Calculus BC

Motion Along a Curve Vectors Worksheet

1. If a particle moves in the xy-plane so that at any time $t > 0$, its position vector is $\langle \ln(t^2 + 5t), 3t^2 \rangle$, find its velocity vector at time $t = 2$.

2. A particle moves in the xy-plane so that at any time t, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.

3. If a particle moves in the xy-plane so that at time t its position vector is $\left\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \frac{\pi}{2}$.

4. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?

5. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1 + t, t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, find the position of the particle at $t = 2$.

6. A particle moves in the xy-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$. Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

7. The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$, $y(t) = \frac{2}{3}t^3$.

   (a) Find the magnitude of the velocity vector at $t = 2$.

   (b) Set up an integral expression to find the total distance traveled by the particle from $t = 0$ to $t = 4$.

   (c) Find $\frac{dy}{dx}$ as a function of x.

   (d) At what time t is the particle on the y-axis? Find the acceleration vector at this time.

8. An object moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time t with the velocity vector $v(t) = \left(\frac{1}{t+1}, 2t\right)$. At time $t = 1$, the object is at (ln 2, 4).

(a) Find the position vector.

(b) Write an equation for the line tangent to the curve when $t = 1$.

(c) Find the magnitude of the velocity vector when $t = 1$.

(d) At what time $t > 0$ does the line tangent to the particle at $(x(t), y(t))$ have a slope of 12?

9. A particle moving along a curve in the *xy*-plane has position $(x(t), y(t))$, with $x(t) = 2t + 3\sin t$ and $y(t) = t^2 + 2\cos t$, where $0 \le t \le 10$. Find the velocity vector at the time when the particle's vertical position is $y = 7$.

10. A particle moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = 1 + \sin(t^3)$. The derivative $\frac{dy}{dt}$ is not explicitly given. For any time t, $t \ge 0$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $t + 3$. Find the acceleration vector of the object at time $t = 2$.

11. An object moving along a curve in the *xy*-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = \cos(e^t)$ and $\frac{dy}{dt} = \sin(e^t)$ for $0 \le t \le 2$. At time $t = 1$, the object is at the point (3, 2).

(a) Find the equation of the tangent line to the curve at the point where $t = 1$.

(b) Find the speed of the object at $t = 1$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 2$.

(d) Find the position of the object at time $t = 2$.

12. The position of a particle at time $t \ge 0$ is given by the parametric equations $x(t) = \frac{(t-2)^3}{3} + 4$ and $y(t) = t^2 - 4t + 4$.

(a) Find the magnitude of the velocity vector at $t = 1$.

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.

(c) When is the particle at rest? What is its position at that time?