Derivatives of Inverse Functions Homework

1) For $f(x) = x^2, x \geq 0$, calculate the value of $(f^{-1})'(x)$ when $x = 4$.

$f(2) = 4$

$f^{-1}(4) = 2$

$f'(x) = 2x$

$$\frac{d}{dx}\left[f^{-1}(4)\right] = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(2)} = \frac{1}{2(2)} = \frac{1}{4}$$

2) For $f(x) = \frac{1}{4}x^3 + x - 1$, calculate the value of $(f^{-1})'(x)$ when $x = 3$.

$f(2) = 3$

$f^{-1}(3) = 2$

$f'(x) = \frac{3}{4}x^2 + 1$

$$\frac{d}{dx}\left[f^{-1}(3)\right] = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(2)} = \frac{1}{\frac{3}{4}(2)^2+1} = \frac{1}{4}$$

3) Find the derivative of the inverse function of $f(x) = e^x + \ln x$ at $x = e$.

$f(1) = e$

$f^{-1}(e) = 1$

$f'(x) = e^x + \frac{1}{x}$

$$\frac{d}{dx}\left[f^{-1}(e)\right] = \frac{1}{f'(f^{-1}(e))} = \frac{1}{f'(1)} = \frac{1}{e^1 + \frac{1}{1}} = \frac{1}{e+1}$$

4) Find the derivative of the inverse function of $y = e^{x^2}, x > 0$.

<u>INVERSE</u>: $x = e^{y^2}$

$\ln x = \ln e^{y^2}$

$\ln x = y^2$

$y = \pm\sqrt{\ln x}$

$f^{-1}(x) = \sqrt{\ln x}$

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{d}{dx}\left[\sqrt{\ln x}\right]$$

$$= \frac{1}{2}(\ln x)^{-1/2} \cdot \frac{1}{x}$$

$$= \frac{1}{2x\sqrt{\ln x}}$$

5) Determine the equation of the tangent line to $f^{-1}(x)$ at the point where x = 3, given the following information: $f(2) = 3$ and $f'(2) = 5$.

$f^{-1}(3) = 2$

$$\frac{d}{dx}\left[f^{-1}(3)\right] = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(2)} = \frac{1}{5}$$

$$y - 2 = \frac{1}{5}(x - 3)$$

6) Determine the equation of the tangent line to $f^{-1}(x)$ at the point $\cancel{(-5, 0)}$ $(-6, 0)$, given the function $f(x) = -5 + 2x - \cos x$. (hint: use alternate form of the derivative)

$f(y) = -5 + 2y - \cos y$

$f'(y) = 2 + \sin y$

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{2 + \sin y} = \frac{1}{2 + \sin(0)} = \frac{1}{2}$$

$$y = \frac{1}{2}(x + 6)$$

7) Find $g'(2)$, where g is the inverse function of $f(x) = x^5 - x^3 + 2x$.

$f(1) = 2$

$g(2) = 1$

$$g'(2) = \frac{1}{f'(g(2))} = \frac{1}{f'(1)} = \frac{1}{5(1)^4 - 3(1)^2 + 2} = \frac{1}{4}$$

$f'(x) = 5x^4 - 3x^2 + 2$

8) Calculate $g'(1)$, where $g(x)$ is the inverse of the function $f(x) = x + e^x$.

$f(0) = 1$

$g(1) = 0$

$$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(0)} = \frac{1}{1 + e^0} = \frac{1}{2}$$

$f'(x) = 1 + e^x$