Integration Review:

Antiderivatives & Indefinite Integrals:

1. $\frac{dy}{dx} = 3x^2 - 1$ $\quad \int dy = \int 3x^2 - 1\, dx$ $\quad \boxed{B}$

$$y = x^3 - x + c \Rightarrow y = x^3 - x - 1$$

$$-1 = C$$

2. $f(x) = \tan x$ $\quad \boxed{D}$

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx = -\int \frac{1}{u}\, du = -\ln|\cos x| + C$$

$u = \cos x$
$du = -\sin x\, dx$

3. $\frac{dy}{dx} = -x + 2$ $\quad \boxed{C}$

$$\int -x + 2\, dx = -\tfrac{1}{2}x^2 + 2x + c \Rightarrow -\tfrac{1}{2}x^2 + 2x - 1$$

$$1 = -\tfrac{1}{2}(4) + 4 + c$$

$$-1 = c$$

4. $\int (x^2 - 2)\, x^{1/2}\, dx = \int x^{5/2} - 2x^{1/2}\, dx$ $\quad \boxed{C}$

$$= \frac{2}{7}x^{7/2} - \frac{4}{3}x^{3/2} + C$$

$$= \frac{2}{7}x^3\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$$

5. $f'(x) \Rightarrow$ line $\Rightarrow y = 2x+3$

$m = \frac{3-0}{0-(-1.5)} = \frac{3}{1.5} = 2$

$$f(+3) = 11 + \int_{-3}^{3} 2x+3$$
$$= 11 + \left[x^2+3x\right]_{-3}^{3}$$
$$= 11 + \left[(9+9) - (9-9)\right] = 29$$

$f(3) = 29$

Riemann Sum and Area Approximations:

1. $\Delta x = 1$ $\quad \int_0^3 (3-x)(x+1)\,dx \approx 1\left[f(0)+f(1)+f(2)\right]$
$$= \left[3+4+3\right] = 10$$
$\boxed{C}$

2. $\int_1^{10} f(x)\,dx \approx 2\cdot f(3) + 2\cdot f(5) + 3\cdot f(8) + 2\cdot f(10)$
$$= 24 + 32 + 69 + 34 = 159$$
$\boxed{D}$

3. $\frac{1}{20}\cdot \sum_{i=1}^{20} \left(\frac{i}{20}\right)^2$ $\quad \Delta x = \frac{1}{20}$ $\quad \int_0^1 x^2\,dx$ $\quad \boxed{C}$

$f(x) = x^2$

$c_i = i \cdot \frac{1}{20}$

$a=0,\ b=1$

4. $\int_0^3 \sqrt{1+x^2}\,dx \approx 1\left[f(\tfrac{1}{2}) + f(\tfrac{3}{2}) + f(\tfrac{5}{2})\right]$ $\quad \boxed{A}$

$\Delta x = 1$
$$= \sqrt{\tfrac{5}{4}} + \sqrt{\tfrac{13}{4}} + \sqrt{\tfrac{29}{4}} = 5.613$$

5. $\frac{1}{30} \cdot \sum_{i=1}^{30} \sqrt{\frac{i}{30}}$

$\Delta x = \frac{1}{30}$

$f(x) = \sqrt{x}$

$c_i = i \cdot \frac{1}{30}$

$\rightarrow a = 0, \; b = 1$

$\int_0^1 \sqrt{x}\, dx$

$\boxed{A}$

6. $\frac{1}{10} \sum_{i=1}^{20} \frac{i}{10}$

$\Delta x = \frac{1}{10} = \frac{2}{20}$

$f(x) = x$

$c_i = i \cdot \frac{1}{10} = i \cdot \frac{2}{20}$

$a = 0, \; b = 2$

$\int_0^2 x\, dx$

$\boxed{B}$

Definite Integrals & Area Under a Curve:

1. $\int_0^3 \frac{dx}{\sqrt{1+x}} = \int_1^4 \frac{du}{\sqrt{u}} = \int_1^4 u^{-1/2}\, du = \left[2u^{1/2}\right]_1^4$

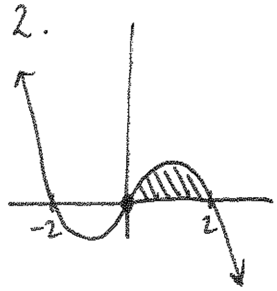
$= 2\sqrt{4} - 2\sqrt{1} = 2$

$u = 1 + x$

$du = dx$

$\boxed{A}$

2.



$f(x) = 4x - x^3$

$= -x(x^2 - 4)$

$= -x(x-2)(x+2)$

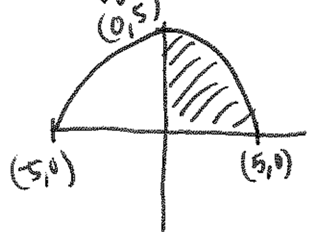
$\int_0^2 4x - x^3\, dx = \left[2x^2 - \frac{1}{4}x^4\right]_0^2$

$= \left[2(4) - \frac{1}{4}(16)\right] - \left[0\right] = 4$

$\boxed{C}$

3. $\int_0^5 \sqrt{25 - x^2}\, dx$

$y = \sqrt{25 - x^2} \Rightarrow x^2 + y^2 = 25$



$\int_0^5 \sqrt{25 - x^2}\, dx = \frac{1}{4}\pi(5)^2 = \frac{25\pi}{4}$

$\boxed{B}$

5. $\int_1^3 x^3\,dx$ $\quad \lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\cdot\frac{2}{n}$ $\quad \boxed{D}$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$c_i = 1+\frac{2i}{n}$

6. $\lim_{n\to\infty}\sum_{i=1}^{n}\left(-1+\frac{3i}{n}\right)^2\cdot\frac{3}{n}$

$\Delta x = \frac{3}{n} \Rightarrow b-a=3$ $\quad \int_{-1}^{2} x^2\,dx$ $\quad \boxed{A}$

$f(x) = x^2$

$c_i = -1+i\cdot\frac{3}{n}$

$a=-1,\ b=2$

7. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{\sqrt{x_i}}\cdot\Delta x$ $\quad \int_a^b x^{-1/2}\,dx = \left[2x^{1/2}\right]_a^b$ $\quad \boxed{C}$

$f(x) = \frac{1}{\sqrt{x}}$ $\quad = 2\sqrt{b}-2\sqrt{a}$

8. $\lim_{n\to\infty}\frac{1}{n}\cdot\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2$ $\quad \Delta x = \frac{1}{n}$ $\quad \int_0^1 x^2\,dx$ $\quad \boxed{C}$

$f(x) = x^2$

$c_i = i\cdot\frac{1}{n}$

$a=0,\ b=1$

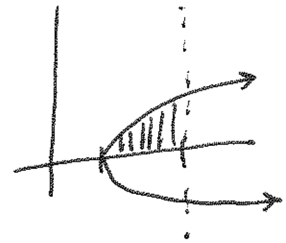
9. $\lim_{n\to\infty}\frac{2}{n}\cdot\sum_{i=1}^{n}\sqrt{\frac{2i}{n}}$ $\quad \Delta x = \frac{2}{n}$ $\quad \int_0^2 \sqrt{x}\,dx$ $\quad \boxed{B}$

$f(x) = \sqrt{x}$

$c_i = i\cdot\frac{2}{n}$

$a=0,\ b=2$

11. (a) $\int_1^2 \sqrt{\frac{x-1}{k}}\,dx$ or $\int_0^{\sqrt{1/k}} ky^2+1\,dy$



$x=2 \Rightarrow 2 = ky^2+1$

$y=\sqrt{1/k}$

(b) $\int_1^2 \sqrt{\frac{x-1}{k}}\,dx = 2$

$\frac{1}{\sqrt{k}} \int_1^2 \sqrt{x-1}\,dx = 2$

$\frac{1}{\sqrt{k}} \left[\frac{2}{3}(x-1)^{3/2} \right]_0^1 = 2$

$\frac{1}{\sqrt{k}} \cdot \frac{2}{3} = 2 \Rightarrow \frac{1}{3} = \sqrt{k} \Rightarrow k = \frac{1}{9}$

(c) $1 = k \cdot 2y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2ky}$

$y - \sqrt{1/k} = \frac{1}{2ky}(x-2)$

$y = \frac{1}{2ky}x - \frac{1}{ky} + \frac{1}{\sqrt{k}}$

$y = \frac{1}{2ky}x - \frac{1}{k(\frac{1}{\sqrt{k}})} + \frac{1}{\sqrt{k}}$

$y = \frac{1}{2ky}x + \underset{\substack{\uparrow \\ \text{y-int} \Rightarrow (0,0)}}{0}$

12. (a) $\int_{-5}^{-2} f(x)\,dx = \frac{-1}{2}(1)(2) + \frac{1}{2}(2)(2) = 1$

(b) $\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\pi(2)^2 = 2\pi$

(c) $\int_2^5 f(x)\,dx = -\frac{1}{2}(3+2)\cdot 1 = -5/2$

(d) $\int_{-5}^{5} |f(x)|\,dx = \frac{1}{2}(1)(2) + \frac{1}{2}(2)(2) + 2\pi + 5/2$

$= 2\pi + 11/2$

Properties of Definite Integrals:

1. $\int_a^b [f(x)-2]\,dx = \int_a^b [f(x)]\,dx - \int_a^b 2\,dx$

$= (2a-5b) - [2x]_a^b$

$= 2a - 5b - 2b + 2a = 4a - 7b$

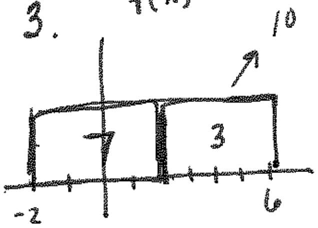
$\boxed{C}$

2. $\int_1^4 f(x)\,dx = \int_1^6 f(x)\,dx - \int_4^6 f(x)\,dx$

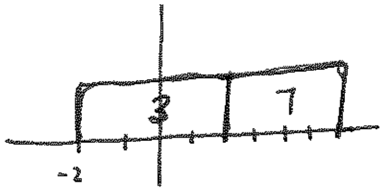
$= \frac{15}{2} - (-5) = \frac{25}{2}$

$\boxed{D}$

3. $f(x)$



$f(4-x) \rightarrow$ Reflection over y-axis & shift right 4



$\boxed{C}$

4. $\int_{-3}^{3} f(x)\,dx = A - B$

$\int_{-1}^{3} f(x)\,dx = -B \Rightarrow 2\int_{-1}^{3} f(x)\,dx = -2B$

$\int_{-3}^{3} f(x)dx - 2\int_{-1}^{3} f(x)\,dx$

$(A-B) - (-2B) = A+B$

$\boxed{B}$

5. (a) $\int_{3}^{5} f(x)\,dx = \int_{1}^{5} f(x)\,dx - \int_{1}^{3} f(x)\,dx = 7 - -3 = 10$

(b) $\int_{1}^{3} [f(x)+3]\,dx = \int_{1}^{3} f(x)\,dx + [3x]_{1}^{3} = -3 + 9 - 3 = 3$

(c) $\int_{5}^{1} 2g(x)\,dx = -2\int_{1}^{5} g(x)\,dx = -18$

(d) $\int_{5}^{5} g(x)\,dx + \int_{5}^{3} f(x)\,dx = 0 - \int_{3}^{5} f(x)\,dx = -10$

(e) $\int_{-1}^{3} f(x+2)\,dx = \int_{1}^{5} f(x)\,dx = 7$

$\downarrow$ left shift by 2

6. (a) $\int_{0}^{4} f(x)\,dx = 1 + \int_{4}^{6} g(x)\,dx = 1 + \int_{4}^{6} f(x) - n\,dx$

$= 1 + (5n-1) - [nx]_{4}^{6} = 5n - 6n + 4n = 3n$

(b) $\int_{0}^{6} g(x)\,dx = \int_{0}^{6} f(x) - n\,dx = \int_{0}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx - \int_{0}^{6} n\,dx$

$= 3n + 5n - 1 - [nx]_{0}^{6} = 2n - 1$

(c) $\int_{0}^{2} f(2x)\,dx = \frac{1}{2}\int_{0}^{4} f(u)\,du = \frac{1}{2}(3n) = \frac{3n}{2}$

$u = 2x$

$du = 2dx \Rightarrow \frac{1}{2}du = dx$

Trapezoidal Rule:

1. $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$$\int_0^2 e^x\,dx \approx \frac{1}{2}\cdot\frac{1}{2}\left[f(0) + 2f(\tfrac{1}{2}) + 2\cdot f(1) + 2\cdot f(\tfrac{3}{2}) + f(2)\right]$$

$$\approx \frac{1}{4}\left[1 + 2\sqrt{e} + 2e + 2e\sqrt{e} + e^2\right]$$

$\boxed{A}$

2. $\Delta x = \frac{\pi - \pi/2}{3} = \frac{\pi/2}{3} = \pi/6$

$$\int_{\pi/2}^{\pi} \sin x\,dx = \frac{1}{2}\cdot\frac{\pi}{6}\left[f(\pi/2) + 2\cdot f(\tfrac{2\pi}{3}) + 2\cdot f(5\pi/6) + f(\pi)\right]$$

$$= \frac{\pi}{12}\left[1 + 2\cdot\left(\tfrac{\sqrt{3}}{2}\right) + 2\cdot\left(\tfrac{1}{2}\right) + 0\right]$$

$\boxed{C}$

3. $$\int_0^6 \ln(x+1)\,dx = \tfrac{1}{2}\cdot 2\left[f(0) + 2\cdot f(2) + 2\cdot f(4) + f(6)\right]$$

$\Delta x = \frac{6-0}{3} = 2$

$$= \left[\ln(1) + 2\cdot\ln(3) + 2\cdot\ln(5) + \ln(7)\right] = \ln(1) + \ln(9) + \ln(25) + \ln(7)$$

$\boxed{D}$

4. Trapezoidal underapproximates → Concave Down

Right Riemann overapprox → Increasing

$\boxed{C}$

5. $$\int_1^{12} f(x)\,dx = \frac{1}{2}\left[2(4+10) + 2(10+14) + 4(14+11) + 3(11+7)\right]$$

$$= \frac{1}{2}\left[28 + 48 + 100 + 54\right] = 115$$

$\boxed{B}$

6. $$\int_0^2 \cos(x^2)\,dx = \frac{1}{2}\left[\frac{1}{2}\right]\left[\cos(0) + 2\cdot\cos(\tfrac{1}{4}) + 2\cdot\cos(1) + 2\cdot\cos(\tfrac{9}{4}) + \cos(4)\right]$$

$$= \frac{1}{4}\left[1 + 1.938 + 1.081 + -1.256 + -.654\right] = \frac{2109}{4000} \approx .527$$

7. $\int_{-2}^{8} f(x)\,dx = \frac{1}{2}\cdot 2\left[f(-2) + 2\cdot f(0) + 2f(2) + 2f(4) + 2f(6) + f(8)\right]$

$= -1 + 0 + 6 + 8 + 6 + 2 = 21$

Fundamental Theorem of Calculus:

1. $f(3) = f(1) + \int_{1}^{3} \frac{\sqrt{x}}{1+x^3}\,dx = 2.397$ $\boxed{B}$

2. $f(5) = f(-1) + \int_{-1}^{5} \cos(x^2-1)\,dx = 3.024$ $\boxed{C}$

3. $g(0) = g(3) + \int_{3}^{0} \sqrt{x^4 - 3x + 4}\,dx = -2.967$ $\boxed{A}$

4. $\int_{2}^{10} f(\frac{1}{2}x)\,dx = 2\int_{1}^{5} f(u)\,dx = 2\left[F(u)\right]_{1}^{5} = 2\left[F(5) - F(1)\right]$ $\boxed{C}$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx \Rightarrow 2du = dx$

5. (a) $f(0) = f(6) + \int_{6}^{0} f'(x)\,dx = f(6) - \int_{0}^{6} f'(x)\,dx = 9 - \left[-3 + 7 - 10\right] = 15$

(b) $f(1) = f(6) + \int_{6}^{1} f'(x)\,dx = f(6) - \int_{1}^{6} f'(x)\,dx = 9 - \left[7 - 10\right] = 12$

(c) $f(3) = f(6) + \int_{6}^{3} f'(x)\,dx = f(6) - \int_{3}^{6} f'(x)\,dx = 9 - \left[-10\right] = 19$

(d) $f(8) = f(6) + \int_{6}^{8} f'(x)\,dx = 9 + 8 = 17$

6. (a) $f(-3) = f(2) + \int_2^{-3} f'(x)\,dx = 3 - \int_{-3}^{2} f'(x)\,dx = 3 - \left[\frac{-1}{2}(4)(2) + \frac{1}{2}(1)(1)\right]$

$= 3 + 4 - \frac{1}{2} = \frac{13}{2}$

$f(7) = f(2) + \int_2^{7} f'(x)\,dx = 3 + \left[\frac{1}{2}\pi(2)^2 + \frac{1}{2}(4+5)\cdot 1\right]$

$= 3 + 2\pi + \frac{9}{2} = \frac{15}{2} + 2\pi$

(b) $f'(2) = 1$ $\qquad y - 3 = x - 2$

(c) f increasing $\Rightarrow$ f' positive $\Rightarrow (1, 7)$

(d) f concave up $\Rightarrow$ f' increasing $\Rightarrow (-1, 2), (2, 4)$

7. (a) $g(0) = \int_{-2}^{0} f(t)\,dt = \frac{1}{2}(2)(2) = 2$

NOTE: $g'(x) = f(x)$
$g''(x) = f'(x)$

$g'(0) = f(0) = 2$

$g''(0) = f'(0) = 1$

(b) g concave up $\Rightarrow g''(x) > 0 \Rightarrow (-2, 1), (7, 9)$

(c) g increasing $\Rightarrow g'(x) > 0 \Rightarrow (-2, 4), (8, 9)$

8. (a) $h(8) = \int_0^8 f(t)\,dt = \frac{-1}{2}(1)(2) + \frac{1}{2}(4)(4) + \frac{-1}{2}(4+2)\cdot 2 + \frac{1}{2}(4)(4) + \frac{-1}{2}(3+2)\cdot 2$

NOTE: $h'(x) = f(x)$
$h''(x) = f'(x)$

$= -1 + 8 - 6 + 8 - 5 = 4$

$h'(6) = f(6) = -2$

$h''(4) = f'(4) = -2$

(b) Extrema on $[0, 8] \Rightarrow h'(x) = 0$ OR ENDpoints $\Rightarrow x = 0, 1, 5, 8$,

x	$h(x)$
0	0
1	−1
5	7
8	4

$h(1) = \int_0^1 f(t)\,dt$

$h(5) = \int_0^5 f(t)\,dt$

$h(8) = \int_0^8 f(t)\,dt$

According to EVT, minimums & maximums occur at critical values or endpoints. For $h(x)$ on $[0, 8]$, there exists a minimum at $x = 1$ and a maximum at $x = 5$.

8. (c) Periodic with a period of 8 $\Rightarrow h(35) = h(3) = \int_0^3 f(t)\,dt = -1+4 = 3$

$\Rightarrow h'(35) = h'(3) = f(3) = 4$

$$y - 3 = 4(x-35)$$

Second Fundamental Theorem of Calculus:

1. $\frac{d}{dx}\left[\int_1^{x^2} \sqrt{3+t^2}\,dt\right] = \sqrt{3+(x^2)^2}\cdot 2x$ $\boxed{C}$

2. $F'(x) = \frac{1}{\sqrt{1-(\sin x)^2}}\cdot \cos x$ $\boxed{B}$

3. $F'(x) = \cos\left((\sqrt{x})^2\right)\cdot \frac{1}{2}x^{-1/2} = \frac{\cos(x)}{2\sqrt{x}}$ $\boxed{B}$

$F'(4) = \frac{\cos(4)}{4}$

4. f inc $\Rightarrow f' > 0$ $\boxed{C}$

$f'(x) = \cos(x^2+2)$

$f'(x)$: $-$ on $(0, 1.647)$, $+$ on $(1.647, 2.420)$, $-$ on $(2.420, 2.999)$, $+$ on $(2.999, \pi)$

cv: $\cos(x^2+2) = 0$

$x^2+2 \neq \pi/2$ $\quad x^2+2 = 3\pi/2 \Rightarrow 1.647$

$x^2+2 = 5\pi/2 \Rightarrow 2.420$

$x^2+2 = 7\pi/2 \Rightarrow 2.999$

5. $f'(x) = g(\sqrt{x})\cdot \frac{1}{2}x^{-1/2}$ $\boxed{B}$

$f'(4) = g(2)\cdot\frac{1}{2\cdot\sqrt{4}} = 2\cdot\frac{1}{4} = \frac{1}{2}$

6. $F'(x) = \frac{\sqrt{x^4+3}}{2x^2}\cdot 2x = \frac{\sqrt{x^4+3}}{x} = \sqrt{\frac{x^4+3}{x^2}} = \sqrt{x^2+3x^{-2}}$

$F''(x) = \frac{1}{2}(x^2+3x^{-2})^{-1/2}\cdot(2x-6x^{-3})$ $\quad F''(1) = \frac{1}{2\sqrt{4}}\cdot(2-6) = \frac{-4}{4} = -1$ $\boxed{A}$

7\. (a) $g(x)$ undefined $\Rightarrow \int_{-3}^{2x-1} f(t)\,dt =$ undefined

$$-3 \le 2x-1 \le 5$$
$$-2 \le 2x \le 6$$
$$-1 \le x \le 3$$

(b) $g'(x) = f(2x-1)\cdot 2$

$g'(3) = f(5)\cdot 2 = 2$

(c) $g(x)$ maximum on $[-1,3] \Rightarrow g'(x)=0$ or endpoints

$$2\cdot f(2x-1) = 0$$
$$2x-1 = 0$$
$$x = 1/2$$

x	$g(x)$
-1	0
$1/2$	< 0
3	

Minimum at $x = 1/2$ since the domain of $g(x)$ is $[-1,3]$, and $g(x)$ is decreasing from $(-3, 1/2)$ and increasing from $(1/2, 3)$.

$g(-1) = \int_{-3}^{-3} f(t)\,dt = 0$

$g(1/2) = \int_{-3}^{0} f(t)\,dt =$ Less than $g(-1)$ since area from $[-3,0]$ is below axis

$g(3) = \int_{-3}^{5} f(t)\,dt =$ Greater than $g(1/2)$ since area from $[0,5]$ is above axis.

8\. (a) $g'(x) = f(x) \qquad g'(1) = f(1) = 0$

(b) $g''(x) = f'(x)$

$f'(x) \ne 0 \quad f'(x) = \text{und} \Rightarrow x = 1, 2, 6$

<u>NOTE</u>: No sign change in $g''(x) = f'(x)$ at $x = 1$

Points of inflection occur at $x = 2$ and $x = 6$ since $g''(x) = f'(x)$ changes sign.

(c) Aroc for g on $[2,8] \Rightarrow \dfrac{g(8)-g(2)}{8-2} \Rightarrow \dfrac{\int_{-2}^{8} f(t)\,dt - \int_{-2}^{2} f(t)\,dt}{8-2} \Rightarrow \dfrac{-\frac{1}{2}(2)(3) + \frac{1}{2}(4)(3)}{6}$

$= 1/2$

(d) Since $g(x)$ is continuous and differentiable on $(2,8)$, MVT guarantees at least one value of c, such that $g'(c) = \dfrac{g(8)-g(2)}{8-2} = 1/2$.

$g'(c) = f(c) = 1/2 \Longrightarrow$ occurs at two x values on $(2,8)$.

Integration by Substitution:

1. $\int \sqrt{x}\, \sin(x^{3/2})\,dx = \frac{2}{3}\int \sin(u)\,du = -\frac{2}{3}\cos(x^{3/2}) + C$ $\boxed{D}$

$u = x^{3/2}$

$du = \frac{3}{2}x^{1/2}dx \Rightarrow \frac{2}{3}du = x^{1/2}dx$

2. $\int_1^3 x\sqrt{2-x}\,dx = -\int_1^{-1}(2-u)\sqrt{u}\,du = \int_{-1}^{1}(2-u)\sqrt{u}\,du$

$u = 2-x \rightarrow x = 2-u$

$du = -dx$

$-du = dx$

Correct answer is not an answer choice

$\int_{-1}^{1}(2-u)\sqrt{u}\,du$

3. $\int_{-1}^{3} f(x+k)\,dx = \int_{k-1}^{k+3} f(u)\,du = 8$ $\boxed{C}$

$u = x+k \qquad x=3 \rightarrow u = 3+k$

$du = dx \qquad x=-1 \rightarrow u = -1+k$

4. $\int_0^6 f(6-x)\,dx = -\int_6^0 f(u)\,du = \int_0^6 f(u)\,du = 12$ $\boxed{A}$

$u = 6-x \qquad x=6 \rightarrow u=0$

$du = -dx \qquad x=0 \rightarrow u=6$

$-du = dx$

6. $\int_0^{\pi/4} \frac{\tan\theta}{\sqrt{\sec\theta}}\,d\theta = \int_1^{\sqrt{2}} \frac{1}{u\sqrt{u}}\,du$ $\boxed{B}$

$u = \sec\theta \qquad x = \pi/4 \rightarrow u = \sqrt{2}$

$du = \sec\theta\tan\theta\,d\theta \qquad x = 0 \rightarrow u = 1$

$du = u \cdot \tan\theta\,d\theta$

$\frac{1}{u}du = \tan\theta\,d\theta$

5. $\int \frac{(1+\sqrt{x})^{3/2}}{\sqrt{x}}\,dx$ $\boxed{B}$

$u = 1+\sqrt{x}$

$du = \frac{1}{2}x^{-1/2}dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$2\int u^{3/2}\,du$

7. $\int_{e}^{e^2} \frac{1-(\ln x)^2}{x}\,dx = \int_1^2 1-u^2\,du$ $\boxed{C}$

$u = \ln x$ $x = e^2 \rightarrow u = 2$

$du = \frac{1}{x}\,dx$ $x = e \rightarrow u = 1$

8. $\int_1^2 x^2 f(x^3)\,dx = \frac{1}{3}\int_1^8 f(u)\,du = \frac{1}{3}(15) = 5$

$u = x^3$ $x = 2 \rightarrow u = 8$

$du = 3x^2\,dx$ $x = 1 \rightarrow u = 1$

$\frac{1}{3}du = x^2\,dx$

Integration Exponential and Logarithmic Functions:

1. $\int_1^3 \frac{x+3}{x^2+6x}\,dx = \frac{1}{2}\int_7^{27} \frac{1}{u}\,du = \frac{1}{2}\left[\ln|u|\right]_7^{27}$ $\boxed{B}$

$u = x^2+6x$

$du = (2x+6)\,dx$

$du = 2(x+3)\,dx$

2. $\int_0^1 \frac{x}{e^{x^2}}\,dx = \int_0^1 x\,e^{-x^2}\,dx = -\frac{1}{2}\int_0^{-1} e^u\,du = \frac{1}{2}\int_{-1}^{0} e^u\,du$ $\boxed{C}$

$u = -x^2$

$du = -2x\,dx$

$= \frac{1}{2}\left[e^u\right]_{-1}^{0}$

$= \frac{1}{2}\left[1 - \frac{1}{e}\right]$

3. $\int_0^{\pi/2} \cos x \cdot e^{\sin x}\,dx = \int_0^1 e^u\,du = \left[e^u\right]_0^1 = e-1$ $\boxed{D}$

$u = \sin x$

$du = \cos x\,dx$

4. $\displaystyle\int_0^{\pi/2} \frac{\cos x}{2+\sin x}\,dx = \int_2^3 \frac{1}{u}\,du = \Big[\ln|u|\Big]_2^3$ $\boxed{B}$

$u = 2+\sin x$

$du = \cos x\,dx$

$= \ln 3 - \ln 2$

$= \ln \frac{3}{2}$

5. $\displaystyle F(3) = F(1) + \int_1^3 \ln(\sin^2 x) + 3\,dx = 6.595$ $\boxed{A}$

6. $\displaystyle\int_0^2 \frac{x^2}{x+1}\,dx = \int_1^3 \frac{(u-1)^2}{u}\,du = \int_1^3 \frac{u^2-2u+1}{u}\,du$

$u = x+1 \rightarrow x = u-1$

$du = dx \qquad x^2 = (u-1)^2$

$\displaystyle = \int_1^3 u - 2 + \frac{1}{u}\,du$ $\boxed{A}$

$\displaystyle = \Big[\tfrac{1}{2}u^2 - 2u + \ln|u|\Big]_1^3$

$= \left(\frac{9}{2} - 6 + \ln 3\right) - \left(\frac{1}{2} - 2 + \ln(1)\right)$

$= \ln 3$

7. $\displaystyle\int_1^e \frac{\cos(\ln x)}{x}\,dx = \int_0^1 \cos(u)\,du = \Big[\sin(u)\Big]_0^1$ $\boxed{D}$

$= \sin(1)$

$u = \ln x$

$du = \frac{1}{x}\,dx$

8. (a) $(1, 7000)$

$\displaystyle\lim_{t\to\infty} S(t) = \lim_{t\to\infty} Ce^{k/t} = \boxed{C = 45000}$

$7000 = Ce^k$

$7000 = 45000e^k$

$\frac{7}{45} = e^k \longrightarrow \boxed{k = \ln\left(\frac{7}{45}\right)}$

(b) $S(t) = 45000e^{\ln(7/45)/t}$

$S(5) = 31016.274$

$S(10) = 37359.501$