AP Calculus

Integration Review

Antiderivatives and Indefinite Integrals

1. If $\frac{dy}{dx} = 3x^2 - 1$, and if $y = -1$ when $x = 1$, then $y =$

(A) $x^3 - x + 1$

(B) $x^3 - x - 1$

(C) $-x^3 + x - 1$

(D) $-x^3 + 1$

2. Which of the following is the antiderivative of $f(x) = \tan x$?

(A) $\sec x + \tan x + C$

(B) $\csc x + \cot x + C$

(C) $\ln|\csc x| + C$

(D) $-\ln|\cos x| + C$

3. A curve has a slope of $-x + 2$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(2,1)$?

(A) $\frac{1}{2}x^2 - 2x - 4$

(B) $2x^2 + x - 8$

(C) $-\frac{1}{2}x^2 + 2x - 1$

(D) $x^2 - 2x + 1$

4. $\int (x^2 - 2)\sqrt{x}\ dx =$

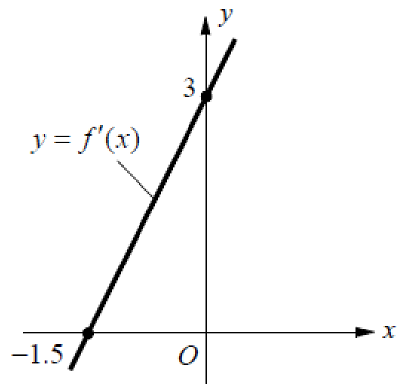
(A) $\frac{2}{5}x^2\sqrt{x} - \frac{2}{3}x\sqrt{x} + C$

(B) $\frac{2}{5}x^2\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$

(C) $\frac{2}{7}x^3\sqrt{x} - \frac{4}{3}x\sqrt{x} + C$

(D) $\frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x^2\sqrt{x} + C$

Free Response Questions



5. The graph of f', the derivative of f, is the line shown in the figure above. If $f(3) = 11$, then $f(-3) =$

Riemann Sum and Area Approximation

1. Using a left Riemann sum with three subintervals $[0,1]$, $[1,2]$, and $[2,3]$, what is the approximation of $\int_0^3 (3-x)(x+1)\,dx$?

(A) 7.5 (B) 9 (C) 10 (D) 11.5

x	1	3	5	8	10
$f(x)$	7	12	16	23	17

2. The function f is continuous on the closed interval $[1,10]$ and has values as shown in the table above. Using a right Riemann sum with four subintervals $[1,3]$, $[3,5]$, $[5,8]$, $[8,10]$, what is the approximation of $\int_1^{10} f(x)\,dx$?

(A) 96 (B) 116 (C) 132 (D) 159

3. The expression $\frac{1}{20}\left[\left(\frac{1}{20}\right)^2+\left(\frac{2}{20}\right)^2+\left(\frac{3}{20}\right)^2+\ldots+\left(\frac{20}{20}\right)^2\right]$ is a Riemann sum approximation for

(A) $\frac{1}{20}\int_0^{20} x^2\,dx$

(B) $\frac{1}{20}\int_0^{1} x^2\,dx$

(C) $\int_0^{1} x^2\,dx$

(D) $\int_0^{1} \frac{1}{x^2}\,dx$

4. Using a midpoint Riemann sum with three subintervals $[0,1]$, $[1,2]$, and $[2,3]$, what is the approximation of $\int_0^3 \sqrt{1+x^2}\,dx$?

(A) 5.613 (B) 6.213 (C) 6.812 (D) 7.195

5. The expression $\frac{1}{30}\left[\sqrt{\frac{1}{30}}+\sqrt{\frac{2}{30}}+\sqrt{\frac{3}{30}}+\ldots+\sqrt{\frac{30}{30}}\right]$ is a Riemann sum approximation for

(A) $\int_0^{1} \sqrt{x}\,dx$

(B) $\frac{1}{30}\int_0^{1} \sqrt{x}\,dx$

(C) $\frac{1}{30}\int_0^{30} \sqrt{x}\,dx$

(D) $\int_0^{1} \frac{1}{\sqrt{x}}\,dx$

6. The expression $\frac{1}{10}\left[\frac{1}{10}+\frac{2}{10}+\frac{3}{10}+\ldots+\frac{20}{10}\right]$ is a Riemann sum approximation for

(A) $\int_0^{2} 2\,dx$

(B) $\int_0^{2} x\,dx$

(C) $\int_0^{2} \frac{x}{10}\,dx$

(D) $\frac{1}{10}\int_0^{1} x\,dx$

Definite Integrals and Area Under a Curve

1. $\int_0^3 \frac{dx}{\sqrt{1+x}} =$

(A) 2 (B) 2.5 (C) 3 (D) 4

2. The area of the region in the first quadrant enclosed by the graph of $f(x) = 4x - x^3$ and the x-axis is

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 4 (D) $\frac{11}{2}$

3. $\int_0^5 \sqrt{25 - x^2}\ dx =$

(A) $\frac{25\pi}{8}$ (B) $\frac{25\pi}{4}$ (C) $\frac{25\pi}{2}$ (D) 25π

5. Which of the following limits is equal to $\int_1^3 x^3\ dx$?

(A) $\lim_{n\to\infty} \sum_{i=1}^{n} (1+\frac{i}{n})^3 \frac{1}{n}$

(B) $\lim_{n\to\infty} \sum_{i=1}^{n} (1+\frac{i}{n})^3 \frac{2}{n}$

(C) $\lim_{n\to\infty} \sum_{i=1}^{n} (1+\frac{2i}{n})^3 \frac{1}{n}$

(D) $\lim_{n\to\infty} \sum_{i=1}^{n} (1+\frac{2i}{n})^3 \frac{2}{n}$

6. Which of the following integrals is equal to $\lim_{n\to\infty} \sum_{i=1}^{n} (-1+\frac{3i}{n})^2 \frac{3}{n}$?

(A) $\int_{-1}^{2} x^2\ dx$

(B) $\int_{-1}^{0} x^2\ dx$

(C) $\int_{-1}^{2} (-1+x)^2\ dx$

(D) $\int_{-1}^{0} (-1+\frac{x}{3})^2\ dx$

7. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $0 < a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{\sqrt{x_i}} \Delta x$?

(A) $\frac{1}{\sqrt{b}} - \frac{1}{\sqrt{a}}$

(B) $\frac{(\sqrt{b}-\sqrt{a})}{2}$

(C) $2(\sqrt{b}-\sqrt{a})$

(D) $\sqrt{b}-\sqrt{a}$

8. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \cdots + \left(\frac{n}{n}\right)^2\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x}\,dx$ (B) $\int_0^1 \frac{1}{x^2}\,dx$ (C) $\int_0^1 x^2\,dx$ (D) $\frac{1}{2}\int_0^1 x^2\,dx$

9. If n is a positive integer, then $\lim_{n\to\infty} \frac{2}{n}\left[\sqrt{\frac{2}{n}} + \sqrt{\frac{4}{n}} + \cdots + \sqrt{\frac{2n}{n}}\right]$ can be expressed as

(A) $\int_0^1 \sqrt{x}\,dx$ (B) $\int_0^2 \sqrt{x}\,dx$ (C) $\int_0^1 \frac{1}{\sqrt{x}}\,dx$ (D) $\int_0^2 \frac{1}{\sqrt{x}}\,dx$

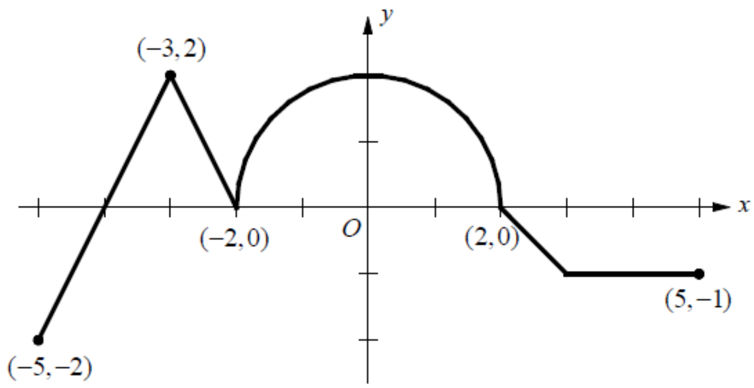
Free Response Questions

11. Let R be the region in the first quadrant bounded by the x-axis, the graph of $x = ky^2 + 1$ ($k > 0$), and the line $x = 2$.

(a) Write an integral expression for the area of the region R.

(b) If the area of the region R is 2, what is the value of k ?

(c) Show that for all $k > 0$, the line tangent to the graph of $x = ky^2 + 1$ at the point $(2, \sqrt{\frac{1}{k}})$ passes through the origin.



12. The graph of $y = f(x)$ consists of four line segments and a semicircle as shown in the figure above. Evaluate each definite integral by using geometric formulas.

(a) $\int_{-5}^{-2} f(x)\,dx$ (b) $\int_{-2}^{2} f(x)\,dx$ (c) $\int_{2}^{5} f(x)\,dx$ (d) $\int_{-5}^{5} |f(x)|\,dx$

Properties of Definite Integrals

1. If $\int_a^b f(x)\,dx = 2a - 5b$, then $\int_a^b [f(x) - 2]\,dx =$

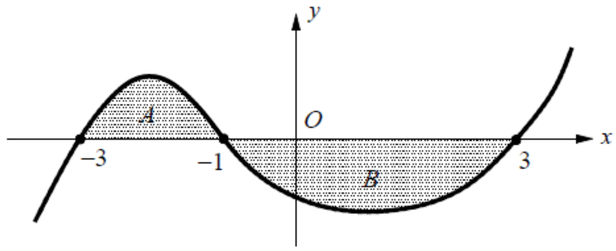
(A) $-7b$ (B) $-3b$ (C) $4a - 7b$ (D) $4a - 3b$

2. If $\int_1^6 f(x)\,dx = \frac{15}{2}$ and $\int_6^4 f(x)\,dx = 5$, then $\int_1^4 f(x)\,dx =$

(A) $\frac{5}{2}$ (B) $\frac{9}{2}$ (C) $\frac{19}{2}$ (D) $\frac{25}{2}$

3. If $\int_{-2}^{6} f(x)\,dx = 10$ and $\int_2^6 f(x)\,dx = 3$, then $\int_2^6 f(4 - x)\,dx =$

(A) 3 (B) 6 (C) 7 (D) 10



4. The graph of $y = f(x)$ is shown in the figure above. If A and B are positive numbers that represent the areas of the shaded regions, what is the value of $\int_{-3}^{3} f(x)\,dx - 2\int_{-1}^{3} f(x)\,dx$, in terms of A and B?

(A) $-A-B$ (B) $A+B$ (C) $A-2B$ (D) $A-B$

Free Response Questions

5. Let f and g be a continuous function on the interval $[1,5]$. Given $\int_{1}^{3} f(x)\,dx = -3$, $\int_{1}^{5} f(x)\,dx = 7$, and $\int_{1}^{5} g(x)\,dx = 9$, find the following definite integrals.

(a) $\int_{3}^{5} f(x)\,dx$ (b) $\int_{1}^{3} [f(x)+3]\,dx$ (c) $\int_{5}^{1} 2g(x)\,dx$

(d) $\int_{5}^{5} g(x)\,dx + \int_{5}^{3} f(x)\,dx$ (e) $\int_{-1}^{3} f(x+2)\,dx$

6. Let f and g be continuous functions with the following properties.

(1) $g(x) = f(x) - n$ where n is a constant.

(2) $\int_{0}^{4} f(x)\,dx - \int_{4}^{6} g(x)\,dx = 1$

(3) $\int_{4}^{6} f(x)\,dx = 5n - 1$

(a) Find $\int_{0}^{4} f(x)\,dx$ in terms of n.

(b) Find $\int_{0}^{6} g(x)\,dx$ in terms of n.

(c) Find the value of k if $\int_{0}^{2} f(2x)\,dx = kn$.

Trapezoidal Rule

1. If four equal subdivisions on $[0,2]$ are used, what is the trapezoidal approximation of $\int_0^2 e^x \, dx$?

(A) $\frac{1}{4}\left[1+2\sqrt{e}+2e+2e\sqrt{e}+e^2\right]$

(B) $\frac{1}{2}\left[1+2\sqrt{e}+2e+2e\sqrt{e}+e^2\right]$

(C) $\frac{1}{4}\left[1+\sqrt{e}+e+e\sqrt{e}+e^2\right]$

(D) $\frac{1}{2}\left[1+\sqrt{e}+e+e\sqrt{e}+e^2\right]$

2. If three equal subdivisions on $\left[\frac{\pi}{2},\pi\right]$ are used, what is the trapezoidal approximation of $\int_{\pi/2}^{\pi} \sin x \, dx$?

(A) $\frac{\pi}{12}(\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\pi)$

(B) $\frac{\pi}{12}(\sin\frac{\pi}{2}+\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\pi)$

(C) $\frac{\pi}{12}(\sin\frac{\pi}{2}+2\sin\frac{2\pi}{3}+2\sin\frac{5\pi}{6}+\sin\pi)$

(D) $\frac{\pi}{6}(\sin\frac{\pi}{2}+\sin\frac{2\pi}{3}+\sin\frac{5\pi}{6}+\sin\pi)$

3. If three equal subdivisions on $[0,6]$ are used, what is the trapezoidal approximation of $\int_0^6 \ln(x+1) \, dx$?

(A) $\frac{1}{3}(\ln 1+\ln 9+\ln 25+\ln 7)$

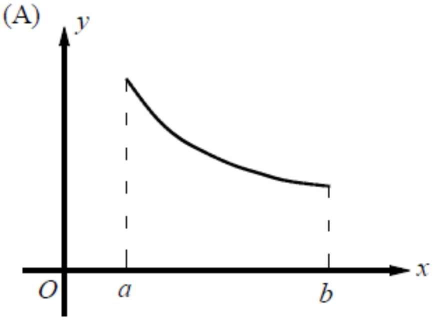
(B) $\frac{1}{2}(\ln 1+\ln 9+\ln 25+\ln 7)$

(C) $\ln 1+\ln 3+\ln 5+\ln 7$

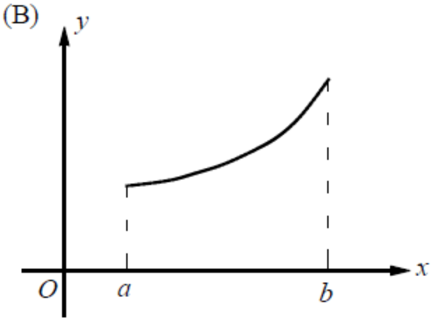
(D) $\ln 1+\ln 9+\ln 25+\ln 7$

4. If a trapezoidal sum underapproximates $\int_a^b f(x)\,dx$, and a right Riemann sum overapproximates $\int_a^b f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

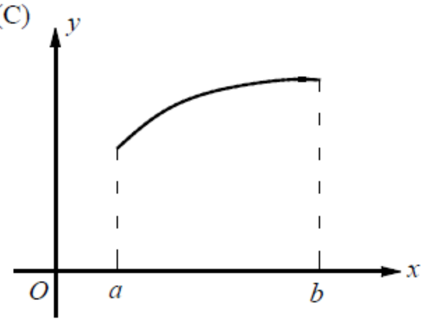
(A)



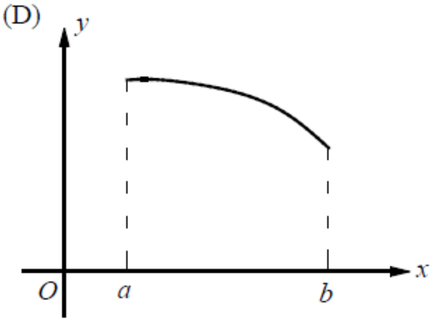
(B)



(C)



(D)



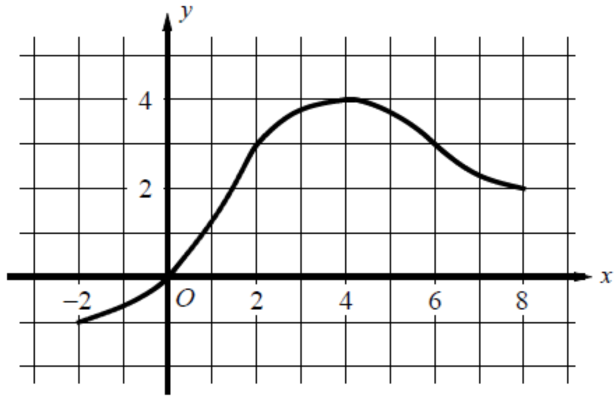
x	1	3	5	9	12
$f(x)$	4	10	14	11	7

5. A function f is continuous on the closed interval $[1,12]$ and has values that are given in the table above. Using subintervals $[1,3]$, $[3,5]$, $[5,9]$, and $[9,12]$, what is the trapezoidal approximation of $\int_1^{12} f(x)\,dx$?

(A) 97 (B) 115 (C) 128 (D) 136

Free Response Questions

6. Find a trapezoidal approximation of $\int_0^2 \cos(x^2)\,dx$ using four subdivisions of length $\Delta x = 0.5$.



7. The graph of a differentiable function f on the closed interval $[-2,8]$ is shown in the figure above. Find a trapezoidal approximation of $\int_{-2}^{8} f(x)\,dx$ using five subdivisions of length $\Delta x = 2$.

Fundamental Theorem of Calculus

1. If f is the antiderivative of $\dfrac{\sqrt{x}}{1+x^3}$ such that $f(1)=2$, then $f(3)=$

(A) 1.845 (B) 2.397 (C) 2.906 (D) 3.234

2. If $f'(x)=\cos(x^2-1)$ and $f(-1)=1.5$, then $f(5)=$

(A) 1.554 (B) 2.841 (C) 3.024 (D) 3.456

3. If $f(x)=\sqrt{x^4-3x+4}$ and g is the antiderivative of f, such that $g(3)=7$, then $g(0)=$

(A) −2.966 (B) −1.472 (C) −0.745 (D) 1.086

4. If f is a continuous function and $F'(x)=f(x)$ for all real numbers x, then $\int_{2}^{10} f(\frac{1}{2}x)\,dx =$

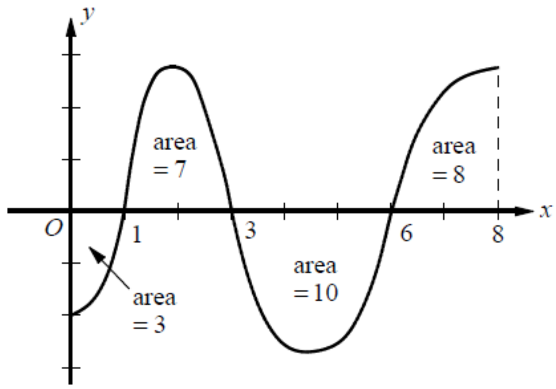
(A) $\frac{1}{2}[F(5)-F(1)]$

(B) $\frac{1}{2}[F(10)-F(2)]$

(C) $2[F(5)-F(1)]$

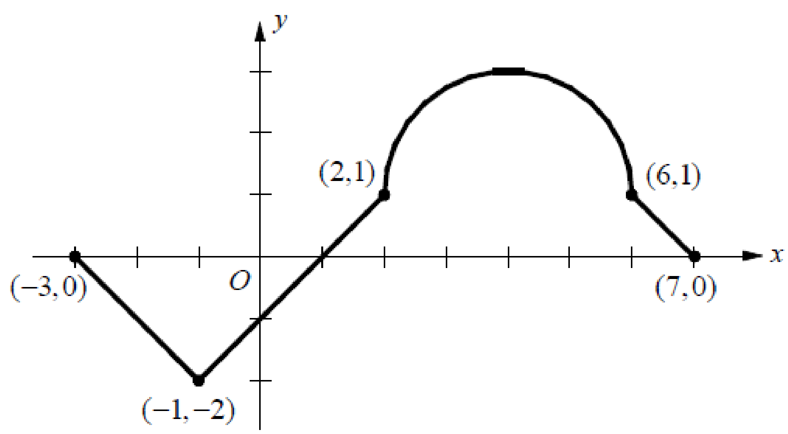
(D) $2[F(10)-F(2)]$

Free Response Questions



5. The figure above shows the graph of f', the derivative of a differentiable function f, on the closed interval $0 \le x \le 8$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. Given $f(6) = 9$, find each of the following.

(a) $f(0)$ (b) $f(1)$ (c) $f(3)$ (d) $f(8)$



graph of f'

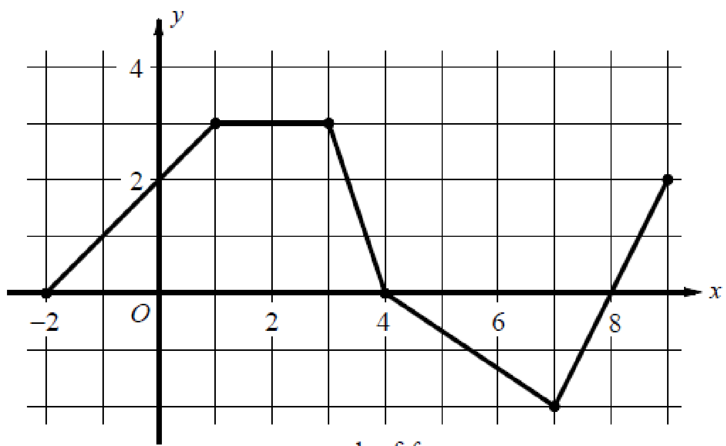
6. Let f be a function defined on the closed interval $[-3, 7]$ with $f(2) = 3$. The graph of f' consists of three line segments and a semicircle, as shown above.

(a) Find $f(-3)$ and $f(7)$.

(b) Find an equation for the line tangent to the graph of f at $(2, 3)$.

(c) On what interval is f increasing? Justify your answer.

(d) On what interval is f concave up? Justify your answer.



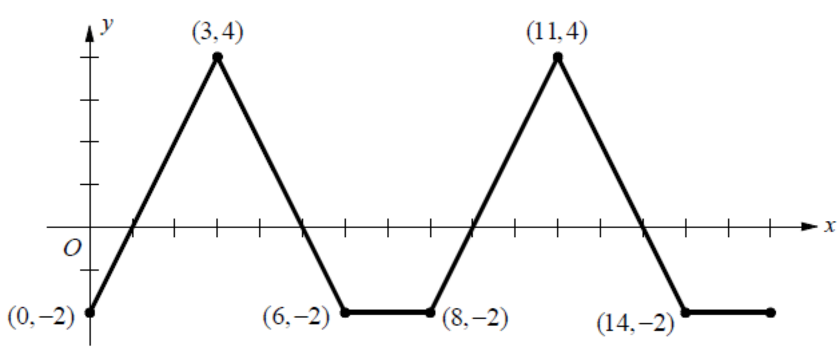
graph of f

7. Let g be the function given by $g(x)=\int_{-2}^{x} f(t)\,dt$. The graph of the function f, shown above, consists of five line segments.

(a) Find $g(0)$, $g'(0)$ and $g''(0)$.

(b) For what values of x, in the open interval $(-2,9)$, is the graph of g concave up?

(c) For what values of x, in the open interval $(-2,9)$, is g increasing?



graph of f

8. The graph above shows two periods of f. The function f is defined for all real numbers x and is periodic with a period of 8. Let h be the function given by $h(x)=\int_{0}^{x} f(t)\,dt$.

(a) Find $h(8)$, $h'(6)$, and $h''(4)$.

(b) Find the values of x at which h has its minimum and maximum on the closed interval $[0,8]$. Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x=35$.

Second Fundamental Theorem of Calculus

1. $\frac{d}{dx}\int_1^{x^2}\sqrt{3+t^2}\,dt =$

(A) $\sqrt{3+x^2}$ (B) $\sqrt{3+x^4}$ (C) $2x\sqrt{3+x^4}$ (D) $2\sqrt{3+x^2}$

2. For $-\frac{\pi}{2}<x<\frac{\pi}{2}$, if $F(x)=\int_0^{\sin x}\frac{dt}{\sqrt{1-t^2}}$, then $F'(x)=$

(A) $\frac{\sin x}{\sqrt{1-x^2}}$ (B) $\frac{\cos x}{\sqrt{1-x^2}}$ (C) 1 (D) $\csc x$

3. If $F(x)=\int_0^{\sqrt{x}}\cos(t^2)\,dt$, then $F'(4)=$

(A) $\cos 2$ (B) $\frac{\cos 4}{4}$ (C) $\frac{\cos 4}{\sqrt{2}}$ (D) $\sqrt{2}\cos 4$

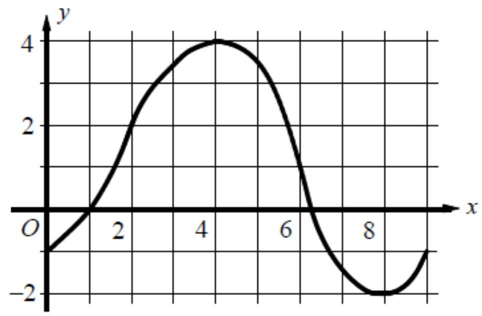
4. Let f be the function given by $f(x)=\int_0^x\cos(t^2+2)\,dt$ for $0\le x\le\pi$. On which of the following intervals is f increasing?

(A) $0\le x\le\frac{\pi}{2}$

(B) $0\le x\le 1.647$

(C) $1.647\le x\le 2.419$

(D) $\frac{\pi}{2}\le x\le\pi$



graph of g

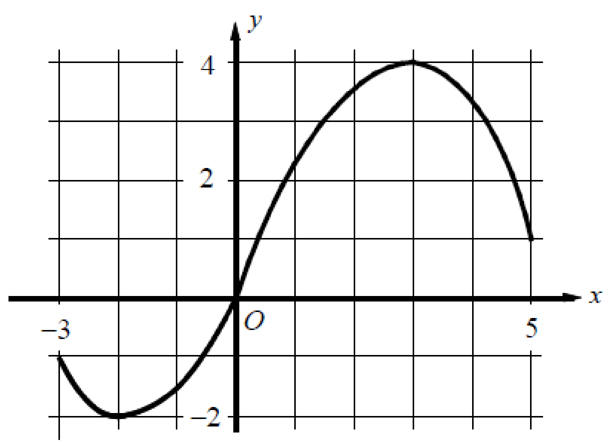
5. The graph of the function g, shown in the figure above, has horizontal tangents at $x = 4$ and $x = 8$. If $f(x) = \int_0^{\sqrt{x}} g(t)\,dt$, what is the value of $f'(4)$?

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) $\frac{3}{2}$

6. If $F(x) = \int_0^{x^2} \frac{\sqrt{t^2+3}}{2t}\,dt$, then $F''(1) =$

(A) -1 (B) 0 (C) 1 (D) $\frac{3}{2}$ (E) $\frac{8}{5}$

Free Response Questions



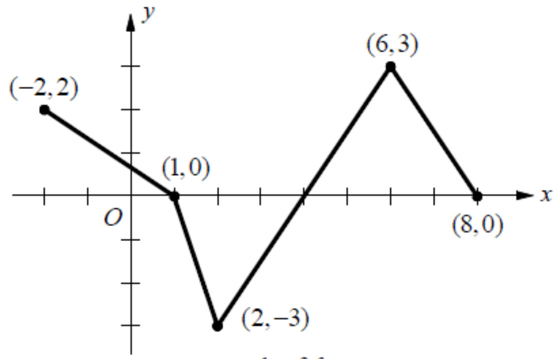
graph of f

7. The graph of a function f, whose domain is the closed interval $[-3, 5]$, is shown above. Let g be the function given by $g(x) = \int_{-3}^{2x-1} f(t)\,dt$.

(a) Find the domain of g.

(b) Find $g'(3)$.

(c) At what value of x is $g(x)$ a maximum? Justify your answer.



graph of f

8. The graph of f, consisting of four line segments, is shown in the figure above. Let g be the function given by $g(x)=\int_{-2}^{x} f(t)\,dt$.

(a) Find $g'(1)$.

(b) Find the x-coordinate for each point of inflection of the graph of g on the interval $-2<x<8$.

(c) Find the average rate of change of g on the interval $2\le x\le 8$.

(d) For how many values of c, where $2<c<8$, is $g'(c)$ equal to the average rate found in part (c)? Explain your reasoning.

Integration by Substitution

1. $\int \sqrt{x}\sin(x^{3/2})\,dx=$

(A) $\frac{2}{3}\cos(x^{3/2})+C$

(B) $\frac{2}{3}\sqrt{x}\cos(x^{3/2})+C$

(C) $-\frac{2}{3}x^{3/2}\cos(x^{3/2})+C$

(D) $-\frac{2}{3}\cos(x^{3/2})+C$

2. If the substitution $u=2-x$ is made, $\int_{1}^{3} x\sqrt{2-x}\,dx=$

(A) $\int_{-1}^{1} u\sqrt{u}\,du$

(B) $-\int_{1}^{3} u\sqrt{u}\,du$

(C) $\int_{1}^{3} (2-u)\sqrt{u}\,du$

(D) $\int_{-1}^{1} (u-2)\sqrt{u}\,du$

3. If $\int_{-1}^{3} f(x+k)\,dx = 8$, where k is a constant, then $\int_{k-1}^{k+3} f(x)\,dx =$

(A) $8-k$ (B) $8+k$ (C) 8 (D) $k-8$

4. If $\int_{0}^{6} f(x)\,dx = 12$, what is the value of $\int_{0}^{6} f(6-x)\,dx$?

(A) 12 (B) 6 (C) 0 (D) –6

5. If the substitution $u = 1+\sqrt{x}$ is made, $\int \frac{(1+\sqrt{x})^{3/2}}{\sqrt{x}}\,dx =$

(A) $\frac{1}{2}\int u^{3/2}\,du$ (B) $2\int u^{3/2}\,du$ (C) $\frac{1}{2}\int \sqrt{u}\,du$ (D) $2\int \sqrt{u}\,du$

6. Using the substitution $u = \sec\theta$, $\int_{0}^{\pi/4} \frac{\tan\theta}{\sqrt{\sec\theta}}\,d\theta$ is equivalent to

(A) $\int_{1}^{\sqrt{2}} \frac{du}{\sqrt{u}}$

(B) $\int_{1}^{\sqrt{2}} \frac{du}{u\sqrt{u}}$

(C) $\int_{1}^{\sqrt{2}} \sqrt{u}\,du$

(D) $\int_{1}^{\sqrt{2}} u\sqrt{u}\,du$

7. If the substitution $u = \ln x$ is made, $\int_e^{e^2} \frac{1-(\ln x)^2}{x}\,dx =$

(A) $\int_e^{e^2} (\frac{1}{u} - u^2)\,du$

(B) $\int_e^{e^2} (\frac{1}{u} - u)\,du$

(C) $\int_1^2 (1-u^2)\,du$

(D) $\int_1^2 (1-u)\,du$

Free Response Questions

8. If f is continuous and $\int_1^8 f(x)\,dx = 15$, find the value of $\int_1^2 x^2 f(x^3)\,dx$.

Integration of Exponential and Logarithmic Functions

1. $\int_1^3 \frac{x+3}{x^2+6x}\,dx =$

(A) $\ln\frac{3}{2}$ (B) $\frac{\ln 27 - \ln 7}{2}$ (C) $\ln 3$ (D) $\frac{\ln 20 - \ln 5}{2}$

2. $\int_0^1 \frac{x}{e^{x^2}}\,dx =$

(A) $e-1$ (B) $(1-\frac{1}{e})$ (C) $\frac{1}{2}(1-\frac{1}{e})$ (D) $\frac{1}{2}(1-\frac{1}{e^2})$

3. $\int_0^{\pi/2} \cos x\, e^{\sin x}\,dx =$

(A) $-e$ (B) $1-e$ (C) $\frac{e}{2}$ (D) $e-1$

4. What is the area of the region in the first quadrant bounded by the curve $y = \frac{\cos x}{2 + \sin x}$ and the vertical line $x = \frac{\pi}{2}$?

(A) $\ln \frac{1}{2}$ (B) $\ln \frac{3}{2}$ (C) $\ln 3$ (D) $\frac{\ln 3}{\ln 2}$

5. Let $F(x)$ be an antiderivetive of $\ln(\sin^2 x) + 3$. If $F(1) = 2$, then $F(3) =$

(A) 6.595 (B) 7.635 (C) 10.036 (D) 12.446

6. $\int_0^2 \frac{x^2}{x+1}\, dx =$

(A) $\ln 3$ (B) $\ln 3 + 2$ (C) $\ln 6$ (D) $\ln 6 + 4$

7. $\int_1^e \frac{\cos(\ln x)}{x}\, dx =$

(A) $\frac{1}{\sin 1}$ (B) $\frac{1}{\cos 1}$ (C) $\sin(e)$ (D) $\sin 1$

Free Response Questions

8. The sales of a new product, after it has been on the market for t years, is given by $S(t) = Ce^{k/t}$.

(a) Find C and k if 7000 units have been sold after one year and $\lim_{t \to \infty} S(t) = 45,000$.

(b) Find the total number of units sold during the year $t = 5$ and $t = 10$.