Calculus BC

Lagrange Remainder Worksheet

Use the Remainder Theorem to bound the error involved in using the specific Taylor polynomial, centered at 0, to approximate *f(x)* at the given value.

1. $P_5(x)$ for $f(x) = \cos x$ at $x = 0.2$

2. $P_4(x)$ for $f(x) = e^x$ at $x = 0.8$

Use the Remainder Theorem to bound the error involved in using the specific Taylor polynomial, at the given center, to approximate *f(x)* at the given value.

3. $P_2(x)$ for $f(x) = x^{5/2}$, centered at 1. Approximate *f(1.7)*.

4. $P_3(x)$ for $f(x) = \frac{1}{1-x}$, centered at 2. Approximate *f(2.4)*.

Determine the degree of the Taylor polynomial, centered at 0, that would be required to approximate the function at the given point to within the stated accuracy.

5. $f(x) = x\ln(1+x)$, at $x = -0.2$, within $1/100$

6. $f(x) = e^{2x}$, at $x = 0.5$, within $1/100$

2004 BC6 parts (a) and (c)

Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree taylor polynomial for f about $x = 0$.

(a) Find $P(x)$.

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(b) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right)\right| < \frac{1}{100}$.

1999 BC4 parts (a) and (b)

The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

(b) The fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \ne -5$.