Calculus AB

Left and Right Riemann Sums

1. a) Approximate the area under the graph of $f(x)=\frac{1}{x}$ from $x=1$ to $x=5$ using the **right endpoints** of four subintervals of equal length. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?
    b) Repeat part a) using **left endpoints**.

2. a) Approximate the area under the graph of $f(x)=x^2+1$ from $x=-1$ to $x=2$ using the **right endpoints** of three subintervals of equal length. Sketch the graph and the rectangles.
    b) Improve your estimate by using six subintervals.
    c) Repeat parts a) and b) using **left endpoints**.

3. a) Approximate the area under the graph of the function shown to the right from $x=-2$ to $x=3$ using the **right endpoints** of five subintervals of equal length.
    b) Repeat part a) using **left endpoints**.
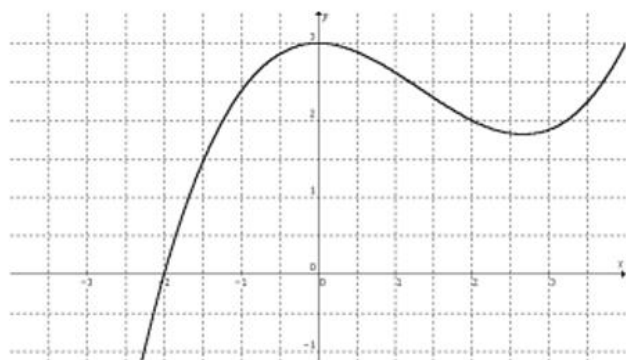
For each problem, use a left-hand Riemann sum to approximate the area under the curve based off of the values in the table.

4.

x	0	2	4	6	9
$f(x)$	6	8	7	8	7

5.

x	0	3	4	5	6	11	14
$f(x)$	−1	−2	−1	0	1	2	3

For each problem, use a right-hand Riemann sum to approximate the area under the curve based off of the values in the table.
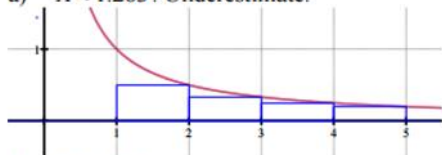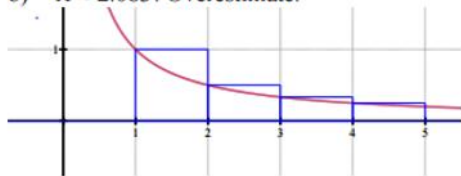
6.

x	0	3	8	12	17	18
$f(x)$	3	2	4	6	5	8

7.

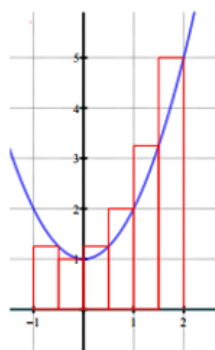| x	0	2	5	26	27
$f(x)$	−11	−9	−4	−1	3

Answers

1. a) $A \approx 1.283$. Underestimate.
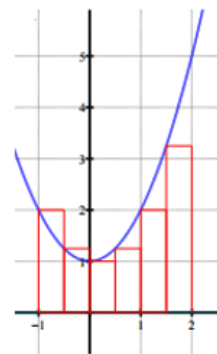

b) $A \approx 2.083$. Overestimate.

2.

a) $A \approx 8$

b) $A \approx 6.875$

c)

3 subintervals:
$A \approx 5$

6 subintervals:
$A \approx 5.375$

3. a) $A \approx 11.7$
   b) $A \approx 10$

4. $A \approx 66$

5. $A \approx 5$

6. $A \approx 83$

7. $A \approx -48$