.44)$ $\rightarrow$ Use $+10.44$ since $0 < \theta < \pi/2$

$\theta = 1.475$

11. (a) $x(2) = 2 + \int_0^2 e^{\sqrt{t}} - \cos(t^2)\,dt = 6.946$ $\quad (6.946, 1.5)$

$y(2) = 1.5$

(b) $\left.\frac{dy}{dx}\right|_{t=2} = \frac{-1/2}{e^{\sqrt{2}} - \cos(4)} = -.105$

(c) $\sqrt{\left[e^{\sqrt{2}} - \cos(4)\right]^2 + \left[-1/2\right]^2} = 4.793$

Note: Magnitude of velocity vector is speed.

(d) $\int_0^3 \sqrt{\left[e^{\sqrt{x}} - \cos(x^2)\right]^2 + \left[\frac{dy}{dt}\right]^2}\,dt$

From $0 < t < 1$, $\frac{dy}{dt} = 1$

$1 < t < 3$, $\frac{dy}{dt} = -1/2$

$\downarrow$

$\int_0^1 \sqrt{\left[e^{\sqrt{x}} - \cos(x^2)\right]^2 + \left[1\right]^2}\,dt + \int_1^3 \sqrt{\left[e^{\sqrt{x}} - \cos(x^2)\right]^2 + \left[-1/2\right]^2}\,dt = 1.533 + 8.539 = 10.072$

Polar Derivatives:

1. $r = \theta - 3\sin\theta$ $\boxed{C}$

$\dfrac{dr}{d\theta} = 1 - 3\cos\theta \qquad \left.\dfrac{dr}{d\theta}\right|_{\theta=\pi} = 1 - 3\cos(\pi) = 4$

2. $r = \dfrac{2}{1-\cos\theta} = 2(1-\cos\theta)^{-1}$ $\boxed{A}$

$\dfrac{dr}{d\theta} = -2(1-\cos\theta)^{-2}\cdot\sin\theta \qquad \left.\dfrac{dr}{d\theta}\right|_{\theta=\pi/2} = \dfrac{-2\sin(\pi/2)}{(1-\cos(\pi/2))^2} = -2$

3. $r = 3\sin\theta \qquad x(\theta) = r\cos\theta = 3\sin\theta\cdot\cos\theta$ $\boxed{B}$

$y(\theta) = r\sin\theta = 3\sin\theta\cdot\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\sin\theta\cdot\cos\theta + 3\cos\theta\cdot\sin\theta}{-3\sin\theta\cdot\sin\theta + 3\cos\theta\cdot\cos\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/3} = \frac{3\cdot\frac{\sqrt3}{2}\cdot\frac12 + 3\cdot\frac12\cdot\frac{\sqrt3}{2}}{-3\cdot\frac{\sqrt3}{2}\cdot\frac{\sqrt3}{2} + 3\cdot\frac12\cdot\frac12} = \frac{\frac{3\sqrt3}{2}}{-\frac32} = -\sqrt3$$

4. $r = \dfrac{8}{1-\cos\theta} \qquad x(\theta) = r\cos\theta$ $\boxed{D}$

$-3 = \dfrac{8}{(1-\cos\theta)}\cdot\cos\theta$

$-3 = \dfrac{8\cos\theta}{1-\cos\theta}$

$-3 + 3\cos\theta = 8\cos\theta$

$-3 = 5\cos\theta$

$-\dfrac{3}{5} = \cos\theta \Rightarrow \theta = 2.214$

5. (a) $r = \sqrt{\theta + \cos(2\theta)}$

$$\frac{dr}{d\theta} = \frac{1}{2}\left(\theta + \cos(2\theta)\right)^{-1/2}\left(1 + -\sin(2\theta)\cdot 2\right)$$

$$= \frac{1 - 2\sin(2\theta)}{2\sqrt{\theta + \cos(2\theta)}}$$

(b) $x(\theta) = \sqrt{\theta + \cos(2\theta)} \cdot \cos\theta$

$$\frac{1}{2} = \cos\theta\sqrt{\theta + \cos(2\theta)} \Rightarrow \theta = 0.910$$

(c) $\frac{dr}{d\theta}$ being negative tells you that the value of r is decreasing. This means the curve is getting closer to the origin.

(d) Least Distance from origin $\Rightarrow$ minimum value of r on $[0, \pi/2]$

$$\frac{dr}{d\theta} = \frac{1 - 2\sin(2\theta)}{2\sqrt{\theta + \cos(2\theta)}} = 0 \Rightarrow 1 - 2\sin(2\theta) = 0$$

$\sin(2\theta) = 1/2$

$2\theta = \pi/6 \quad 2\theta = 5\pi/6$

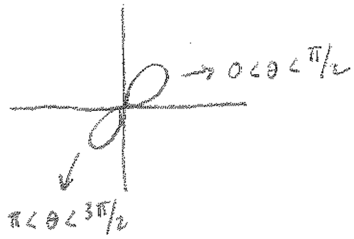
$\theta = \pi/12 \quad \theta = 5\pi/12$

θ	r
0	1
$\pi/12$	1.062
$5\pi/12$	0.666
$\pi/2$	0.756

According to EVT, the absolute minimum value of $r(\theta)$ on $[0, \pi/2]$ occurs either at an endpoint or critical value. The curve is at its least distance from the origin when $\theta = 5\pi/12$.

Polar Area:

1. $r^2 = 6\sin(2\theta) \Rightarrow r = \pm\sqrt{6\sin(2\theta)}$



$\boxed{C}$

$$2\left[\frac{1}{2}\int_0^{\pi/2}\left[r(\theta)\right]^2 d\theta\right]$$

$$\int_0^{\pi/2} r^2(\theta)\,d\theta = \int_0^{\pi/2} 6\sin(2\theta)\,d\theta = 3\int_0^{\pi}\sin u\,du = -3\cos u\Big]_0^{\pi}$$

$$= -3(-1) - -3(1)$$

$$= 6$$

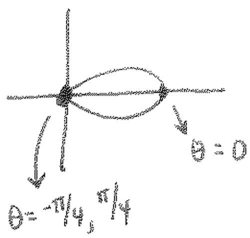
2. $r = 2\cos(2\theta)$ $\qquad r(0) = 2$

$r = 0 \Rightarrow 2\cos(2\theta) = 0$

$\cos(2\theta) = 0$

$2\theta = \pi/2 \quad 2\theta = \pi/2$

$\theta = \pi/4 \quad \theta = \pi/4$



$\boxed{B}$

$$\frac{1}{2}\int_{-\pi/4}^{\pi/4}\left[2\cos(2\theta)\right]^2 d\theta \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$2\int_{-\pi/4}^{\pi/4}\cos^2(2\theta)\,d\theta$$

$$2\int_{-\pi/4}^{\pi/4}\frac{1}{2}\left(1+\cos(4\theta)\right)d\theta$$

$$\int_{-\pi/4}^{\pi/4} 1+\cos(4\theta)\,d\theta$$

$$\left[\theta + \frac{1}{4}\sin(4\theta)\right]_{-\pi/4}^{\pi/4} = \left(\pi/4 + 0\right) - \left(-\frac{\pi}{4} + 0\right) = \frac{\pi}{2}$$

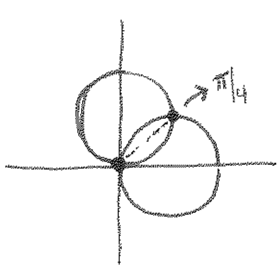
3. $r_1 = \cos\theta \quad r_2 = \sin\theta$

$r_1 = 0 \Rightarrow \theta = \pi/2, -\pi/2$

$r_2 = 0 \Rightarrow \theta = 0, \pi$

$\cos\theta = \sin\theta$

$\theta = \pi/4,$



$$\frac{1}{2}\int_0^{\pi/4}\sin^2\theta\,d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2}\cos^2\theta\,d\theta \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2} \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$\boxed{A}$

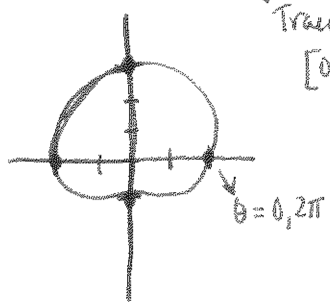
$$\frac{1}{4}\int_0^{\pi/4} 1-\cos(2\theta)\,d\theta + \frac{1}{4}\int_{\pi/4}^{\pi/2} 1+\cos(2\theta)\,d\theta$$

$$\left[\frac{1}{4}\theta - \frac{1}{8}\sin(2\theta)\right]_0^{\pi/4} + \left[\frac{1}{4}\theta + \frac{1}{8}\sin(2\theta)\right]_{\pi/4}^{\pi/2}$$

$$\left(\frac{\pi}{16} - \frac{1}{8}\right) - (0) + \left(\frac{\pi}{8} + 0\right) - \left(\frac{\pi}{16} + \frac{1}{8}\right) = \frac{\pi - 2}{8}$$

4. $r = 2 + \sin\theta$

Traced out on $[0, 2\pi]$



$$\frac{1}{2}\int_0^{2\pi}\left[2+\sin\theta\right]^2 d\theta$$

$$\frac{1}{2}\int_0^{2\pi} 4 + 4\sin\theta + \sin^2\theta \, d\theta$$

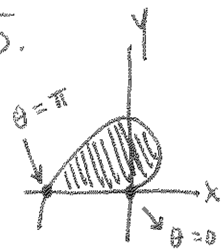
$$\frac{1}{2}\left[4\theta - 4\cos\theta\right]_0^{2\pi} + \frac{1}{2}\int_0^{2\pi}\frac{1-\cos(2\theta)}{2}\,d\theta$$

$$\frac{1}{2}\left[(8\pi - 4) - (0-4)\right] + \frac{1}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{2\pi}$$

$$4\pi + \frac{1}{4}\left[(2\pi - 0) - (0-0)\right] = 4\pi + \frac{\pi}{2} = \frac{9\pi}{2}$$

$\boxed{D}$

5.



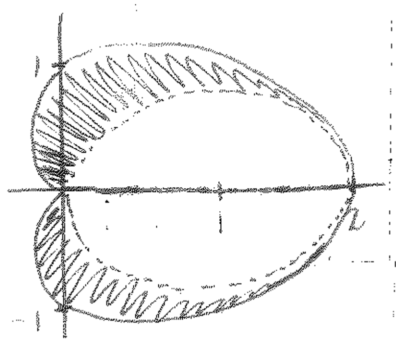
$$\frac{1}{2}\int_0^{\pi}\left[\theta\right]^2 d\theta = \frac{1}{2}\left[\frac{\theta^3}{3}\right]_0^{\pi} = \frac{1}{2}\left[\frac{\pi^3}{3} - 0\right]$$

$$= \frac{\pi^3}{6}$$

$\boxed{B}$

6. $r_1 = 1 + \cos\theta$ —

$r_2 = 2\cos\theta$ - - -



Area inside r_1

$$\frac{1}{2}\int_0^{2\pi}(1+\cos\theta)^2\,d\theta$$

Area inside r_2 $\Rightarrow$ Area of a circle of radius 1

$$\pi(1)^2 = \pi$$

SHADED Area $= \frac{1}{2}\int_0^{2\pi}(1+\cos\theta)^2\,d\theta - \pi$

$\boxed{D}$

7. (a) $P \Rightarrow \; 2+\cos(2\theta) = 2$ and $0<\theta<\frac{\pi}{2}$

$$\cos(2\theta) = 0$$
$$2\theta = \pi/2$$
$$\theta = \pi/4$$

$P: \left(2, \pi/4\right) \qquad \ell: \theta = \pi/4$

(b) $$R_1 = \frac{1}{2}\int_0^{\pi/4}\left[2+\cos(2\theta)\right]^2 d\theta - \frac{1}{2}\int_0^{\pi/4}\left[2\right]^2$$
$$= \frac{1}{2}\left[\int_0^{\pi/4}\left[2+\cos(2\theta)\right]^2 - \left[2\right]^2 d\theta\right]$$

(c) $$R_2 = \frac{1}{2}\int_{\pi/4}^{\pi/2}\left[2\right]^2 d\theta - \frac{1}{2}\int_{\pi/4}^{\pi/2}\left[2+\cos(2\theta)\right]^2 d\theta$$
$$= \frac{1}{2}\left[\int_{\pi/4}^{\pi/2}\left[2\right]^2 - \left[2+\cos(2\theta)\right]^2\right] d\theta$$

(d) $$R_3 = \frac{1}{2}\int_{\pi/2}^{3\pi/4}\left[2+\cos(2\theta)\right]^2 d\theta + \frac{1}{2}\int_{3\pi/4}^{\pi}\left[2\right]^2 d\theta$$
$$= \frac{1}{2}\int_{\pi/2}^{3\pi/4} 4+4\cos(2\theta)+\cos^2(2\theta)\, d\theta + \frac{1}{2}\left[4\theta\right]_{3\pi/4}^{\pi}$$
$$= \frac{1}{2}\left[4\theta+2\sin(2\theta)\right]_{\pi/2}^{3\pi/4} + \frac{1}{4}\int_{\pi/2}^{3\pi/4} 1+\cos(4\theta)\, d\theta + \frac{1}{2}\left[4\pi-3\pi\right]$$
$$= \frac{1}{2}\left[(3\pi+-2)-(2\pi+0)\right] + \frac{1}{4}\left[\theta+\frac{1}{4}\sin(4\theta)\right]_{\pi/2}^{3\pi/4} + \frac{\pi}{2}$$
$$= \frac{\pi}{2}-1 + \frac{1}{4}\left[\left(\frac{3\pi}{4}+0\right)-\left(\frac{\pi}{2}+0\right)\right] + \frac{\pi}{2} = \frac{\pi}{2}-1+\frac{\pi}{16}+\frac{\pi}{2} = \frac{17\pi}{16}-1$$

(e) Distance $= 2+\cos(2\theta)-2 = \cos(2\theta)$

$$\frac{d}{d\theta}\left[\cos(2\theta)\right] = -2\sin(2\theta)$$

$$-2\sin(2\cdot\pi/6) = -2\cdot\frac{\sqrt{3}}{2} = -\sqrt{3}$$

8. (a) $\frac{1}{2}\int_{\pi/2}^{\pi} \left[2+2\cos(\theta)\right]^2 d\theta$

(b) $x(\theta) = r\cdot\cos\theta = (2+2\cos\theta)\cos\theta \qquad y(\theta) = r\cdot\sin\theta = (2+2\cos\theta)\sin\theta$

$\frac{dx}{d\theta} = (2+2\cos\theta)\cdot -\sin\theta + \cos\theta(-2\sin\theta)$

$= -2\sin\theta - 2\sin\theta\cos\theta - 2\sin\theta\cos\theta$

$= -2\sin\theta - 4\sin\theta\cos\theta$

$\frac{dy}{d\theta} = (2+2\cos\theta)\cdot\cos\theta + \sin\theta(-2\sin\theta)$

$= 2\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

(c) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos\theta + 2\cos^2\theta - 2\sin^2\theta}{-2\sin\theta - 4\sin\theta\cos\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\pi/2} = \frac{2(0)+2(0)^2-2(1)^2}{-2(1)-4(1)(0)} = \frac{-2}{-2} = 1$

$x(\pi/2) = (2+2(0))\cdot 0 = 0$

$y(\pi/2) = (2+2(0))\cdot 1 = 2$

$y - 2 = x$