Integration Techniques:

1. $\displaystyle\int \frac{1+\sin x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\,dx = \int \sec^2 x + \tan x \sec x\,dx$ $\boxed{B}$

$= \tan x + \sec x + C$

2. $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx = \int \frac{e^x \cdot e^x}{1+e^x}\,dx = \int \frac{u-1}{u}\,du = \int 1 - \frac{1}{u}\,du$ $\boxed{D}$

$u = 1+e^x \longrightarrow e^x = u-1$

$du = e^x \cdot dx$

$= u - \ln|u| + C$

$= 1+e^x - \ln|1+e^x| + C$

$= e^x - \ln|1+e^x| + C$

3. $\displaystyle\int 2\tan x \cdot \ln(\cos x)\,dx = 2\int \ln(\cos x) \cdot \frac{\sin x}{\cos x}\,dx = -2\int u\,du$ $\boxed{C}$

$u = \ln(\cos x)$

$du = \frac{1}{\cos x} \cdot -\sin x\,dx$

$-du = \frac{\sin x}{\cos x}\,dx$

$= -2 \cdot \frac{1}{2} u^2 + C$

$= -(\ln(\cos x))^2 + C$

4. OMIT

5. OMIT

6. $\pi \int_0^{\pi/4} \left[\frac{\sin x}{\sqrt{\cos x}}\right]^2 dx = \pi \int_0^{\pi/4} \frac{\sin^2 x}{\cos x}\, dx$

$= \pi \int_0^{\pi/4} \sec x \tan x \, dx$

$= \pi \Big[\sec x\Big]_0^{\pi/4}$

$= \pi \left[\sqrt{2} - 1\right]$

Trigonometric Substitutions:

1. $\int \frac{x^3}{\sqrt{x^2+4}}\, dx$

$x = 2\tan\theta$

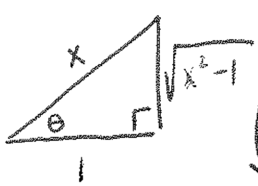
$dx = 2\sec^2\theta \, d\theta$

$\boxed{D}$

$\int \frac{8\tan^3\theta}{\sqrt{4\tan^2\theta + 4}} \cdot 2\sec^2\theta \, d\theta = \int \frac{16\tan^3\theta}{2\sqrt{\sec^2\theta}} \cdot \sec^2\theta \, d\theta = 8\int \tan^3\theta \cdot \sec\theta \, d\theta$

2. $\int_{\sqrt{2}}^{2} \frac{1}{x\sqrt{x^2-1}}\, dx = \int_{\pi/4}^{\pi/3} \frac{1}{\sec\theta\sqrt{\sec^2\theta - 1}} \cdot \sec\theta \tan\theta \, d\theta$

$\boxed{B}$



$\boxed{x = \sec\theta}$

$dx = \sec\theta \tan\theta \, d\theta$

$2 = \sec\theta \to \theta = \pi/3$

$\sqrt{2} = \sec\theta \to \theta = \pi/4$

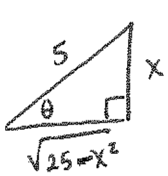
$= \int_{\pi/4}^{\pi/3} \frac{1}{\sec\theta \cdot \sqrt{\tan^2\theta}} \cdot \sec\theta \tan\theta \, d\theta$

$= \int_{\pi/4}^{\pi/3} d\theta$

$= \Big[\theta\Big]_{\pi/4}^{\pi/3} = \pi/3 - \pi/4 = \frac{\pi}{12}$

3. $\int \frac{1}{x^2\sqrt{25 - x^2}}\, dx = \int \frac{1}{25\sin^2\theta \sqrt{25 - 25\sin^2\theta}} \cdot 5\cos\theta \, d\theta$

$\boxed{C}$



$x = 5\sin\theta$

$dx = 5\cos\theta \, d\theta$

$= \int \frac{1}{25\sin^2\theta\sqrt{1-\sin^2\theta}} \cdot \cos\theta \, d\theta$

$= \int \frac{1}{25\sin^2\theta\sqrt{\cos^2\theta}} \cdot \cos\theta \, d\theta$

$= \int \frac{1}{25} \cdot \csc^2\theta \, d\theta = \frac{-1}{25}\cot\theta + C = \frac{-\sqrt{25-x^2}}{25x} + C$

4. $\int \frac{\sqrt{x^2-1}}{x^4}\,dx = \int \frac{\sqrt{\sec^2\theta - 1}}{\sec^4\theta} \cdot \sec\theta \tan\theta \, d\theta$ $\boxed{D}$

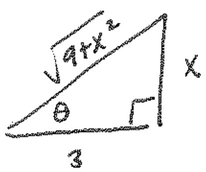
$x = \sec\theta$

$dx = \sec\theta \tan\theta \, d\theta$

$$= \int \frac{\sqrt{\tan^2\theta}}{\sec^4\theta} \cdot \sec\theta \tan\theta \, d\theta$$

$$= \int \frac{\tan^2\theta}{\sec^3\theta}\, d\theta = \int \sin^2\theta \cos\theta \, d\theta$$

5. $\frac{1}{4}\int_0^4 \frac{4}{\sqrt{9+x^2}}\,dx = \frac{1}{4}\int \frac{4}{\sqrt{9+9\tan^2\theta}} \cdot 3\sec^2\theta \, d\theta$ $\boxed{C}$



$x = 3\tan\theta$

$dx = 3\sec^2\theta \, d\theta$

$$= \frac{1}{3}\int \frac{3\sec^2\theta}{\sqrt{1+\tan^2\theta}}\, d\theta$$

$$= \int \frac{\sec^2\theta}{\sqrt{\sec^2\theta}}\, d\theta$$

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta|$$

$$\left[\ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right|\right]_0^4$$

$\ln|3| - \ln|1|$

$\ln 3$

6. (a) $\frac{1}{3}\int_0^3 (9-x^2)^{3/2}\,dx = 15.904$

(b) $\frac{1}{3}\int_0^{\pi/2} (9-9\sin^2\theta)^{3/2} \cdot 3\cos\theta \cdot d\theta = \frac{1}{3}\int_0^{\pi/2} 9^{3/2} \cdot (1-\sin^2\theta)^{3/2} \cdot 3\cos\theta \, d\theta$

$$= 27\int_0^{\pi/2} (\cos^2\theta)^{3/2} \cos\theta \, d\theta$$

$$= 27\int_0^{\pi/2} \cos^4\theta \, d\theta$$

$x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$

$0 = 3\sin\theta \rightarrow \theta = 0$

$3 = 3\sin\theta \rightarrow \theta = \pi/2$

Partial Fractions:

1. $$\int \frac{dx}{x^2+x-6}\,dx = \int \frac{1}{(x+3)(x-2)}\,dx = \frac{1}{5}\int \frac{1}{x-2} - \frac{1}{x+3}\,dx$$ $\boxed{C}$

$$\frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$= \frac{1}{5}\left[\ln|x-2| - \ln|x+3|\right] + C$$

$$1 = (A+B)x + (3B-2A)$$

$$= \frac{1}{5}\ln\left|\frac{x-2}{x+3}\right| + C$$

$A+B=0$ $\quad$ $3B-2A=1$

$A=-1/5$ $\quad$ $B=1/5$

2. $$\int_4^7 \frac{5}{(x-2)(2x+1)}\,dx = \int_4^7 \frac{1}{x-2} - \frac{2}{2x+1}\,dx = \Big[\ln|x-2| - \ln|2x+1|\Big]_4^7$$ $\boxed{C}$

$$= \left[\ln\left|\frac{x-2}{2x+1}\right|\right]_4^7$$

$$\frac{5}{(x-2)(2x+1)} = \frac{A}{x-2} + \frac{B}{2x+1}$$

$$5 = (2A+B)x + (A-2B)$$

$$= \ln\left|\frac{5}{15}\right| - \ln\left|\frac{2}{9}\right|$$

$2A+B=0$

$-2(A-2B=5)$

$5B=-10$ $\quad$ $A=1$

$B=-2$

$$= \ln\left|\frac{1/3}{2/9}\right| = \ln\left|\frac{3}{2}\right|$$

3. $$\int \frac{x}{x^2+5x+6}\,dx = \int \frac{x}{(x+2)(x+3)}\,dx = \int \frac{3}{x+3} - \frac{2}{x+2}\,dx$$ $\boxed{A}$

$$\frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3}$$

$$= 3\ln|x+3| - 2\ln|x+2| + C$$

$$x = (A+B)x + (3A+2B)$$

$A+B=1$ $\quad$ $3A+2B=0$

$-2(A+B=1)$

$B=3$ $\quad$ $A=-2$

4. $\displaystyle\int \frac{2e^{2x}}{(e^x-1)(e^x+1)}\,dx = \int \frac{2u}{(u-1)(u+1)}\,du = \int \frac{1}{u-1} + \frac{1}{u+1}\,du$ $\boxed{D}$

$u = e^x$

$du = e^x dx$

$$\frac{2u}{(u-1)(u+1)} = \frac{A}{u-1} + \frac{B}{u+1}$$

$$2u = (A+B)u + (A-B)$$

$$A+B=2 \quad A-B=0$$

$$A=1 \quad B=1$$

$$= \ln|u-1| + \ln|u+1| + C$$

$$= \ln|e^x-1| + \ln|e^x+1| + C$$

5. (a) $\displaystyle\int \frac{\sin\theta}{\cos\theta(\cos\theta-1)}\,d\theta = -\int \frac{1}{x(x-1)}\,dx$

$x = \cos\theta$

$dx = -\sin\theta\, d\theta$

(b) $\displaystyle\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$$1 = (A+B)x + (-A)$$

$$-A=1 \quad A+B=0$$

$$A=-1 \quad B=1$$

$$-\int \frac{1}{x(x-1)}\,dx = \int \frac{1}{x} - \frac{1}{x-1}\,dx$$

$$= \ln|x| - \ln|x-1| + C$$

$$= \ln|\cos\theta| - \ln|\cos\theta-1| + C$$

$$= \ln\left|\frac{\cos\theta}{\cos\theta-1}\right| + C$$

Integration by Parts:

1. $\displaystyle\int x\sin(2x)\,dx = -\frac{x}{2}\cos(2x) + \frac{1}{2}\int\cos(2x)\,dx$ $\boxed{C}$

$u=x \quad dv = \sin(2x)\,dx$

$du = dx \quad v = -\frac{1}{2}\cos(2x)$

$$= -\frac{x}{2}\cos(2x) + \frac{1}{2}\left[\frac{1}{2}\sin(2x)\right] + C$$

$$= -\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C$$

2. $\int_0^2 x e^x\,dx = xe^x - \int e^x\,dx = \left[xe^x - e^x\right]_0^2$ $\boxed{B}$

$u = x \quad dv = e^x\,dx$

$du = dx \quad v = e^x$

$= (2e^2 - e^2) - (0 - 1)$

$= e^2 + 1$

3. $\int x^2 \cos(3x)\,dx = \frac{x^2}{3}\sin(3x) - \frac{2}{3}\int x\sin(3x)\,dx$ $\boxed{B}$

$u = x^2 \quad dv = \cos(3x)\,dx$

$du = 2x\,dx \quad v = \frac{1}{3}\sin(3x)$

4. $\int x^2 \ln x\,dx = \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$ $\boxed{C}$

$u = \ln x \quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx \quad v = \frac{1}{3}x^3$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

5. $\int_0^{\pi/4} x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$ $\boxed{C}$

$u = x \quad dv = \sec^2 x\,dx$

$du = dx \quad v = \tan x$

$= \left[x\tan x + \ln|\cos x|\right]_0^{\pi/4} = \left(\frac{\pi}{4} + \ln\left|\frac{\sqrt{2}}{2}\right|\right) - (0)$

$= \frac{\pi}{4} + \ln\sqrt{2} - \ln 2$

$= \frac{\pi}{4} + \frac{1}{2}\ln 2 - \ln 2 = \frac{\pi}{4} - \frac{\ln 2}{2}$

6. $\int \sec^3 x\,dx = \int \sec^2 x \cdot \sec x\,dx = \sec x\tan x - \int \sec x\tan^2 x\,dx$ $\boxed{D}$

$u = \sec x \quad dv = \sec^2 x\,dx$

$du = \sec x\tan x\,dx \quad v = \tan x$

$= \sec x\tan x - \int \sec x(\sec^2 x - 1)\,dx$

$= \sec x\tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$\int \sec^3 x\,dx = \sec x\tan x + \ln|\sec x + \tan x| - \int \sec^3 x\,dx$

$2\int \sec^3 x\,dx = \sec x\tan x + \ln|\sec x + \tan x| + C$

7. $\int f(x)\cos(nx)\,dx$ $= \frac{1}{n}f(x)\sin(nx) - \frac{1}{n}\int f'(x)\sin(nx) + C$ $\boxed{A}$

$u = f(x)$ $dv = \cos(nx)\,dx$

$du = f'(x)\,dx$ $v = \frac{1}{n}\sin(nx)$

8. $\int \arccos x\,dx$ $= x\arccos x + \int \frac{x}{\sqrt{1-x^2}}\,dx$ $\boxed{D}$

$u = \arccos x$ $dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}}\,dx$ $v = x$

9. $\int_1^3 f(x)\cdot g'(x)\,dx$ $= \left[f(x)\cdot g(x)\right]_1^3 - \int_1^3 f'(x)g(x)\,dx = 8$ $\boxed{A}$

$u = f(x)$ $dv = g'(x)\,dx$

$du = f'(x)\,dx$ $v = g(x)$

$\int_1^3 f'(x)g(x)\,dx = \left[f(x)\cdot g(x)\right]_1^3 - 8$

$= f(3)\cdot g(3) - f(1)\cdot g(1) - 8$

$= -2 + 6 - 8 = -4$

10. $\int_0^1 \arcsin x\,dx$ $= \left[x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx\right]_0^1$

$u = \arcsin x$ $dv = dx$

$du = \frac{1}{\sqrt{1-x^2}}$ $v = x$

$w = 1-x^2$

$dw = -2x\,dx$

$-\frac{1}{2}dw = x\,dx$

$= x\arcsin x + \frac{1}{2}\int u^{-1/2}\,du$

$= x\arcsin x + u^{1/2}$

$= \left[x\arcsin x + \sqrt{1-x^2}\right]_0^1$

$= \left(\frac{\pi}{2} + 0\right) - \left(1\right) = \frac{\pi}{2} - 1$

Improper Integrals:

1. $$\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx = \lim_{b\to\infty}\int_2^b (x-1)^{-1/2}\,dx$$

$$= \lim_{b\to\infty}\left[2(x-1)^{1/2}\right]_2^b$$

$$= \lim_{b\to\infty}\left[2(b-1)^{1/2} - 2\right] = \infty$$

$\boxed{D}$

2. $$\int_0^\infty \frac{1}{(x+3)(x+4)}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{x+3} - \frac{1}{x+4}\,dx$$

$$\frac{1}{(x+3)(x+4)} = \frac{A}{x+3} + \frac{B}{x+4}$$

$$1 = (A+B)x + (4A+3B)$$

$$4A+3B=1$$
$$-3(A+B=0)$$
$$A=1 \quad B=-1$$

$$= \lim_{b\to\infty}\left[\ln|x+3| - \ln|x+4|\right]_0^b$$

$$= \lim_{b\to\infty}\left[\ln\left|\frac{b+3}{b+4}\right| - \ln\left|\frac{3}{4}\right|\right] =$$

$\boxed{B}$

3. $$\int_0^4 \frac{dx}{(x-1)^{2/3}} = \lim_{b\to 1^-}\int_0^b \frac{dx}{(x-1)^{2/3}} + \lim_{a\to 1^+}\int_a^4 \frac{dx}{(x-1)^{2/3}}$$

$\to x \neq 1$

$$= \lim_{b\to 1^-}\left[3(x-1)^{1/3}\right]_0^b + \lim_{a\to 1^+}\left[3(x-1)^{1/3}\right]_a^4$$

$$= \lim_{b\to 1^-}\left[3(b-1)^{1/3} + 3\right] + \lim_{a\to 1^+}\left[3\sqrt{3} - 3(a-1)^{1/3}\right]$$

$$= 3 + 3\sqrt{3}$$

$\boxed{C}$

4. $\int_0^\infty x^2 e^{-x^3}\,dx = \lim_{b\to\infty} -\frac{1}{3}\int e^u\,du$

$u = -x^3$

$du = -3x^2\,dx$

$= \lim_{b\to\infty} -\frac{1}{3}\left[e^{-x^3}\right]_0^b$

$= \lim_{b\to\infty} -\frac{1}{3}\left[e^{-b^3} - e^0\right] = \frac{1}{3}$

$\boxed{A}$

5. $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to 0^+}\int_a^1 x^{-1/2}\cdot\ln x\,dx = \lim_{a\to 0^+}\left[2x^{1/2}\ln x - 2\int x^{-1/2}\,dx\right]_a^1$

$x \neq 0$

$u = \ln x \quad dv = x^{-1/2}\,dx$

$du = \frac{1}{x}\,dx \quad v = 2x^{1/2}$

$= \lim_{a\to 0^+}\left[2x^{1/2}\ln x - 4x^{1/2}\right]_a^1$

$= \lim_{a\to 0^+}\left[2\sqrt{x}(\ln x - 2)\right]_a^1$

$= \lim_{a\to 0^+}\left[(-4) - (2\sqrt{a}(\ln a - 2))\right] = -4$

$\boxed{B}$

consider:

$\lim_{a\to 0} 2\sqrt{a}\cdot\ln a = \lim_{a\to 0}\frac{2\ln a}{a^{-1/2}}$

L'Hops $= \lim_{a\to 0}\frac{2\cdot\frac{1}{a}}{-\frac{1}{2}a^{-3/2}}$

$= \lim_{a\to 0} -4a^{1/2} = 0$

6. $k\int_0^1 e^{-x^{1/2}}\cdot x^{-1/2}\,dx = k\cdot\lim_{a\to 0^+}\int_a^1 e^{-x^{1/2}}\cdot x^{-1/2}\,dx = -2k\cdot\lim_{a\to 0^+}\int e^u\,du$

$x \neq 0$

$u = -x^{1/2}$

$du = -\frac{1}{2}x^{-1/2}\,dx$

$-2\,du = x^{-1/2}\,dx$

$= -2k\cdot\lim_{a\to 0^+}\left[e^{-\sqrt{x}}\right]_a^1$

$= -2k\cdot\lim_{a\to 0^+}\left[e^{-1} - e^{-\sqrt{a}}\right]$

$= -2k\left(\frac{1}{e} - 1\right)$

CORRECT ANSWER CHOICE NOT AVAILABLE

$-2k\left(\frac{1}{e} - 1\right) = 1$

$-2k\left(\frac{1-e}{e}\right) = 1$

$k\left(\frac{2e-2}{e}\right) = 1$

$k = \frac{e}{2e-2}$

7. (a) $\int_1^\infty \frac{x}{\sqrt{x^2+1}}\,dx = \lim_{b\to\infty} \int_1^b \frac{x}{\sqrt{x^2+1}}\,dx = \lim_{b\to\infty} \frac{1}{2}\int_2^{b^2+1} \frac{1}{\sqrt{u}}\,du$

$u = x^2+1 \quad u = b^2+1$

$du = 2x\,dx \quad u = 2$

$\frac{1}{2}du = x\,dx$

$= \lim_{b\to\infty} \left[u^{1/2}\right]_2^{b^2+1}$

$= \lim_{b\to\infty} \left[\sqrt{b^2+1} - \sqrt{2}\right]$

$= \infty - \sqrt{2} = \infty$

(b) $\lim_{b\to\infty} \left[\frac{1}{b-1}\int_1^b \frac{x}{\sqrt{x^2+1}}\,dx\right]$

$\lim_{b\to\infty}\left[\frac{1}{b-1}\left[\sqrt{b^2+1} - \sqrt{2}\right]\right] = \lim_{b\to\infty}\left[\frac{\sqrt{b^2+1}}{b-1} - \frac{\sqrt{2}}{b-1}\right]$

$= 1 - 0 = 1$