Limits and Continuity

Rate of Change

1. The traffic flow at a particular intersection is modeled by the function f defined by $f(t) = 25 + 6\cos(\frac{x}{3})$ for $0 \le t \le 120$. What is the average rate of change of the traffic flow over the time interval $30 \le t \le 40$.

(A) 0.743 (B) 0.851 (C) 0.935 (D) 1.176

2. The rate of change of the altitude of a hot air balloon rising from the ground is given by $y(t) = t^3 - 3t^2 + 3t$ for $0 \le t \le 10$. What is the average rate of change in altitude of the balloon over the time interval $0 \le t \le 10$.

(A) 56 (B) 73 (C) 85 (D) 94

Free Response Questions

t (sec)	0	10	20	30	40	50	60	70	80	90
$f(t)$ (ft/sec)	0	28	43	67	82	85	74	58	42	35

3. The table above shows the velocity of a car moving on a straight road. The car's velocity v is measured in feet per second.

(a) Find the average velocity of the car from $t = 60$ to $t = 90$.

(b) The instantaneous rate of change of f (See Ch. 2.1 for an explanation of instantaneous rate of change) with respect to x at $x = a$ can be approximated by finding the average rate of change of f near $x = a$. Approximate the instantaneous rate of change of f at $x = 40$ using two points, $x = 30$ and $x = 50$.

Limit of a Function and One Sided Limits

1. $\lim_{x \to \frac{\pi}{6}} \sec^2 x =$

(A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{4}{3}$ (D) $\frac{2\sqrt{3}}{3}$

2. If $f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 1, & x = 1 \end{cases}$, then $\lim_{x \to 1} f(x) =$

(A) 1 (B) 2 (C) 3 (D) 4

3. $\lim_{x \to 1} \frac{|x-1|}{1-x} =$

(A) -2 (B) -1 (C) 1 (D) nonexistent

4. Let f be a function given by $f(x) = \begin{cases} 3 - x^2, & \text{if } x < 0 \\ 2 - x, & \text{if } 0 \leq x < 2 \\ \sqrt{x-2}, & \text{if } x > 2 \end{cases}$.

Which of the following statements are true about f ?

I. $\lim_{x \to 0} f(x) = 2$

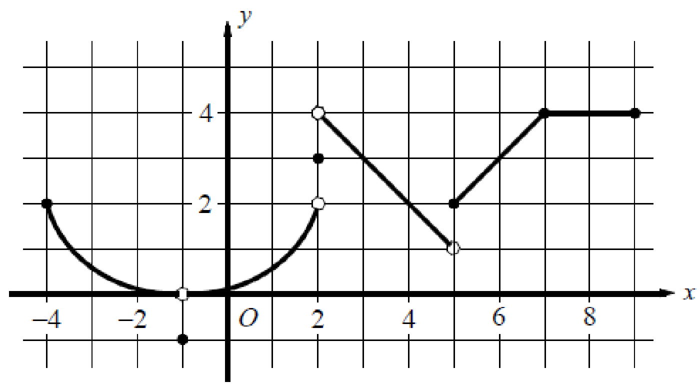
II. $\lim_{x \to 2} f(x) = 0$

III. $\lim_{x \to 1} f(x) = \lim_{x \to 6} f(x)$

(A) I only (B) II only (C) II and III only (D) I , II, and III

Free Response Questions

Questions 5-11 refer to the following graph.



The figure above shows the graph of $y = f(x)$ on the closed interval $[-4, 9]$.

5. Find $\lim_{x \to -1} \cos(f(x))$.

6. Find $\lim_{x \to 2^-} f(x)$.

7. Find $\lim_{x \to 2^+} f(x)$.

8. Find $\lim_{x \to 2} f(x)$.

9. Find $f(2)$.

10. Find $\lim_{x \to 5^-} \arctan(f(x))$.

11. Find $\lim_{x \to 5^+} [x f(x)]$.

Calculating Limits Using the Limit Laws

1. $\lim_{x \to \pi/3} \frac{\sin(\frac{\pi}{3} - x)}{\frac{\pi}{3} - x} =$

(A) -1 (B) 0 (C) $\frac{\sqrt{3}}{2}$ (D) 1

2. $\lim_{x \to 0} \frac{\sin 3x}{\sin 2x} =$

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{2}$ (D) nonexistent

3. $\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x} =$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) nonexistent

4. $\lim_{x \to 1} \frac{\sqrt{3+x} - 2}{x^3 - 1} =$

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\sqrt{3}$ (D) nonexistent

5. $\lim_{\theta \to 0} \frac{\theta + \theta \cos\theta}{\sin\theta \cos\theta} =$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

6. $\lim_{x \to 0} \frac{\tan 3x}{x} =$

(A) 0 (B) $\frac{1}{3}$ (C) 1 (D) 3

7. $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} =$

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) -9 (D) 9

Free Response Questions

8. If $\lim_{x \to 0} \frac{\sqrt{2+ax} - \sqrt{2}}{x} = \sqrt{2}$ what is the value of a ?

9. Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, if $f(x) = \sqrt{2x+1}$.

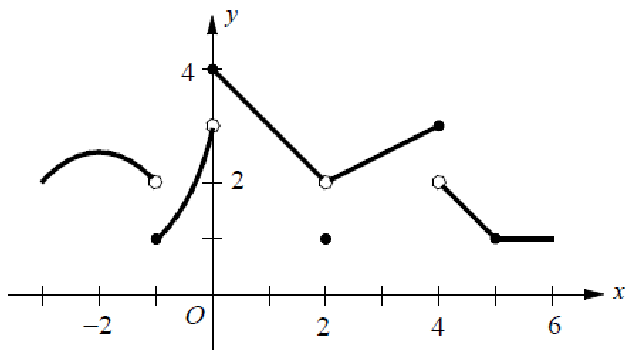
10. Find $\lim_{x \to 0} \frac{f(x) - g(x)}{\sqrt{g(x) + 7}}$, if $\lim_{x \to 0} f(x) = 2$ and $\lim_{x \to 0} g(x) = -3$.

11. Find $\lim_{x \to \sqrt{3}} g(x)$, if $\lim_{x \to \sqrt{3}} \frac{1}{x^2 + g(x)} = \frac{1}{5}$.

Continuity and Intermediate Value Theorem

1. Let f be a function defined by $f(x)=\begin{cases}\dfrac{x^2-a^2}{x-a}, & \text{if } x\neq a\\ 4, & \text{if } x=a\end{cases}$. If f is continuous for all real numbers x, what is the value of a?

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2



2. The graph of a function f is shown above. If $\lim_{x\to a} f(x)$ exists and f is not continuous at $x=a$, then $a=$

(A) -1 (B) 0 (C) 2 (D) 4

3. If $f(x)=\begin{cases}\dfrac{\sqrt{3x-1}-\sqrt{2x}}{x-1}, & \text{for } x\neq 1\\ a, & \text{for } x=1\end{cases}$, and if f is continuous at $x=1$, then $a=$

(A) $\frac{1}{4}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\sqrt{2}$ (D) 2

4. Let f be a continuous function on the closed interval $[-2,7]$. If $f(-2)=5$ and $f(7)=-3$, then the Intermediate Value Theorem guarantees that

(A) $f'(c)=0$ for at least one c between -2 and 7

(B) $f'(c)=0$ for at least one c between -3 and 5

(C) $f(c)=0$ for at least one c between -3 and 5

(D) $f(c)=0$ for at least one c between -2 and 7

Free Response Questions

5. Let g be a function defined by $g(x)=\begin{cases} \dfrac{\pi \sin x}{x}, & \text{if } x<0 \\ a-bx, & \text{if } 0\le x<1 \\ \arctan x, & \text{if } x\ge 1 \end{cases}$.

   If g is continuous for all real numbers x, what are the values of a and b?

6. Evaluate $\lim\limits_{a\to 0}\dfrac{-1+\sqrt{1+a}}{a}$.

7. What is the value of a, if $\lim\limits_{x\to 0}\dfrac{\sqrt{ax+9}-3}{x}=1$?

Limits and Asymptotes

1. $\lim_{x \to \infty} \frac{3+2x^2-x^4}{3x^4-5} =$

(A) -2 (B) $-\frac{1}{3}$ (C) $\frac{1}{5}$ (D) 1

2. What is $\lim_{x \to -\infty} \frac{x^3+x-8}{2x^3+3x-1} =$

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 2

3. Which of the following lines is an asymptote of the graph of $f(x) = \frac{x^2+5x+6}{x^2-x-12}$?

I. $x = -3$

II. $x = 4$

III. $y = 1$

(A) II only (B) III only (C) II and III only (D) I, II, and III

4. If the horizontal line $y = 1$ is an asymptote for the graph of the function f, which of the following statements must be true?

(A) $\lim_{x \to \infty} f(x) = 1$

(B) $\lim_{x \to 1} f(x) = \infty$

(C) $f(1)$ is undefined

(D) $f(x) = 1$ for all x

5. If $x = 1$ is the vertical asymptote and $y = -3$ is the horizontal asymptote for the graph of the function f, which of the following could be the equation of the curve?

(A) $f(x) = \dfrac{-3x^2}{x-1}$

(B) $f(x) = \dfrac{-3(x-1)}{x+3}$

(C) $f(x) = \dfrac{-3(x^2-1)}{x-1}$

(D) $f(x) = \dfrac{-3(x^2-1)}{(x-1)^2}$

6. What are all horizontal asymptotes of the graph of $y = \dfrac{6+3e^x}{3-3e^x}$ in the xy-plane?

(A) $y = -1$ only

(B) $y = 2$ only

(C) $y = -1$ and $y = 2$

(D) $y = 0$ and $y = 2$

Free Response Questions

7. Let $f(x) = \dfrac{3x-1}{x^3-8}$.

(a) Find the vertical asymptote(s) of f. Show the work that leads to your answer.

(b) Find the horizontal asymptote(s) of f. Show the work that leads to your answer.

8. Let $f(x) = \dfrac{\sin x}{x^2+2x}$.

(a) Find the vertical asymptote(s) of f. Show the work that leads to your answer.

(b) Find the horizontal asymptote(s) of f. Show the work that leads to your answer.