Motion Along a Curve Vectors :

1. $v(2) = \langle \frac{9}{14}, 12 \rangle$

2. $a(1) = \langle 20, 24 \rangle$

3. $v(\frac{\pi}{2}) = \langle -3, 3\pi \rangle$

4. $v(t) = \langle 0, 0 \rangle \Rightarrow t = 3$

5. Position $\Big|_{t=2} = (9, 4)$

6. $5t + 3\sin t = 25 \Rightarrow t = 5.446$

   $v(5.446) = \langle 7.009, -2.227 \rangle$

7. (a) $\sqrt{(2t)^2 + (2t^2)^2}\Big|_{t=2} = 4\sqrt{5}$

   (b) Distance $= \int_0^4 \sqrt{4t^2 + 4t^4}\, dt$

   (c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t = \sqrt{x+2}$

   (d) Particle on y-axis when $t = \sqrt{2}$, and $\frac{dy}{dx} = 2t - 1$

8. (a) $x(t) = \int \frac{1}{t+1}\, dt = \ln(t+1) + C$. Since $x(1) = \ln 2$, $C = 0$.

   $y(t) = \int 2t\, dt = t^2 + C$. Since $y(1) = 4$, $C = 3$

   Position $= \langle \ln(t+1), t^2 + 3 \rangle$

   (b) When $t = 1$, $\frac{dy}{dx} = \frac{2}{\frac{1}{2}} = 4$. Tangent: $y - 4 = 4(x - \ln 2)$

   (c) Magnitude $= \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2}\Big|_{t=1} = \frac{\sqrt{17}}{2}$

   (d) $\frac{dy}{dx} = \frac{2t}{\frac{1}{t+1}} = 2t(t+1)$. $2t(t+1) = 12 \Rightarrow t = 2$

9. $t^2 + 2\cos t = 7 \Rightarrow t = 2.996 \quad v(2.996) = \langle -0.968, 5.704 \rangle$

10. $\frac{dy}{dt} = \frac{dx}{dt}(t+3) = (1+\sin t^3)(t+3) \quad a(2) = \langle -1.746, -6.741 \rangle$

11. (a) $\frac{dy}{dx} = \frac{\sin(e^t)}{\cos(e^t)}\Bigg|_{t=1} = -0.451$ Tangent: $y-2 = -0.451(x-3)$

(b) Speed $= \sqrt{[\cos(e^t)]^2 + [\sin(e^t)]^2}\,\Bigg|_{t=1} = 1$

(c) Distance $= \int_0^2 \sqrt{[\cos(e^t)]^2 + [\sin(e^t)]^2}\,dt = 2$

(d) $x(2) = 3 + \int_1^2 \cos(e^t)\,dt \quad y(2) = 2 + \int_1^2 \sin(e^t)\,dt$

Position : $\langle 2.896, 1.676 \rangle$

12. (a) Magnitude $= \sqrt{(t-2)^4 + (2t-4)^2}\,\Bigg|_{t=1} = \sqrt{5}$

(b) Distance $= \int_0^1 \sqrt{(t-2)^4 + (2t-4)^2}\,dt = 3.816$

(c) Rest $\Rightarrow v(t) = \langle (t-2)^2, 2t-4 \rangle = \langle 0, 0 \rangle \Rightarrow t = 2$

Position $= \langle 4, 0 \rangle$