Complete Solutions Guide for

CALCULUS

EIGHTH EDITION

Larson / Hostetler / Edwards

Volume I
Chapters P–6

Bruce H. Edwards
University of Florida

Houghton Mifflin Company Boston New York

Printed in the U.S.A.

ISBN-10: 0-618-52793-1
ISBN-13: 978-0-618-52793-9

6789- EB -10 09 08

CONTENTS

CHAPTER P
Preparation for Calculus

CHAPTER P
Preparation for Calculus

Section P.1 Graphs and Models

1. $y = -\frac{1}{2}x + 2$

   x-intercept: $(4, 0)$

   y-intercept: $(0, 2)$

   Matches graph (b).

2. $y = \sqrt{9 - x^2}$

   x-intercepts: $(-3, 0), (3, 0)$

   y-intercept: $(0, 3)$

   Matches graph (d).

3. $y = 4 - x^2$

   x-intercepts: $(2, 0), (-2, 0)$

   y-intercept: $(0, 4)$

   Matches graph (a).

4. $y = x^3 - x$

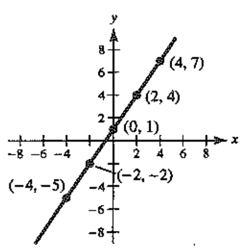
   x-intercepts: $(0, 0), (-1, 0), (1, 0)$

   y-intercept: $(0, 0)$

   Matches graph (c).

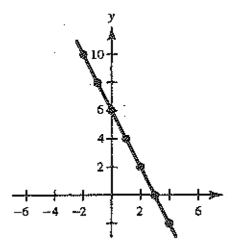
5. $y = \frac{3}{2}x + 1$

x	-4	-2	0	2	4
y	-5	-2	1	4	7



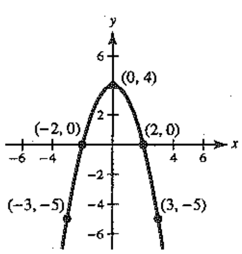
6. $y = 6 - 2x$

x	-2	-1	0	1	2	3	4
y	10	8	6	4	2	0	-2



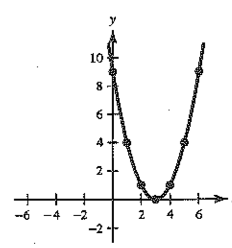
7. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



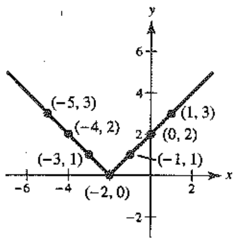
8. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



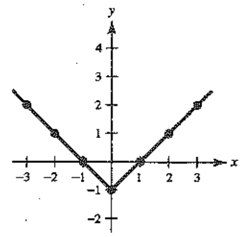
9. $y = |x + 2|$

x	-5	-4	-3	-2	-1	0	1
y	3	2	1	0	1	2	3



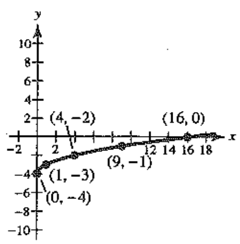
10. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



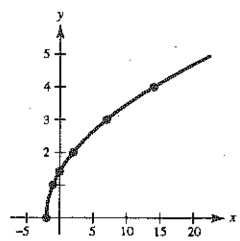
11. $y = \sqrt{x} - 4$

x	0	1	4	9	16
y	-4	-3	-2	-1	0



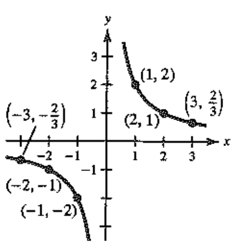
12. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



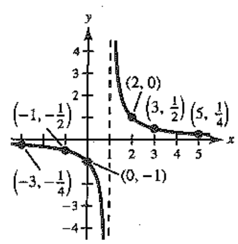
13. $y = \dfrac{2}{x}$

x	-3	-2	-1	0	1	2	3
y	$-\frac{2}{3}$	-1	-2	Undef.	2	1	$\frac{2}{3}$



14. $y = \dfrac{1}{x - 1}$

x	-3	-1	0	1	2	3	5
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



15.

Xmin = -3
Xmax = 5
Xscl = 1
Ymin = -3
Ymax = 5
Yscl = 1

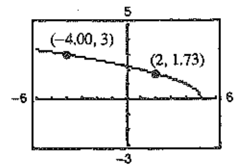
Note that $y = 4$ when $x = 0$.

16.

Xmin = -30
Xmax = 30
Xscl = 5
Ymin = -10
Ymax = 40
Yscl = 5

Note that $y = 10$ when $x = 0$ or $x = 10$.

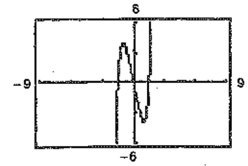
17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73)$ $\left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3)$ $\left(3 = \sqrt{5 - (-4)}\right)$

18. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

19. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2; (0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1; (-2, 0), (1, 0)$

20. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0; (0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2; (0, 0), (\pm 2, 0)$

21. $y = x^2\sqrt{25 - x^2}$

y-intercept: $y = 0^2\sqrt{25 - 0^2}$

$y = 0; (0, 0)$

x-intercepts: $0 = x^2\sqrt{25 - x^2}$

$0 = x^2\sqrt{(5 - x)(5 + x)}$

$x = 0, \pm 5; (0, 0); (\pm 5, 0)$

22. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1; (1, 0)$

23. $y = \dfrac{3\left(2 - \sqrt{x}\right)}{x}$

y-intercept: None. x cannot equal 0.

x-intercept: $0 = \dfrac{3\left(2 - \sqrt{x}\right)}{x}$

$0 = 2 - \sqrt{x}$

$x = 4; (4, 0)$

24. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0; (0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3; (0, 0), (-3, 0)$

25. $x^2y - x^2 + 4y = 0$

y-intercept:

$0^2(y) - 0^2 + 4y = 0$

$y = 0; (0, 0)$

x-intercept:

$x^2(0) - x^2 + 4(0) = 0$

$x = 0; (0, 0)$

26. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1; (0, -1)$

x-intercept:

$$0 = 2x - \sqrt{x^2 + 1}$$
$$2x = \sqrt{x^2 + 1}$$
$$4x^2 = x^2 + 1$$
$$3x^2 = 1$$
$$x^2 = \frac{1}{3}$$
$$x = \pm\frac{\sqrt{3}}{3}$$
$$x = \frac{\sqrt{3}}{3}; \left(\frac{\sqrt{3}}{3}, 0\right)$$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

27. Symmetric with respect to the y-axis since

$$y = (-x)^2 - 2 = x^2 - 2.$$

28. $y = x^2 - x$

No symmetry with respect to either axis or the origin.

29. Symmetric with respect to the x-axis since

$$(-y)^2 = y^2 = x^3 - 4x.$$

30. Symmetric with respect to the origin since

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

31. Symmetric with respect to the origin since

$$(-x)(-y) = xy = 4.$$

32. Symmetric with respect to the x-axis since

$$x(-y)^2 = xy^2 = -10.$$

33. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

34. Symmetric with respect to the origin since

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

35. Symmetric with respect to the origin since

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

36. $y = \dfrac{x^2}{x^2 + 1}$ is symmetric with respect to the y-axis

since $y = \dfrac{(-x)^2}{(-x)^2 + 1} = \dfrac{x^2}{x^2 + 1}.$

37. $y = |x^3 + x|$ is symmetric with respect to the y-axis

since $y = |(-x)^3 + (-x)| = |-(x^3 + x)| = |x^3 + x|.$

38. $|y| - x = 3$ is symmetric with respect to the x-axis

since $|-y| - x = 3$

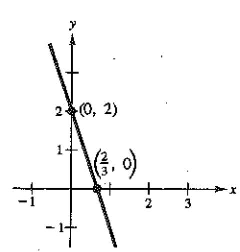
$|y| - x = 3.$

39. $y = -3x + 2$

Intercepts:

$\left(\frac{2}{3}, 0\right), (0, 2)$

Symmetry: none

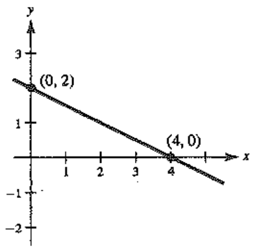


40. $y = -\frac{x}{2} + 2$

Intercepts:

$(4, 0), (0, 2)$

Symmetry: none

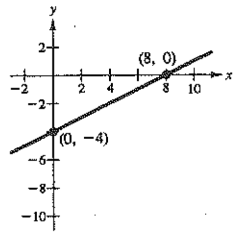


41. $y = \frac{1}{2}x - 4$

Intercepts:

$(8, 0), (0, -4)$

Symmetry: none

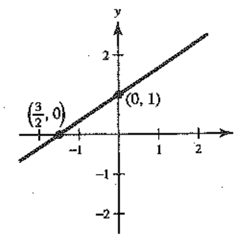


42. $y = \frac{2}{3}x + 1$

Intercepts:

$(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none

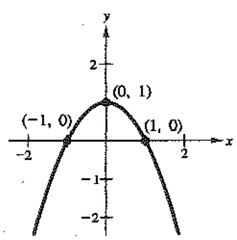


43. $y = 1 - x^2$

Intercepts:

$(1, 0), (-1, 0), (0, 1)$

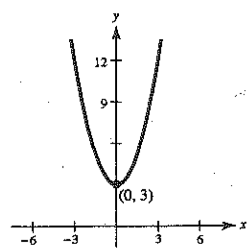
Symmetry: y-axis



44. $y = x^2 + 3$

Intercept: $(0, 3)$

Symmetry: y-axis

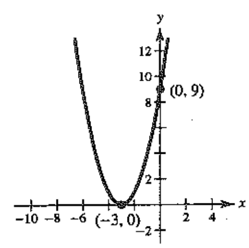


45. $y = (x + 3)^2$

Intercepts:

$(-3, 0), (0, 9)$

Symmetry: none

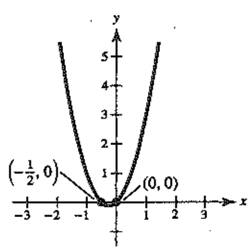


46. $y = 2x^2 + x = x(2x + 1)$

Intercepts:

$(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none

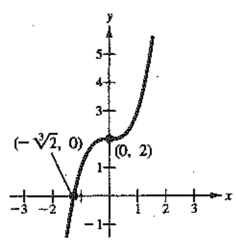


47. $y = x^3 + 2$

Intercepts:

$\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none

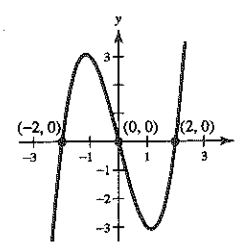


48. $y = x^3 - 4x$

Intercepts:

$(0, 0), (2, 0), (-2, 0)$

Symmetry: origin



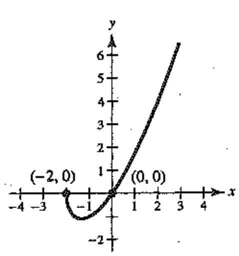
49. $y = x\sqrt{x + 2}$

Intercepts:

$(0, 0), (-2, 0)$

Symmetry: none

Domain: $x \geq -2$



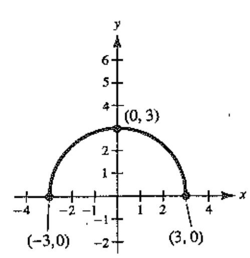
50. $y = \sqrt{9 - x^2}$

Intercepts:

$(-3, 0), (3, 0), (0, 3)$

Symmetry: y-axis

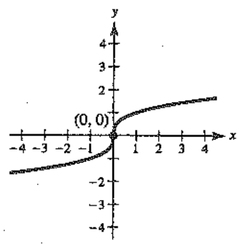
Domain: $[-3, 3]$



51. $x = y^3$

Intercept: $(0, 0)$

Symmetry: origin

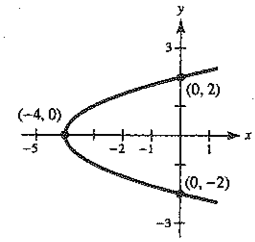


52. $x = y^2 - 4$

Intercepts:

$(0, 2), (0, -2), (-4, 0)$

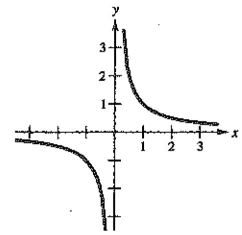
Symmetry: x-axis



53. $y = \dfrac{1}{x}$

Intercepts: none

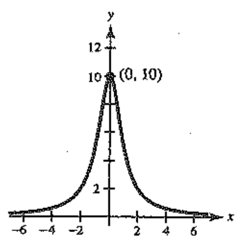
Symmetry: origin



54. $y = \dfrac{10}{x^2 + 1}$

Intercept: $(0, 10)$

Symmetry: y-axis

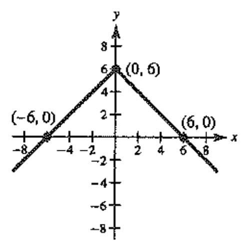


55. $y = 6 - |x|$

Intercepts:

$(0, 6), (-6, 0), (6, 0)$

Symmetry: y-axis

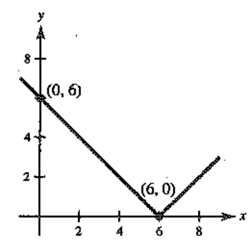


56. $y = |6 - x|$

Intercepts:

$(0, 6), (6, 0)$

Symmetry: none



57. $y^2 - x = 9$

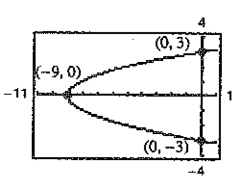
$y^2 = x + 9$

$y = \pm\sqrt{x + 9}$

Intercepts:

$(0, 3), (0, -3), (-9, 0)$

Symmetry: x-axis



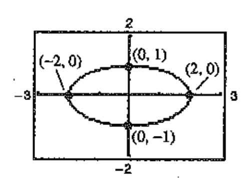
58. $x^2 + 4y^2 = 4 \Longrightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

Intercepts:

$(-2, 0), (2, 0), (0, -1), (0, 1)$

Symmetry: origin and both axes

Domain: $[-2, 2]$



59. $x + 3y^2 = 6$

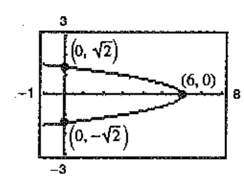
$3y^2 = 6 - x$

$y = \pm\sqrt{2 - \dfrac{x}{3}}$

Intercepts:

$(6, 0), (0, \sqrt{2}), (0, -\sqrt{2})$

Symmetry: x-axis



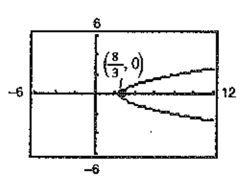
60. $3x - 4y^2 = 8$

$4y^2 = 3x - 8$

$y = \pm\sqrt{\frac{3}{4}x - 2}$

Intercept: $\left(\frac{8}{3}, 0\right)$

Symmetry: x-axis



61. $x + y = 2 \Longrightarrow y = 2 - x$

$2x - y = 1 \Longrightarrow y = 2x - 1$

$2 - x = 2x - 1$

$3 = 3x$

$1 = x$

The corresponding y-value is $y = 1$.

Point of intersection: $(1, 1)$

62. $2x - 3y = 13 \Rightarrow y = \dfrac{2x - 13}{3}$

$5x + 3y = 1 \Rightarrow y = \dfrac{1 - 5x}{3}$

$\dfrac{2x - 13}{3} = \dfrac{1 - 5x}{3}$

$2x - 13 = 1 - 5x$

$7x = 14$

$x = 2$

The corresponding y-value is $y = -3$.

Point of intersection: $(2, -3)$

63. $x^2 + y = 6 \Rightarrow y = 6 - x^2$

$x + y = 4 \Rightarrow y = 4 - x$

$6 - x^2 = 4 - x$

$0 = x^2 - x - 2$

$0 = (x - 2)(x + 1)$

$x = 2, -1$

The corresponding y-values are $y = 2$ (for $x = 2$) and $y = 5$ (for $x = -1$).

Points of intersection: $(2, 2), (-1, 5)$

64. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$3 - x = (x - 1)^2$

$3 - x = x^2 - 2x + 1$

$0 = x^2 - x - 2 = (x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ and $y = 1$.

Points of intersection: $(-1, -2), (2, 1)$

65. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$5 - x^2 = (x - 1)^2$

$5 - x^2 = x^2 - 2x + 1$

$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$

$x = -1$ or $x = 2$

The corresponding y-values are $y = -2$ and $y = 1$.

Points of intersection: $(-1, -2), (2, 1)$

66. $x^2 + y^2 = 25 \Rightarrow y^2 = 25 - x^2$

$2x + y = 10 \Rightarrow y = 10 - 2x$

$25 - x^2 = (10 - 2x)^2$

$25 - x^2 = 100 - 40x + 4x^2$

$0 = 5x^2 - 40x + 75 = 5(x - 3)(x - 5)$

$x = 3$ or $x = 5$

The corresponding y-values are $y = 4$ and $y = 0$.

Points of intersection: $(3, 4), (5, 0)$

67. $y = x^3$

$y = x$

$x^3 = x$

$x^3 - x = 0$

$x(x + 1)(x - 1) = 0$

$x = 0, x = -1$, or $x = 1$

The corresponding y-values are $y = 0, y = -1$, and $y = 1$.

Points of intersection: $(0, 0), (-1, -1), (1, 1)$

68. $y = x^3 - 4x$

$y = -(x + 2)$

$x^3 - 4x = -(x + 2)$

$x^3 - 3x + 2 = 0$

$(x - 1)^2(x + 2) = 0$

$x = 1$ or $x = -2$

The corresponding y-values are $y = -3$ and $y = 0$.

Points of intersection: $(1, -3), (-2, 0)$

69. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$

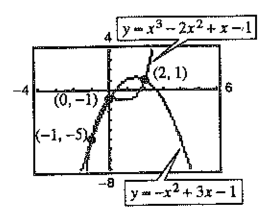
$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$

$x^3 - x^2 - 2x = 0$

$x(x - 2)(x + 1) = 0$

$x = -1, 0, 2$

$(-1, -5), (0, -1), (2, 1)$



70. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$

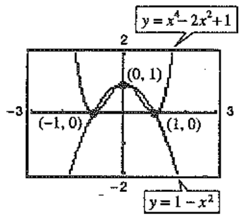
$1 - x^2 = x^4 - 2x^2 + 1$

$0 = x^4 - x^2$

$0 = x^2(x + 1)(x - 1)$

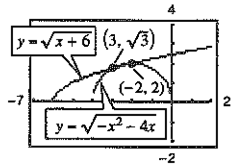
$x = -1, 0, 1$

$(-1, 0), (0, 1), (1, 0)$



71. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



Points of intersection: $(-2, 2), \left(-3, \sqrt{3}\right) \approx (-3, 1.732)$

Analytically, $\sqrt{x + 6} = \sqrt{-x^2 - 4x}$

$x + 6 = -x^2 - 4x$

$x^2 + 5x + 6 = 0$

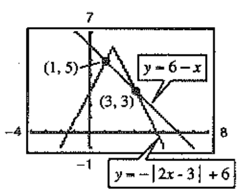
$(x + 3)(x + 2) = 0$

$x = -3, y = \sqrt{3} \Rightarrow \left(-3, \sqrt{3}\right)$

$x = -2, y = 2 \Rightarrow (-2, 2)$.

72. $y = -|2x - 3| + 6$

$y = 6 - x$



Points of intersection: $(3, 3), (1, 5)$

Analytically, $-|2x - 3| + 6 = 6 - x$

$|2x - 3| = x$

$2x - 3 = x$ or $2x - 3 = -x$

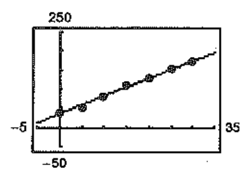
$x = 3$ or $x = 1$.

Hence, $(3, 3), (1, 5)$.

73. (a) Using a graphing utility, you obtain

$y = -0.007t^2 + 4.82t + 35.4$.

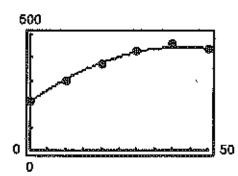
(b)



(c) For 2010, $t = 40$ and $y = 217$.

74. (a) $y = -0.13t^2 + 11.1t + 207$

(b)



(c) For 2010, $t = 60$ and

$y = -0.13(60^2) + 11.1(60) + 207 = 405$ acres.

75.

$C = R$

$5.5\sqrt{x} + 10{,}000 = 3.29x$

$\left(5.5\sqrt{x}\right)^2 = (3.29x - 10{,}000)^2$

$30.25x = 10.8241x^2 - 65{,}800x + 100{,}000{,}000$

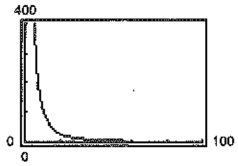
$0 = 10.8241x^2 - 65{,}830.25x + 100{,}000{,}000$ Use the Quadratic Formula.

$x \approx 3133$ units

The other root, $x \approx 2949$, does not satisfy the equation $R = C$.

This problem can also be solved by using a graphing utility and finding the intersection of the graphs of C and R.

76. $y = \dfrac{10{,}770}{x^2} - 0.37$



If the diameter is doubled, the resistance is changed by approximately a factor of $\frac{1}{4}$. For instance, $y(20) \approx 26.555$ and $y(40) \approx 6.36125$.

77. $y = (x + 2)(x - 4)(x - 6)$ (other answers possible)

78. $y = \left(x + \frac{5}{2}\right)(x - 2)\left(x - \frac{3}{2}\right)$ (other answers possible)

79. (i) $y = kx + 5$ matches (b).

Use $(1, 7)$: $7 = k(1) + 5 \Longrightarrow k = 2$, thus, $y = 2x + 5$.

(ii) $y = x^2 + k$ matches (d).

Use $(1, -9)$:

$-9 = (1)^2 + k \Longrightarrow k = -10$, thus, $y = x^2 - 10$.

(iii) $y = kx^{3/2}$ matches (a).

Use $(1, 3)$: $3 = k(1)^{3/2} \Longrightarrow k = 3$, thus, $y = 3x^{3/2}$.

(iv) $xy = k$ matches (c).

Use $(1, 36)$: $(1)(36) = k \Longrightarrow k = 36$, thus, $xy = 36$.

80. (a) If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Since $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. Hence, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

(b) Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Since $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

81. False; x-axis symmetry means that if $(1, -2)$ is on the graph, then $(1, 2)$ is also on the graph.

82. True

83. True; the x-intercepts are

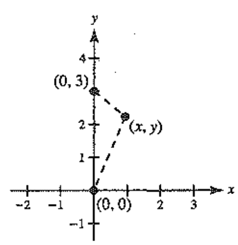
$$\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right).$$

84. True; the x-intercept is

$$\left(-\frac{b}{2a}, 0\right).$$

85.
$$\begin{aligned}
2\sqrt{(x - 0)^2 + (y - 3)^2} &= \sqrt{(x - 0)^2 + (y - 0)^2}\\
4[x^2 + (y - 3)^2] &= x^2 + y^2\\
4x^2 + 4y^2 - 24y + 36 &= x^2 + y^2\\
3x^2 + 3y^2 - 24y + 36 &= 0\\
x^2 + y^2 - 8y + 12 &= 0\\
x^2 + (y - 4)^2 &= 4
\end{aligned}$$

Circle of radius 2 and center $(0, 4)$



86.

$$\begin{aligned}
\text{Distance from the origin} &= K \times \text{Distance from } (2, 0)\\
\sqrt{x^2 + y^2} &= K\sqrt{(x - 2)^2 + y^2},\ K \neq 1\\
x^2 + y^2 &= K^2(x^2 - 4x + 4 + y^2)
\end{aligned}$$

$(1 - K^2)x^2 + (1 - K^2)y^2 + 4K^2x - 4K^2 = 0$

Note: This is the equation of a circle!

Section P.2 Linear Models and Rates of Change

1. $m = 1$

2. $m = 2$

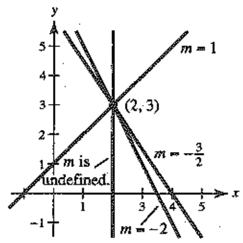
3. $m = 0$

4. $m = -1$

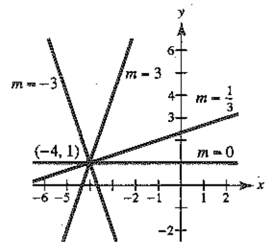
5. $m = -12$

6. $m = \frac{40}{3}$

7.

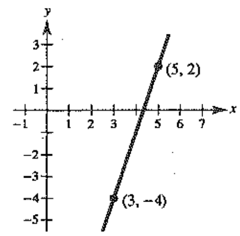


8.



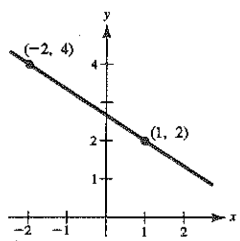
9. $m = \dfrac{2 - (-4)}{5 - 3}$

$= \dfrac{6}{2} = 3$



10. $m = \dfrac{4 - 2}{-2 - 1}$

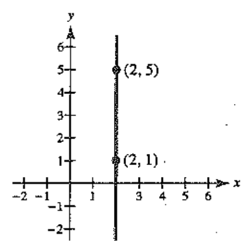
$= -\dfrac{2}{3}$



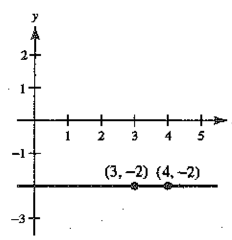
11. $m = \dfrac{5 - 1}{2 - 2}$

$= \dfrac{4}{0}$

undefined

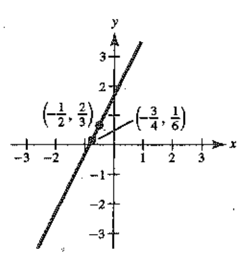


12. $m = \dfrac{-2 - (-2)}{4 - 3} = 0$



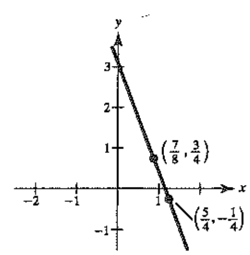
13. $m = \dfrac{2/3 - 1/6}{-1/2 - (-3/4)}$

$= \dfrac{1/2}{1/4} = 2$



14. $m = \dfrac{(3/4) - (-1/4)}{(7/8) - (5/4)}$

$= \dfrac{1}{-3/8} = -\dfrac{8}{3}$



15. Since the slope is 0, the line is horizontal and its equation is $y = 1$. Therefore, three additional points are $(0, 1)$, $(1, 1)$, and $(3, 1)$.

16. Since the slope is undefined, the line is vertical and its equation is $x = -3$. Therefore, three additional points are $(-3, 2)$, $(-3, 3)$, and $(-3, 5)$.

17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

Therefore, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

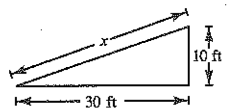
18. The equation of this line is

$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and $(0, 2)$.

19. (a) Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{1}{3}$

(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$

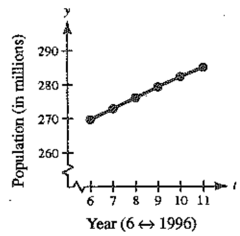
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

20. (a) $m = 400$ indicates that the revenues increase by 400 in one day.

(b) $m = 100$ indicates that the revenues increase by 100 in one day.

(c) $m = 0$ indicates that the revenues do not change from one day to the next.

21. (a)



(b) The slopes of the line segments are:

$$\frac{272.9 - 269.7}{7 - 6} = 3.2$$

$$\frac{276.1 - 272.9}{8 - 7} = 3.2$$

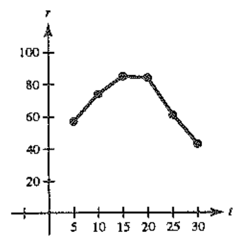
$$\frac{279.3 - 276.1}{9 - 8} = 3.2$$

$$\frac{282.3 - 279.3}{10 - 9} = 3.0$$

$$\frac{285.0 - 282.3}{11 - 10} = 2.7$$

The population increased least rapidly between 2000 and 2001.

22. (a)



(b) The slopes are:

$$\frac{74 - 57}{10 - 5} = 3.4$$

$$\frac{85 - 74}{15 - 10} = 2.2$$

$$\frac{84 - 85}{20 - 15} = -0.2$$

$$\frac{61 - 84}{25 - 20} = -4.6$$

$$\frac{43 - 61}{30 - 25} = -3.6$$

The rate changed most rapidly between 20 and 25 seconds. The change is -4.6 mph/sec.

23. $x + 5y = 20$

$$y = -\tfrac{1}{5}x + 4$$

Therefore, the slope is $m = -\frac{1}{5}$ and the y-intercept is $(0, 4)$.

24. $6x - 5y = 15$

$$y = \tfrac{6}{5}x - 3$$

Therefore, the slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

25. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

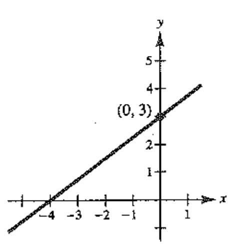
26. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

27. $y = \frac{3}{4}x + 3$

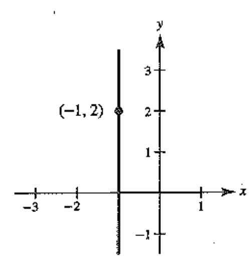
$$4y = 3x + 12$$

$$0 = 3x - 4y + 12$$



28. $x = -1$

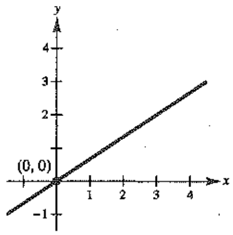
$$x + 1 = 0$$



29. $y = \frac{2}{3}x$

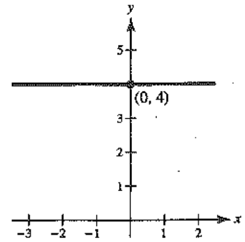
$3y = 2x$

$2x - 3y = 0$



30. $y = 4$

$y - 4 = 0$

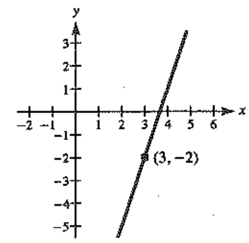


31. $y + 2 = 3(x - 3)$

$y + 2 = 3x - 9$

$y = 3x - 11$

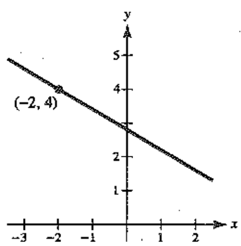
$y - 3x + 11 = 0$



32. $y - 4 = -\frac{3}{5}(x + 2)$

$5y - 20 = -3x - 6$

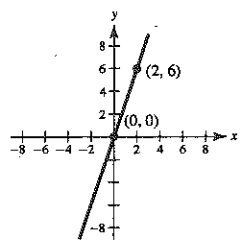
$3x + 5y - 14 = 0$



33. $m = \frac{6 - 0}{2 - 0} = 3$

$y - 0 = 3(x - 0)$

$y = 3x$

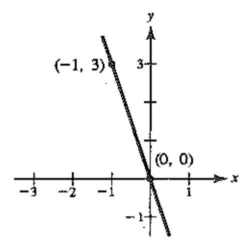


34. $m = \frac{3 - 0}{-1 - 0} = -3$

$y - 0 = -3(x - 0)$

$y = -3x$

$3x + y = 0$

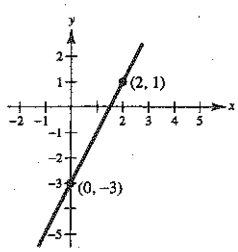


35. $m = \frac{1 - (-3)}{2 - 0} = 2$

$y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$0 = 2x - y - 3$

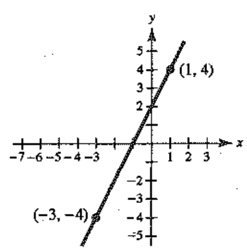


36. $m = \frac{4 - (-4)}{1 - (-3)} = \frac{8}{4} = 2$

$y - 4 = 2(x - 1)$

$y - 4 = 2x - 2$

$0 = 2x - y + 2$

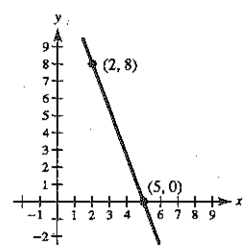


37. $m = \frac{8 - 0}{2 - 5} = -\frac{8}{3}$

$y - 0 = -\frac{8}{3}(x - 5)$

$y = -\frac{8}{3}x + \frac{40}{3}$

$3y + 8x - 40 = 0$

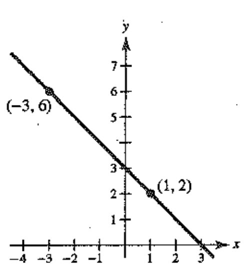


38. $m = \frac{6 - 2}{-3 - 1} = \frac{4}{-4} = -1$

$y - 2 = -1(x - 1)$

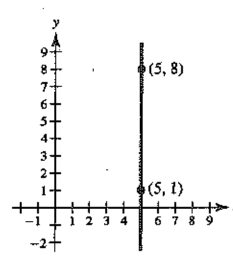
$y - 2 = -x + 1$

$x + y - 3 = 0$



39. $m = \frac{8 - 1}{5 - 5}$ Undefined

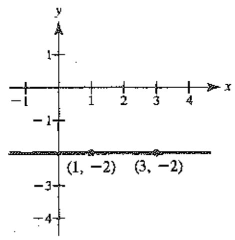
Vertical line $x = 5$



40. $m = 0$

$y = -2$

$y + 2 = 0$

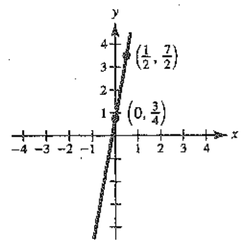


41. $m = \dfrac{7/2 - 3/4}{1/2 - 0} = \dfrac{11/4}{1/2} = \dfrac{11}{2}$

$y - \dfrac{3}{4} = \dfrac{11}{2}(x - 0)$

$y = \dfrac{11}{2}x + \dfrac{3}{4}$

$22x - 4y + 3 = 0$



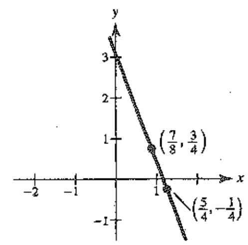
42. $m = \dfrac{(3/4) - (-1/4)}{(7/8) - (5/4)}$

$= \dfrac{1}{-3/8} = -\dfrac{8}{3}$

$y + \dfrac{1}{4} = \dfrac{-8}{3}\left(x - \dfrac{5}{4}\right)$

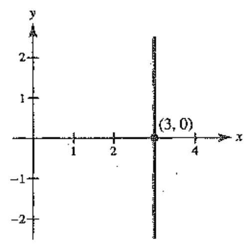
$12y + 3 = -32x + 40$

$32x + 12y - 37 = 0$



43. $x = 3$

$x - 3 = 0$

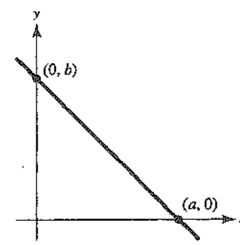


44. $m = -\dfrac{b}{a}$

$y = \dfrac{-b}{a}x + b$

$\dfrac{b}{a}x + y = b$

$\dfrac{x}{a} + \dfrac{y}{b} = 1$



45. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$3x + 2y - 6 = 0$

46. $\dfrac{x}{-2/3} + \dfrac{y}{-2} = 1$

$\dfrac{-3x}{2} - \dfrac{y}{2} = 1$

$3x + y = -2$

$3x + y + 2 = 0$

47. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{1}{a} + \dfrac{2}{a} = 1$

$\dfrac{3}{a} = 1$

$a = 3 \Rightarrow x + y = 3$

$x + y - 3 = 0$

48. $\dfrac{x}{a} + \dfrac{y}{a} = 1$

$\dfrac{-3}{a} + \dfrac{4}{a} = 1$

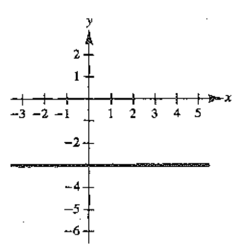
$\dfrac{1}{a} = 1$

$a = 1 \Rightarrow x + y = 1$

$x + y - 1 = 0$

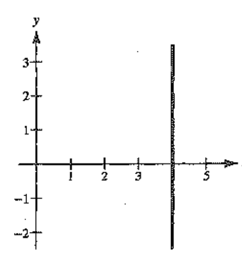
49. $y = -3$

$y + 3 = 0$

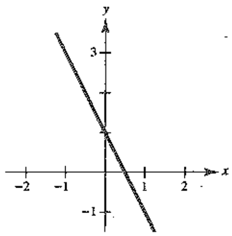


50. $x = 4$

$x - 4 = 0$

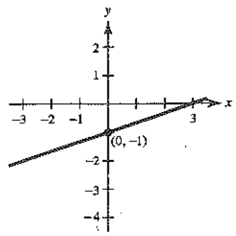


51. $y = -2x + 1$



52. $y = \frac{1}{3}x - 1$

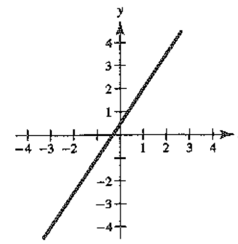
$3y - x + 3 = 0$



53. $y - 2 = \frac{3}{2}(x - 1)$

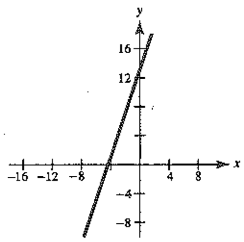
$y = \frac{3}{2}x + \frac{1}{2}$

$2y - 3x - 1 = 0$



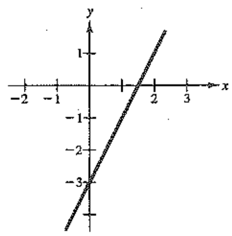
54. $y - 1 = 3(x + 4)$

$y = 3x + 13$



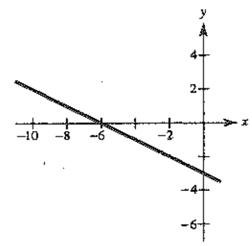
55. $2x - y - 3 = 0$

$y = 2x - 3$

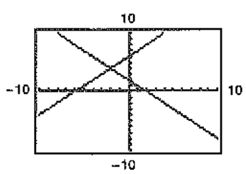


56. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

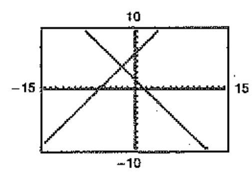


57. (a)



The lines do not appear perpendicular.

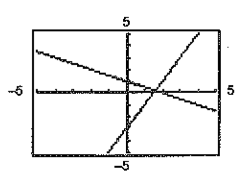
(b)



The lines appear perpendicular.

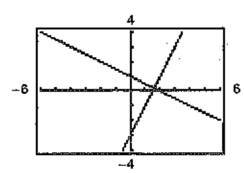
The lines are perpendicular because their slopes 1 and -1 are negative reciprocals of each other. You must use a square setting in order for perpendicular lines to appear perpendicular. Answers depend on calculator used.

58. (a)



The lines do not appear perpendicular.

(b)



The lines appear perpendicular.

The lines are perpendicular because their slopes 2 and $-\frac{1}{2}$ are negative reciprocals of each other. You must use a square setting in order for perpendicular lines to appear perpendicular. Answers depend on calculator used.

59. $4x - 2y = 3$

$y = 2x - \frac{3}{2}$

$m = 2$

(a) $y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$2x - y - 3 = 0$

(b) $y - 1 = -\frac{1}{2}(x - 2)$

$2y - 2 = -x + 2$

$x + 2y - 4 = 0$

60. $x + y = 7$

$y = -x + 7$

$m = -1$

(a) $y - 2 = -1(x + 3)$

$y - 2 = -x - 3$

$x + y + 1 = 0$

(b) $y - 2 = 1(x + 3)$

$y - 2 = x + 3$

$x - y + 5 = 0$

61. $5x - 3y = 0$

$y = \frac{5}{3}x$

$m = \frac{5}{3}$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$24y - 21 = 40x - 30$

$24y - 40x + 9 = 0$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$40y - 35 = -24x + 18$

$40y + 24x - 53 = 0$

62. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m = -\frac{3}{4}$

(a) $y - 4 = -\frac{3}{4}(x + 6)$

$4y - 16 = -3x - 18$

$3x + 4y + 2 = 0$

(b) $y - 4 = \frac{4}{3}(x + 6)$

$3y - 12 = 4x + 24$

$4x - 3y + 36 = 0$

63. The given line is vertical.

(a) $x = 2 \Longrightarrow x - 2 = 0$

(b) $y = 5 \Longrightarrow y - 5 = 0$

64. (a) $y = 0$

(b) $x = -1 \Longrightarrow x + 1 = 0$

65. The slope is 125. $V = 2540$ when $t = 4$.

$V = 125(t - 4) + 2540 = 125t + 2040$

66. The slope is 4.5. $V = 156$ when $t = 4$.

$V = 4.5(t - 4) + 156 = 4.5t + 138$

67. The slope is -2000. $V = 20{,}400$ when $t = 4$.

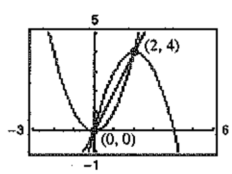
$V = -2000(t - 4) + 20{,}400 = -2000t + 28{,}400$

68. The slope is -5600. $V = 245{,}000$ when $t = 4$.

$V = -5600(t - 4) + 245{,}000$

$= -5600t + 267{,}400$

69.



You can use the graphing utility to determine that the points of intersection are (0, 0) and (2, 4). Analytically,

$x^2 = 4x - x^2$

$2x^2 - 4x = 0$

$2x(x - 2) = 0$

$x = 0 \Longrightarrow y = 0 \Longrightarrow (0, 0)$

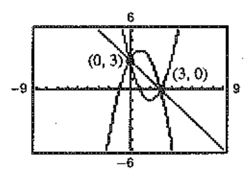
$x = 2 \Longrightarrow y = 4 \Longrightarrow (2, 4).$

The slope of the line joining (0, 0) and (2, 4) is $m = (4 - 0)/(2 - 0) = 2$. Hence, an equation of the line is

$y - 0 = 2(x - 0)$

$y = 2x.$

70. $y = x^2 - 4x + 3,\ y = -x^2 + 2x + 3$



You can use the graphing utility to determine that the points of intersection are (0, 3) and (3, 0). Analytically,

$x^2 - 4x + 3 = -x^2 + 2x + 3$

$2x^2 - 6x = 0$

$2x(x - 3) = 0$

$x = 0 \Longrightarrow y = 3 \Longrightarrow (0, 3)$

$x = 3 \Longrightarrow y = 0 \Longrightarrow (3, 0).$

The slope of the line joining (0, 3) and (3, 0) is $m = (0 - 3)/(3 - 0) = -1$. Hence, an equation of the line is

$y - 3 = -1(x - 0)$

$y = -x + 3.$

71. $m_1 = \dfrac{1-0}{-2-(-1)} = -1$

$m_2 = \dfrac{-2-0}{2-(-1)} = -\dfrac{2}{3}$

$m_1 \neq m_2$

The points are not collinear.

72. $m_1 = \dfrac{-6-4}{7-0} = -\dfrac{10}{7}$

$m_2 = \dfrac{11-4}{-5-0} = -\dfrac{7}{5}$

$m_1 \neq m_2$

The points are not collinear.

73. Equations of perpendicular bisectors:

$$y - \frac{c}{2} = \frac{a-b}{c}\left(x - \frac{a+b}{2}\right)$$

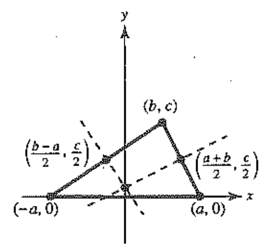
$$y - \frac{c}{2} = \frac{a+b}{-c}\left(x - \frac{b-a}{2}\right)$$

Setting the right-hand sides of the two equations equal and solving for x yields $x = 0$.

Letting $x = 0$ in either equation gives the point of intersection:

$$\left(0, \frac{-a^2+b^2+c^2}{2c}\right).$$

This point lies on the third perpendicular bisector, $x = 0$.



74. Equations of medians:

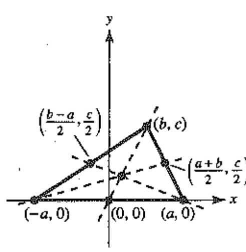
$$y = \frac{c}{b}x$$

$$y = \frac{c}{3a+b}(x+a)$$

$$y = \frac{c}{-3a+b}(x-a)$$

Solving simultaneously, the point of intersection is

$$\left(\frac{b}{3}, \frac{c}{3}\right).$$



75. Equations of altitudes:

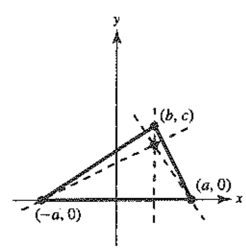
$$y = \frac{a-b}{c}(x+a)$$

$$x = b$$

$$y = -\frac{a+b}{c}(x-a)$$

Solving simultaneously, the point of intersection is

$$\left(b, \frac{a^2-b^2}{c}\right).$$



76. The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(b, \dfrac{a^2-b^2}{c}\right)$ is:

$$m_1 = \frac{[(a^2-b^2)/c] - (c/3)}{b-(b/3)} = \frac{(3a^2-3b^2-c^2)/(3c)}{(2b)/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

The slope of the line segment from $\left(\dfrac{b}{3}, \dfrac{c}{3}\right)$ to $\left(0, \dfrac{-a^2+b^2+c^2}{2c}\right)$ is:

$$m_2 = \frac{[(-a^2+b^2+c^2)/(2c)] - (c/3)}{0-(b/3)} = \frac{(-3a^2+3b^2+3c^2-2c^2)/(6c)}{-b/3} = \frac{3a^2-3b^2-c^2}{2bc}$$

$m_1 = m_2$

Therefore, the points are collinear.

77. Find the equation of the line through the points (0, 32) and (100, 212).

$$m = \tfrac{180}{100} = \tfrac{9}{5}$$

$$F - 32 = \tfrac{9}{5}(C - 0)$$

$$F = \tfrac{9}{5}C + 32$$

or

$$C = \tfrac{1}{9}(5F - 160)$$

$$5F - 9C - 160 = 0$$

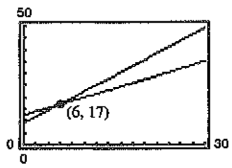
For $F = 72°$, $C \approx 22.2°$.

78. $C = 0.34x + 150$

If $x = 137$, $C = 0.34(137) + 150 = \$196.58$.

79. (a) $W_1 = 0.75x + 12.50$

$W_2 = 1.30x + 9.20$

(b) 

Using a graphing utility, the point of intersection is (6, 17).
Analytically,

$$0.75x + 12.50 = 1.30x + 9.20$$

$$3.3 = 0.55x \Longrightarrow x = 6$$

$$y = 0.75(6) + 12.50 = 17.$$

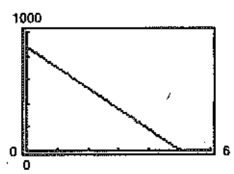
(c) Both jobs pay \$17 per hour if 6 units are produced. For someone who can produce more than 6 units per hour, the second offer would pay more. For a worker who produces less than 6 units per hour, the first offer pays more.

80. (a) Depreciation per year:

$\frac{875}{5} = \$175$

$y = 875 - 175x$

where $0 \le x \le 5$.



(b) $y = 875 - 175(2) = \$525$

(c) $200 = 875 - 175x$

$175x = 675$

$x \approx 3.86$ years

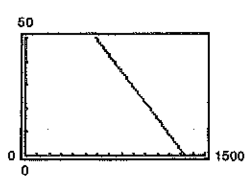
81. (a) Two points are (50, 580) and (47, 625). The slope is

$$m = \frac{625 - 580}{47 - 50} = -15.$$

$$p - 580 = -15(x - 50)$$

$$p = -15x + 750 + 580 = -15x + 1330$$

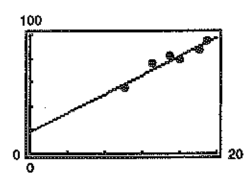
$$\text{or } x = \frac{1}{15}(1330 - p)$$

(b) 

If $p = 655$, $x = \dfrac{1}{15}(1330 - 655) = 45$ units.

(c) If $p = 595$, $x = \dfrac{1}{15}(1330 - 595) = 49$ units.

82. (a) $y = 18.91 + 3.97x$ (x = quiz score, y = test score)

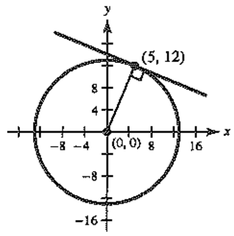
(b) 

(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

83. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).



Slope of the line joining (5, 12) and (0, 0) is

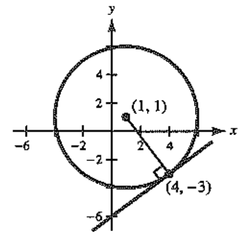
$\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$
$$y = \frac{-5}{12}x + \frac{169}{12}$$

$12y + 5x - 169 = 0.$

84. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$\frac{1 + 3}{1 - 4} = \frac{-4}{3}$.

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$
$$y = \frac{3}{4}x - 6$$

$4y - 3x + 24 = 0$

85. $4x + 3y - 10 = 0 \Rightarrow d = \frac{|4(0) + 3(0) - 10|}{\sqrt{4^2 + 3^2}} = \frac{10}{5} = 2$

86. $4x + 3y - 10 = 0 \Rightarrow d = \frac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \frac{7}{5}$

87. $x - y - 2 = 0 \Rightarrow d = \frac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

88. $x + 1 = 0 \Rightarrow d = \frac{|1(6) + (0)(2) + 1|}{\sqrt{1^2 + 0^2}} = 7$

89. A point on the line $x + y = 1$ is (0, 1). The distance from the point (0, 1) to $x + y - 5 = 0$ is

$$d = \frac{|1(0) + 1(1) - 5|}{\sqrt{1^2 + 1^2}} = \frac{|1 - 5|}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

90. A point on the line $3x - 4y = 1$ is $(-1, -1)$. The distance from the point $(-1, -1)$ to $3x - 4y - 10 = 0$ is

$$d = \frac{|3(-1) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|-3 + 4 - 10|}{5} = \frac{9}{5}.$$

91. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.)

—CONTINUED—

91. —CONTINUED—

The slope of the line $Ax + By + C = 0$ is $-A/B$. The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$

$$Ay - Ay_1 = Bx - Bx_1$$

$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow \quad A^2x + ABy = -AC \qquad (1)$$

$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \quad \underline{B^2x - ABy} = \underline{B^2x_1 - ABy_1} \qquad (2)$$

$$(A^2 + B^2)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$

$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow \quad ABx + B^2y = -BC \qquad (3)$$

$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y} = \underline{-ABx_1 + A^2y_1} \qquad (4)$$

$$(A^2 + B^2)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$

$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

The distance between (x_1, y_1) and this point gives us the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$d = \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2}$$

$$= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2}$$

$$= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2}$$

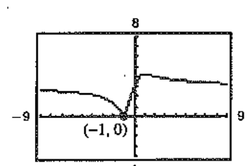
$$= \sqrt{\frac{(A^2 + B^2)(C + Ax_1 + By_1)^2}{(A^2 + B^2)^2}}$$

$$= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

92. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}}$$

$$= \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$



The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point $(3, 1)$.

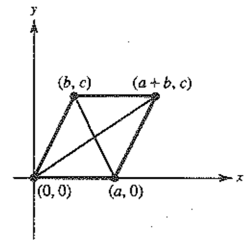
93. For simplicity, let the vertices of the rhombus be $(0, 0)$, $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure. The slopes of the diagonals are then

$$m_1 = \frac{c}{a+b} \text{ and } m_2 = \frac{c}{b-a}.$$

Since the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and we have

$$m_1 m_2 = \frac{c}{a+b} \cdot \frac{c}{b-a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



94. For simplicity, let the vertices of the quadrilateral be $(0, 0)$, $(a, 0)$, (b, c), and (d, e), as shown in the figure. The midpoints of the sides are

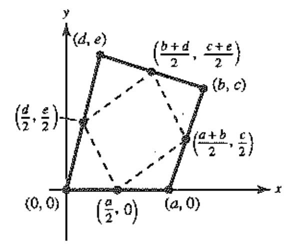
$$\left(\frac{a}{2}, 0\right), \left(\frac{a+b}{2}, \frac{c}{2}\right), \left(\frac{b+d}{2}, \frac{c+e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a+b}{2} - \frac{a}{2}} = \frac{\frac{c+e}{2} - \frac{e}{2}}{\frac{b+d}{2} - \frac{d}{2}} = \frac{c}{b}$$

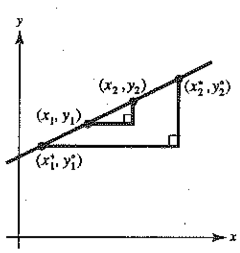
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c+e}{2}}{\frac{a+b}{2} - \frac{b+d}{2}} = -\frac{e}{a-d}$$

Therefore, the figure is a paralleogram.



95. Consider the figure below in which the four points are collinear. Since the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



96. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. Hence, $m_2 = m_3 \Longrightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

97. True.

$$ax + by = c_1 \Longrightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Longrightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Longrightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Longrightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

98. False; if m_1 is positive, then $m_2 = -1/m_1$ is negative.

Section P.3 Functions and Their Graphs

1. (a) Domain of f: $-4 \le x \le 4$

Range of f: $-3 \le y \le 5$

Domain of g: $-3 \le x \le 3$

Range of g: $-4 \le y \le 4$

(b) $f(-2) = -1$

$g(3) = -4$

(c) $f(x) = g(x)$ for $x = -1$

(d) $f(x) = 2$ for $x = 1$

(e) $g(x) = 0$ for $x = -1$, 1 and 2

2. (a) Domain of f: $-5 \le x \le 5$

Range of f: $-4 \le y \le 4$

Domain of g: $-4 \le x \le 5$

Range of g: $-4 \le y \le 2$

(b) $f(-2) = -2$

$g(3) = 2$

(c) $f(x) = g(x)$ for $x = -2$ and $x = 4$

(d) $f(x) = 2$ for $x = -4, 4$

(e) $g(x) = 0$ for $x = -1$

3. (a) $f(0) = 2(0) - 3 = -3$

(b) $f(-3) = 2(-3) - 3 = -9$

(c) $f(b) = 2b - 3$

(d) $f(x-1) = 2(x-1) - 3 = 2x - 5$

4. (a) $f(-2) = \sqrt{-2+3} = \sqrt{1} = 1$

(b) $f(6) = \sqrt{6+3} = \sqrt{9} = 3$

(c) $f(-5) = \sqrt{-5+3} = \sqrt{-2}$, undefined

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 3}$

5. (a) $g(0) = 3 - 0^2 = 3$

(b) $g(\sqrt{3}) = 3 - (\sqrt{3})^2 = 3 - 3 = 0$

(c) $g(-2) = 3 - (-2)^2 = 3 - 4 = -1$

(d) $g(t-1) = 3 - (t-1)^2 = -t^2 + 2t + 2$

6. (a) $g(4) = 4^2(4-4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c-4) = c^3 - 4c^2$

(d) $g(t+4) = (t+4)^2(t+4-4)$

$= (t+4)^2 t = t^3 + 8t^2 + 16t$

7. (a) $f(0) = \cos(2(0)) = \cos 0 = 1$

(b) $f\left(-\frac{\pi}{4}\right) = \cos\left(2\left(-\frac{\pi}{4}\right)\right) = \cos\left(-\frac{\pi}{2}\right) = 0$

(c) $f\left(\frac{\pi}{3}\right) = \cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\frac{2\pi}{3} = -\frac{1}{2}$

8. (a) $f(\pi) = \sin \pi = 0$

(b) $f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

(c) $f\left(\frac{2\pi}{3}\right) = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

9. $$\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2, \Delta x \ne 0$$

10. $$\frac{f(x) - f(1)}{x-1} = \frac{3x - 1 - (3-1)}{x-1} = \frac{3(x-1)}{x-1} = 3, x \ne 1$$

11. $$\frac{f(x) - f(2)}{x-2} = \frac{(1/\sqrt{x-1} - 1)}{x-2}$$

$$= \frac{1 - \sqrt{x-1}}{(x-2)\sqrt{x-1}} \cdot \frac{1 + \sqrt{x-1}}{1 + \sqrt{x-1}} = \frac{2-x}{(x-2)\sqrt{x-1}(1+\sqrt{x-1})} = \frac{-1}{\sqrt{x-1}(1+\sqrt{x-1})}, x \ne 2$$

12. $$\frac{f(x) - f(1)}{x-1} = \frac{x^3 - x - 0}{x-1} = \frac{x(x+1)(x-1)}{x-1} = x(x+1), x \ne 1$$

13. $h(x) = -\sqrt{x+3}$

Domain: $x + 3 \ge 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

14. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

15. $f(t) = \sec \dfrac{\pi t}{4}$

$\dfrac{\pi t}{4} \ne \dfrac{(2k+1)\pi}{2} \Rightarrow t \ne 4k + 2$

Domain: all $t \ne 4k + 2$, k an integer

Range: $(-\infty, -1] \cup [1, \infty)$

16. $h(t) = \cot t$

Domain: all $t \ne k\pi$, k an integer

Range: $(-\infty, \infty)$

17. $f(x) = \dfrac{1}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

18. $g(x) = \dfrac{2}{x-1}$

Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

19. $f(x) = \sqrt{x} + \sqrt{1-x}$

$x \ge 0$ and $1 - x \ge 0$

$x \ge 0$ and $x \le 1$

Domain: $0 \le x \le 1$

20. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \ge 0$

$(x-2)(x-1) \ge 0$

Domain: $x \ge 2$ or $x \le 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

21. $g(x) = \dfrac{2}{1 - \cos x}$

$1 - \cos x \ne 0$

$\cos x \ne 1$

Domain: all $x \ne 2n\pi$, n an integer

22. $h(x) = \dfrac{1}{\sin x - (1/2)}$

$\sin x - \dfrac{1}{2} \ne 0$

$\sin x \ne \dfrac{1}{2}$

Domain: all $x \ne \dfrac{\pi}{6} + 2n\pi$,

$\dfrac{5\pi}{6} + 2n\pi$, n integer

23. $f(x) = \dfrac{1}{|x+3|}$

$|x+3| \ne 0$

$x + 3 \ne 0$

Domain: all $x \ne -3$

24. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \ne 0$

$(x-2)(x+2) \ne 0$

Domain: all $x \ne \pm 2$

25. $f(x) = \begin{cases} 2x + 1, x < 0 \\ 2x + 2, x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \ge 0$ for all t)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

26. $f(x) = \begin{cases} x^2 + 2, x \le 1 \\ 2x^2 + 2, x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

27. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

28. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

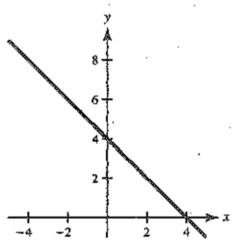
Domain: $[-4, \infty)$

Range: $[0, \infty)$

29. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

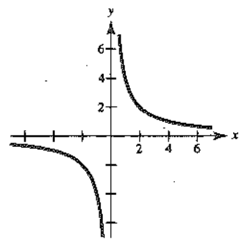
Range: $(-\infty, \infty)$



30. $g(x) = \dfrac{4}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

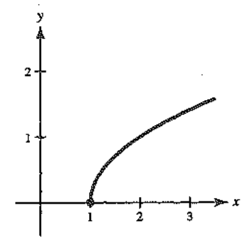
Range: $(-\infty, 0) \cup (0, \infty)$



31. $h(x) = \sqrt{x - 1}$

Domain: $[1, \infty)$

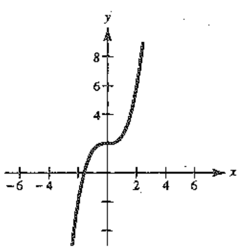
Range: $[0, \infty)$



32. $f(x) = \frac{1}{2}x^3 + 2$

Domain: $(-\infty, \infty)$

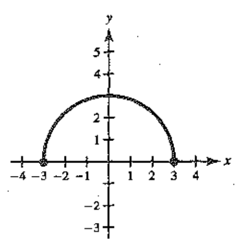
Range: $(-\infty, \infty)$



33. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



34. $f(x) = x + \sqrt{4 - x^2}$

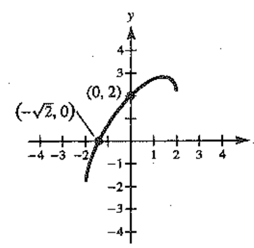
Domain: $[-2, 2]$

Range:

$[-2, 2\sqrt{2}] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

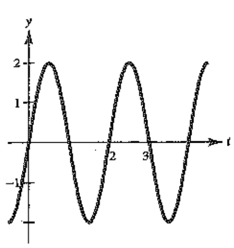
x-intercept: $(-\sqrt{2}, 0)$



35. $g(t) = 2 \sin \pi t$

Domain: $(-\infty, \infty)$

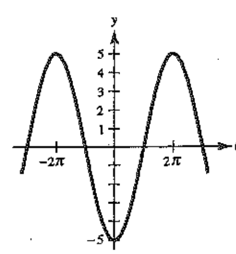
Range: $[-2, 2]$



36. $h(\theta) = -5 \cos \dfrac{\theta}{2}$

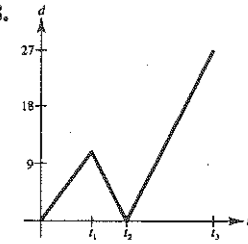
Domain: $(-\infty, \infty)$

Range: $[-5, 5]$



37. The student travels $\frac{2-0}{4-0} = \frac{1}{2}$ mi/min during the first 4 minutes. The student is stationary for the following 2 minutes. Finally, the student travels $\frac{6-2}{10-6} = 1$ mi/min during the final 4 minutes.

38.



39. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

40. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

41. y is a function of x. Vertical lines intersect the graph at most once.

42. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

43. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

y is not a function of x since there are two values of y for some x.

44. $x^2 + y = 4 \Rightarrow y = 4 - x^2$

y is a function of x since there is one value of y for each x.

45. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x since there are two values of y for some x.

46. $x^2y - x^2 + 4y = 0 \Rightarrow y = \frac{x^2}{x^2 + 4}$

y is a function of x since there is one value of y for each x.

47. $y = f(x + 5)$ is a horizontal shift 5 units to the left. Matches d.

48. $y = f(x) - 5$ is a vertical shift 5 units downward. Matches b.

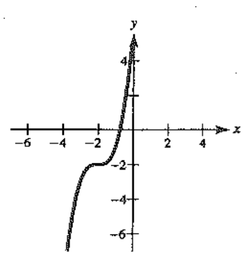
49. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

50. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

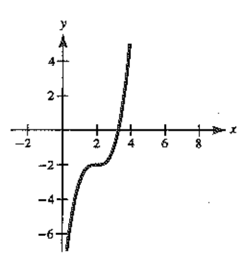
51. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

52. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

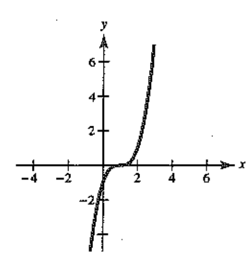
53. (a) The graph is shifted 3 units to the left.



(b) The graph is shifted 1 unit to the right.



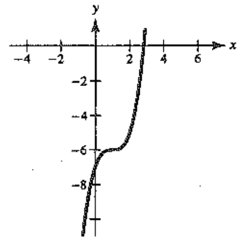
(c) The graph is shifted 2 units upward.



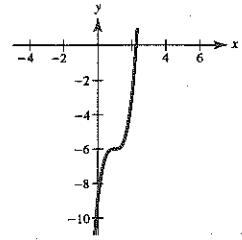
—CONTINUED—

53. —CONTINUED—

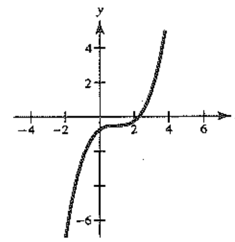
(d) The graph is shifted 4 units downward.



(e) The graph is stretched vertically by a factor of 3.



(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.

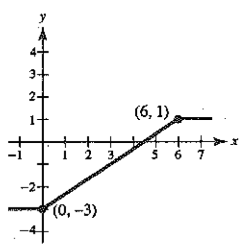


54. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

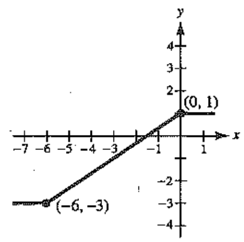
$g(0) = f(-4) = -3$

Shift f right 4 units



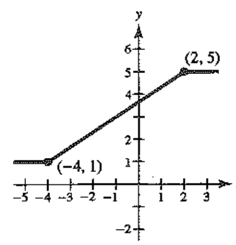
(b) $g(x) = f(x + 2)$

Shift f left 2 units



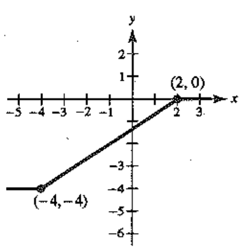
(c) $g(x) = f(x) + 4$

Vertical shift upwards 4 units



(d) $g(x) = f(x) - 1$

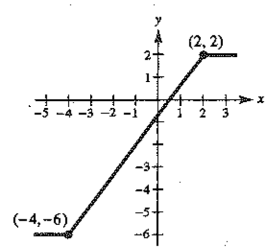
Vertical shift down 1 unit



(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

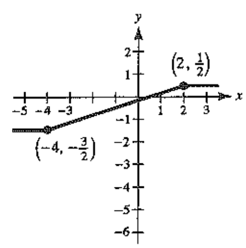
$g(-4) = 2f(-4) = -6$



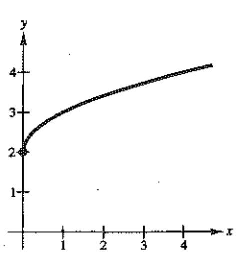
(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

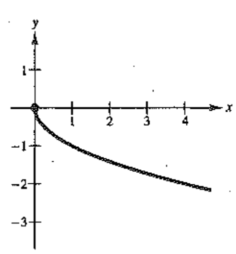


55. (a) $y = \sqrt{x} + 2$



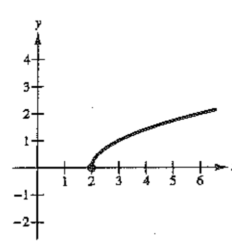
Vertical shift 2 units upward

(b) $y = -\sqrt{x}$



Reflection about the x-axis

(c) $y = \sqrt{x - 2}$



Horizontal shift 2 units to the right

56. (a) $h(x) = \sin(x + (\pi/2)) + 1$ is a horizontal shift $\pi/2$ units to the left, followed by a vertical shift 1 unit upwards.

(b) $h(x) = -\sin(x - 1)$ is a horizontal shift 1 unit to the right followed by a reflection about the x-axis.

57. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \quad (x \geq 0)$

58. $f(x) = \sin x, g(x) = \pi x$

(a) $f(g(2)) = f(2\pi) = \sin(2\pi) = 0$

(b) $f(g(1/2)) = f(\pi/2) = \sin(\pi/2) = 1$

(c) $g(f(0)) = g(0) = 0$

(d) $g(f(\pi/4)) = g(\sin(\pi/4)) = g(\sqrt{2}/2) = \pi(\sqrt{2}/2) = \dfrac{\pi\sqrt{2}}{2}$

(e) $f(g(x)) = f(\pi x) = \sin(\pi x)$

(f) $g(f(x)) = g(\sin x) = \pi \sin x$

59. $f(x) = x^2, g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x, \quad x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

60. $f(x) = x^2 - 1, g(x) = \cos x$

$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^2 x - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = \cos(x^2 - 1)$

Domain: $(-\infty, \infty)$

No, $f \circ g \neq g \circ f$.

61. $f(x) = \dfrac{3}{x}, g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3}{x}\right) = \left(\dfrac{3}{x}\right)^2 - 1 = \dfrac{9}{x^2} - 1 = \dfrac{9 - x^2}{x^2}$

Domain: all $x \neq 0$

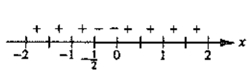
No, $f \circ g \neq g \circ f$.

62. $(f \circ g)(x) = f(\sqrt{x + 2}) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

63. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1) = f(-4)$, which is undefined

64. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

65. $F(x) = \sqrt{2x - 2}$

Let $h(x) = x - 1$, $g(x) = 2x$ and $f(x) = \sqrt{x}$.

Then, $(f \circ g \circ h)(x) = f(g(x - 1)) = f(2(x - 1)) = \sqrt{2(x - 1)} = \sqrt{2x - 2} = F(x)$.

[Other answers possible]

66. $F(x) = -4 \sin(1 - x)$

Let $f(x) = -4x$, $g(x) = \sin x$ and $h(x) = 1 - x$.

Then, $(f \circ g \circ h)(x) = f(g(1 - x)) = f(\sin(1 - x)) = -4 \sin(1 - x) = F(x)$.

[Other answers possible]

67. $f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

Even

68. $f(-x) = \sqrt[3]{-x} = -\sqrt[3]{x} = -f(x)$

Odd

69. $f(-x) = (-x)\cos(-x) = -x \cos x = -f(x)$

Odd

70. $f(-x) = \sin^2(-x) = \sin(-x)\sin(-x) = (-\sin x)(-\sin x) = \sin^2 x$

Even

71. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

72. (a) If f is even, then $(-4, 9)$ is on the graph.

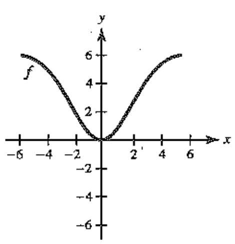
(b) If f is odd, then $(-4, -9)$ is on the graph.

73. f is even because the graph is symmetric about the y-axis.

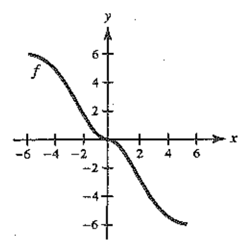
g is neither even nor odd.

h is odd because the graph is symmetric about the origin.

74. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



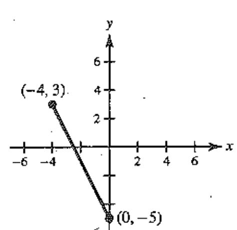
75.

$$\text{Slope} = \frac{3 + 5}{-4 - 0} = -2$$

$$y + 5 = -2(x - 0)$$

$$y = -2x - 5$$

$$f(x) = -2x - 5, \quad -4 \le x \le 0$$

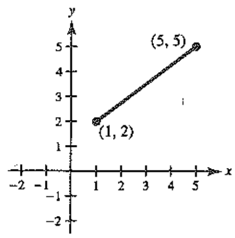


76. $\text{Slope} = \dfrac{5-2}{5-1} = \dfrac{3}{4}$

$$y - 2 = \frac{3}{4}(x - 1)$$

$$y = \frac{3}{4}x + \frac{5}{4}$$

$$f(x) = \frac{3}{4}x + \frac{5}{4}, \quad 1 \le x \le 5$$

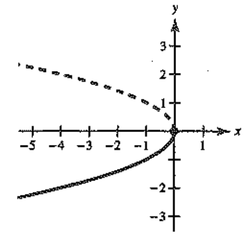


77. $x + y^2 = 0$

$$y^2 = -x$$

$$y = -\sqrt{-x}$$

$$f(x) = -\sqrt{-x}, \quad x \le 0$$

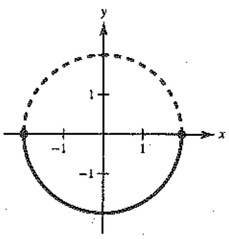


78. $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = -\sqrt{4 - x^2}$$

$$f(x) = -\sqrt{4 - x^2}, \quad -2 \le x \le 2$$



79. Matches (ii). The function is $g(x) = cx^2$. Since $(1, -2)$ satisfies the equation, $c = -2$. Thus, $g(x) = -2x^2$.

80. Matches (i). The function is $f(x) = cx$. Since $(1, 1/4)$ satisfies the equation, $c = 1/4$. Thus, $f(x) = (1/4)x$.

81. Matches (iv). The function is $r(x) = c/x$, since it must be undefined at $x = 0$. Since $(1, 32)$ satisfies the equation, $c = 32$. Thus, $r(x) = 32/x$.

82. Matches (iii). The function is $h(x) = c\sqrt{|x|}$. Since $(1, 3)$ satisfies the equation, $c = 3$. Thus, $h(x) = 3\sqrt{|x|}$.

83. (a) $T(4) = 16°$, $T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the program would turn on (and off) one hour later.

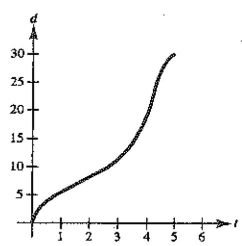
(c) If $H(t) = T(t) - 1$, then the overall temperature would be reduced 1 degree.

84. (a) For each time t, there corresponds a depth d.

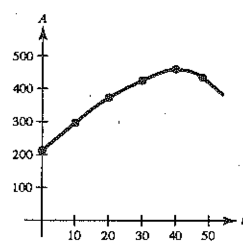
(b) Domain: $0 \le t \le 5$

Range: $0 \le d \le 30$

(c)

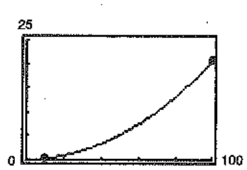


85. (a)



(b) $A(15) \approx 345$ acres/farm

86. (a)



(b) $H\left(\dfrac{x}{1.6}\right) = 0.002\left(\dfrac{x}{1.6}\right)^2 + 0.005\left(\dfrac{x}{1.6}\right) - 0.029$

$$= 0.00078125x^2 + 0.003125x - 0.029$$

87. $f(x) = |x| + |x - 2|$

If $x < 0$, then $f(x) = -x - (x - 2) = -2x + 2 = 2(1 - x)$.

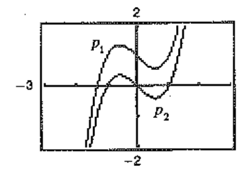
If $0 \le x < 2$, then $f(x) = x - (x - 2) = 2$.

If $x \ge 2$, then $f(x) = x + (x - 2) = 2x - 2 = 2(x - 1)$.

Thus,

$$f(x) = \begin{cases} 2(1 - x), & x < 0 \\ 2, & 0 \le x < 2. \\ 2(x - 1), & x \ge 2 \end{cases}$$

88. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, we have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Since the graph has no breaks, the graph must cross the x-axis at least one time.



89. $f(-x) = a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x)$

$= -[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x]$

$= -f(x)$

Odd

90. $f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$

$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

$= f(x)$

Even

91. Let $F(x) = f(x)g(x)$ where f and g are even. Then

$F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

Thus, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = [-f(x)][-g(x)] = f(x)g(x) = F(x)$.

Thus, $F(x)$ is even.

92. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

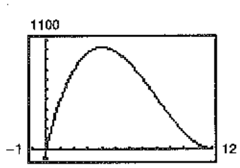
$F(-x) = f(-x)g(-x) = f(x)[-g(x)] = -f(x)g(x) = -F(x)$.

Thus, $F(x)$ is odd.

93. (a) $V = x(24 - 2x)^2 = 4x(12 - x^2)$

Domain: $0 < x < 12$

(b)



The dimensions for maximum volume are $4 \times 16 \times 16$ cm.

(c)

x	*length and width*	*volume*
1	$24 - 2(1)$	$1[24 - 2(1)]^2 = 484$
2	$24 - 2(2)$	$2[24 - 2(2)]^2 = 800$
3	$24 - 2(3)$	$3[24 - 2(3)]^2 = 972$
4	$24 - 2(4)$	$4[24 - 2(4)]^2 = 1024$
5	$24 - 2(5)$	$5[24 - 2(5)]^2 = 980$
6	$24 - 2(6)$	$6[24 - 2(6)]^2 = 864$

The dimensions for maximum volume appear to be $4 \times 16 \times 16$ cm.

94. By equating slopes, $\dfrac{y-2}{0-3} = \dfrac{0-2}{x-3}$

$$y - 2 = \frac{6}{x-3}$$

$$y = \frac{6}{x-3} + 2 = \frac{2x}{x-3},$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

95. False; let $f(x) = x^2$.

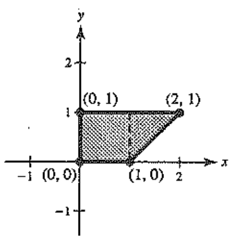
Then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

96. True

97. True, the function is even.

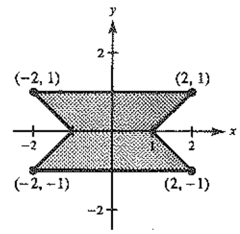
98. False; let $f(x) = x^2$. Then $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. Thus, $3f(x) \neq f(3x)$.

99. First consider the portion of R in the first quadrant: $x \geq 0$, $0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

[49th competition, Problem A1, 1988]

100. Let $g(x) = c$ be a constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

Thus, $f(c) = c$. Since this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section P.4 Fitting Models to Data

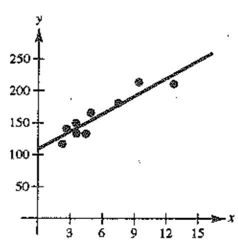
1. Quadratic function

2. Trigonometric function

3. Linear function

4. No relationship

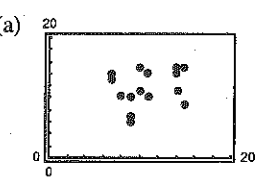
5. (a), (b)



Yes. The cancer mortality increases linearly with increased exposure to the carcinogenic substance.

(c) If $x = 3$, then $y \approx 136$.

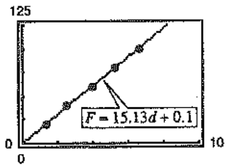
6. (a)



No, the relationship does not appear to be linear.

(b) Quiz scores are dependent on several variables such as study time, class attendance, etc. These variables may change from one quiz to the next.

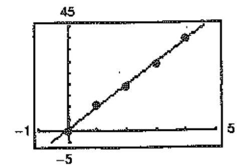
7. (a) $d = 0.066F$ or $F = 15.1d + 0.1$

(b) 

The model fits well.

(c) If $F = 55$, then $d \approx 0.066(55) = 3.63$ cm.

8. (a) $s = 9.7t + 0.4$

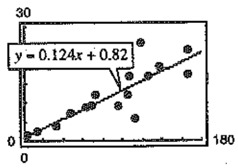
(b) 

The model fits well.

(c) If $t = 2.5$, $s = 24.65$ meters/second.

9. (a) Using a graphing utility, $y = 0.124x + 0.82$.

$r \approx 0.838$ correlation coefficient

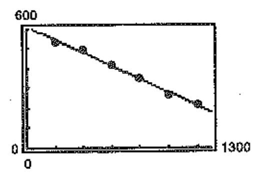
(b) 

(c) The data indicates that greater per capita electricity consumption tends to correspond to greater per capita gross national product.

The data for Hong Kong, Venezuela and South Korea differ most from the linear model.

(d) Removing the data (118, 25.59), (113, 5.74) and (167, 17.3), you obtain the model $y = 0.134x + 0.28$ with $r \approx 0.968$.

10. (a) Linear model: $H = -0.3323t + 612.9333$

(b) 

The fit is very good.

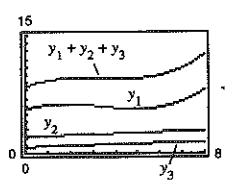
(c) When $t = 500$,

$$H = -0.3323(500) + 612.9333 \approx 446.78.$$

11. (a) $y_1 = 0.0343t^3 - 0.3451t^2 + 0.8837t + 5.6061$

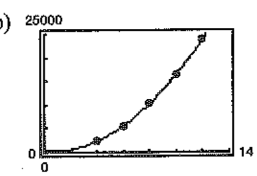
$y_2 = 0.1095t + 2.0667$

$y_3 = 0.0917t + 0.7917$

(b) 

For $t = 12$, $y_1 + y_2 + y_3 \approx 31.06$ cents/mile.

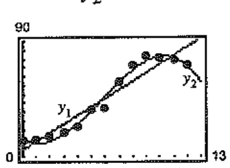
12. (a) $S = 180.89x^2 - 205.79x + 272$

(b) 

(c) When $x = 2$, $S \approx 583.98$ pounds.

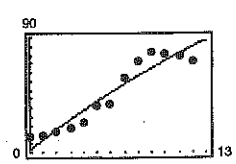
13. (a) Linear: $y_1 = 4.83t + 28.6$

Cubic: $y_2 = -0.1289t^3 + 2.235t^2 - 4.86t + 35.2$

(b) 

(c) The cubic model is better.

(d) $y = -0.084t^2 + 5.84t + 26.7$



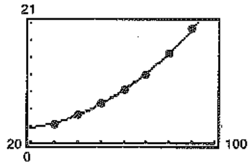
(e) For $t = 14$: Linear model $y_1 \approx 96.2$ million

Cubic model $y_2 \approx 51.5$ million

(f) Answers will vary.

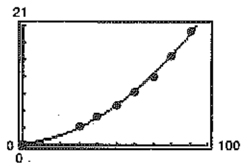
14. (a) $t = 0.00271s^2 - 0.0529s + 2.671$

(b)



(c) The curve levels off for $s < 20$.

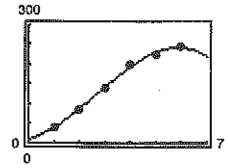
(d) $t = 0.002s^2 + 0.0346s + 0.183$



(e) The model is better for low speeds.

15. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

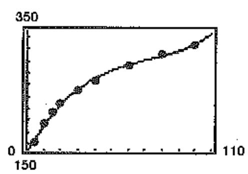
(b)



(c) If $x = 4.5$, $y \approx 214$ horsepower.

16. (a) $T = 2.9856 \times 10^{-4}p^3 - 0.0641p^2 + 5.2826p + 143.1$

(b)



(c) For $T = 300°\text{F}$, $p \approx 68.29$ pounds per square inch.

(d) The model is based on data up to 100 pounds per square inch.

17. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

(b) The amplitude is approximately

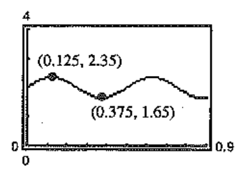
$$(2.35 - 1.65)/2 = 0.35.$$

The period is approximately

$$2(0.375 - 0.125) = 0.5.$$

(c) One model is $y = 0.35 \sin(4\pi t) + 2$.

(d)

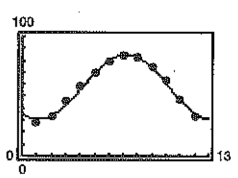


18. (a) $H(t) = 84.4 + 4.28 \sin\left(\frac{\pi t}{6} + 3.86\right)$

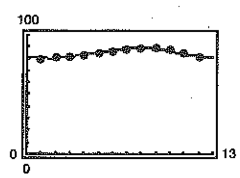
One model is

$$C(t) = 58 + 27 \sin\left(\frac{\pi t}{6} + 4.1\right).$$

(b)



(c)



(d) The average in Honolulu is 84.4.

The average in Chicago is 58.

(e) The period is 12 months (1 year).

(f) Chicago has greater variability $(27 > 4.28)$.

19. Answers will vary.

20. Answers will vary.

Review Exercises for Chapter P

1. $y = 2x - 3$

$x = 0 \Rightarrow y = 2(0) - 3 = -3 \Rightarrow (0, -3)$ y-intercept

$y = 0 \Rightarrow 0 = 2x - 3 \Rightarrow x = \frac{3}{2} \Rightarrow \left(\frac{3}{2}, 0\right)$ x-intercept

2. $y = (x - 1)(x - 3)$

$x = 0 \Rightarrow y = (0 - 1)(0 - 3) = 3 \Rightarrow (0, 3)$ y-intercept

$y = 0 \Rightarrow 0 = (x - 1)(x - 3) \Rightarrow x = 1, 3 \Rightarrow (1, 0), (3, 0)$ x-intercepts

3. $y = \dfrac{x - 1}{x - 2}$

$x = 0 \Rightarrow y = \dfrac{0 - 1}{0 - 2} = \dfrac{1}{2} \Rightarrow \left(0, \dfrac{1}{2}\right)$ y-intercept

$y = 0 \Rightarrow 0 = \dfrac{x - 1}{x - 2} \Rightarrow x = 1 \Rightarrow (1, 0)$ x-intercept

4. $xy = 4$

$x = 0$ and $y = 0$ are both impossible. No intercepts.

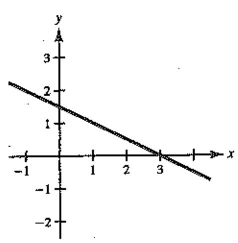
5. Symmetric with respect to y-axis since

$$(-x)^2y - (-x)^2 + 4y = 0$$
$$x^2y - x^2 + 4y = 0.$$

6. Symmetric with respect to y-axis since

$$y = (-x)^4 - (-x)^2 + 3$$
$$y = x^4 - x^2 + 3.$$

7. $y = -\frac{1}{2}x + \frac{3}{2}$

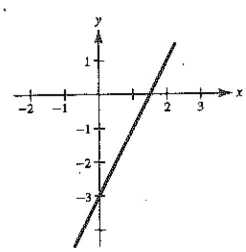


8. $4x - 2y = 6$

$y = 2x - 3$

Slope: 2

y-intercept: $(0, -3)$



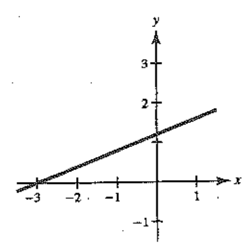
9. $-\frac{1}{3}x + \frac{5}{6}y = 1$

$-\frac{2}{5}x + y = \frac{6}{5}$

$y = \frac{2}{5}x + \frac{6}{5}$

Slope: $\frac{2}{5}$

y-intercept: $\frac{6}{5}$



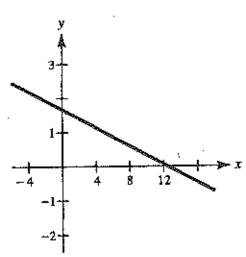
10. $0.02x + 0.15y = 0.25$

$2x + 15y = 25$

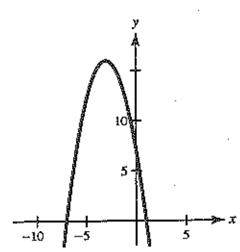
$y = -\frac{2}{15}x + \frac{5}{3}$

Slope: $-\frac{2}{15}$

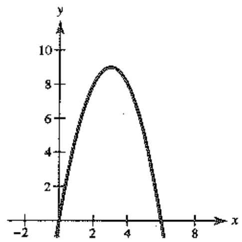
y-intercept: $\left(0, \frac{5}{3}\right)$



11. $y = 7 - 6x - x^2$

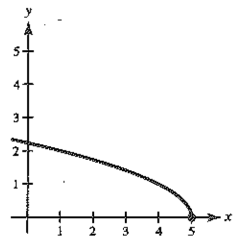


12. $y = x(6 - x)$

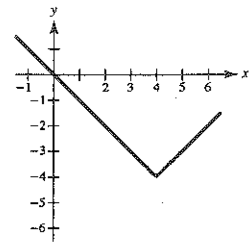


13. $y = \sqrt{5 - x}$

Domain: $(-\infty, 5]$



14. $y = |x - 4| - 4$



15. $y = 4x^2 - 25$

Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -30
Ymax = 10
Yscl = 5

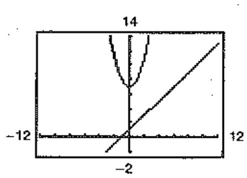
16. $y = 8\sqrt[3]{x - 6}$

Xmin = -40
Xmax = 40
Xscl = 10
Ymin = -40
Ymax = 40
Yscl = 10

17.
$$\begin{aligned} 3x - 4y &= 8 \\ 4x + 4y &= 20 \\ \hline 7x \quad &= 28 \\ x &= 4 \\ y &= 1 \end{aligned}$$

Point: $(4, 1)$

18.



$$y = x + 1$$
$$(x + 1) - x^2 = 7$$
$$0 = x^2 - x + 6$$

No real solution
No points of intersection
The graphs of $y = x + 1$ and $y = x^2 + 7$ do not intersect.

19. You need factors $(x + 2)$ and $(x - 2)$. Multiply by x to obtain origin symmetry.

$$y = x(x + 2)(x - 2)$$
$$= x^3 - 4x$$

20. $y = kx^3$

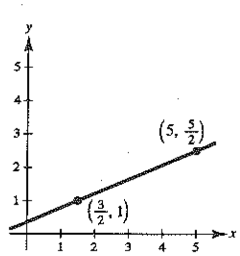
(a) $4 = k(1)^3 \Longrightarrow k = 4$ and $y = 4x^3$

(b) $1 = k(-2)^3 \Longrightarrow k = -\frac{1}{8}$ and $y = -\frac{1}{8}x^3$

(c) $0 = k(0)^3 \Longrightarrow$ any k will do!

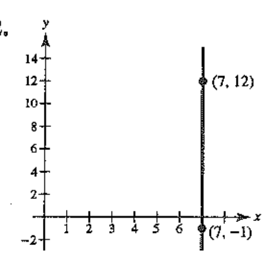
(d) $-1 = k(-1)^3 \Longrightarrow k = 1 \Longrightarrow y = x^3$

21.



$$\text{Slope} = \frac{(5/2) - 1}{5 - (3/2)} = \frac{3/2}{7/2} = \frac{3}{7}$$

22.



The line is vertical and has no slope.

23. $\dfrac{1 - t}{1 - 0} = \dfrac{1 - 5}{1 - (-2)}$

$$1 - t = -\frac{4}{3}$$
$$t = \frac{7}{3}$$

24. $\dfrac{3-(-1)}{-3-t} = \dfrac{3-6}{-3-8}$

$\dfrac{4}{-3-t} = \dfrac{-3}{-11}$

$-44 = 9 + 3t$

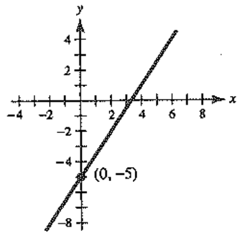
$-53 = 3t$

$t = -\dfrac{53}{3}$

25. $y - (-5) = \frac{3}{2}(x - 0)$

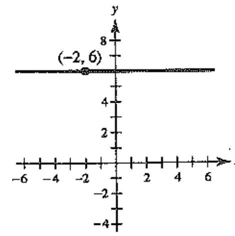
$y = \frac{3}{2}x - 5$

$2y - 3x + 10 = 0$



26. $y - 6 = 0(x - (-2))$

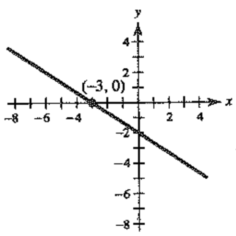
$y = 6$ Horizontal line



27. $y - 0 = -\frac{2}{3}(x - (-3))$

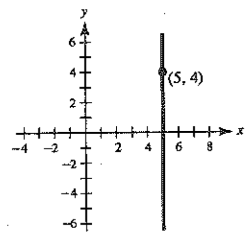
$y = -\frac{2}{3}x - 2$

$3y + 2x + 6 = 0$



28. m is undefined. Line is vertical.

$x = 5$



29. (a) $y - 4 = \dfrac{7}{16}(x + 2)$

$16y - 64 = 7x + 14$

$0 = 7x - 16y + 78$

(b) Slope of line is $\dfrac{5}{3}$.

$y - 4 = \dfrac{5}{3}(x + 2)$

$3y - 12 = 5x + 10$

$0 = 5x - 3y + 22$

(c) $m = \dfrac{4 - 0}{-2 - 0} = -2$

$y = -2x$

$2x + y = 0$

(d) $x = -2$

$x + 2 = 0$

30. (a) $y - 3 = -\dfrac{2}{3}(x - 1)$

$3y - 9 = -2x + 2$

$2x + 3y - 11 = 0$

(b) Slope of perpendicular line is 1.

$y - 3 = 1(x - 1)$

$y = x + 2$

$0 = x - y + 2$

(c) $m = \dfrac{4 - 3}{2 - 1} = 1$

$y - 3 = 1(x - 1)$

$y = x + 2$

$0 = x - y + 2$

(d) $y = 3$

$y - 3 = 0$

31. The slope is -850. $V = -850t + 12{,}500$.

$V(3) = -850(3) + 12{,}500 = \$9950$

32. (a) $C = 9.25t + 13.50t + 36{,}500$

$= 22.75t + 36{,}500$

(b) $R = 30t$

(c) $30t = 22.75t + 36{,}500$

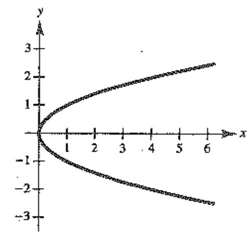
$7.25t = 36{,}500$

$t \approx 5034.48$ hours to break even

33. $x - y^2 = 0$

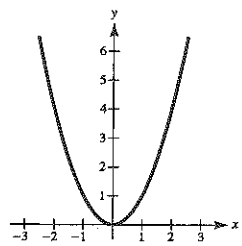
$y = \pm\sqrt{x}$

Not a function of x since there are two values of y for some x.



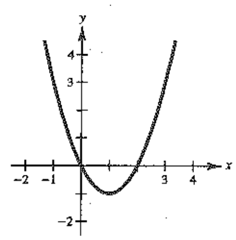
34. $x^2 - y = 0$

Function of x since there is one value for y for each x.



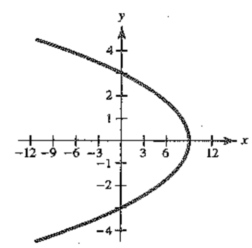
35. $y = x^2 - 2x$

Function of x since there is one value of y for each x.



36. $x = 9 - y^2$

Not a function of x since there are two values of y for some x.



37. $f(x) = \dfrac{1}{x}$

(a) $f(0)$ does not exist.

(b) $$\frac{f(1+\Delta x) - f(1)}{\Delta x} = \frac{\dfrac{1}{1+\Delta x} - \dfrac{1}{1}}{\Delta x} = \frac{1 - 1 - \Delta x}{(1+\Delta x)\Delta x}$$

$$= \frac{-1}{1+\Delta x},\ \Delta x \neq -1, 0$$

38. (a) $f(-4) = (-4)^2 + 2 = 18$ (because $-4 < 0$)

(b) $f(0) = |0 - 2| = 2$

(c) $f(1) = |1 - 2| = 1$

39. (a) Domain: $36 - x^2 \geq 0 \Longrightarrow -6 \leq x \leq 6$ or $[-6, 6]$

Range: $[0, 6]$

(b) Domain: all $x \neq 5$ or $(-\infty, 5) \cup (5, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

(c) Domain: all x or $(-\infty, \infty)$

Range: all y or $(-\infty, \infty)$

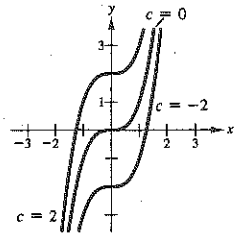
40. $f(x) = 1 - x^2$ and $g(x) = 2x + 1$

(a) $f(x) - g(x) = (1 - x^2) - (2x + 1) = -x^2 - 2x$

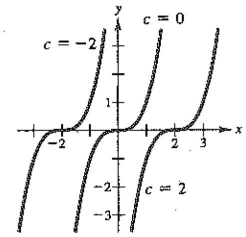
(b) $f(x)g(x) = (1 - x^2)(2x + 1) = -2x^3 - x^2 + 2x + 1$

(c) $g(f(x)) = g(1 - x^2) = 2(1 - x^2) + 1 = 3 - 2x^2$

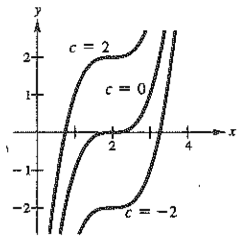
41. (a) $f(x) = x^3 + c, c = -2, 0, 2$



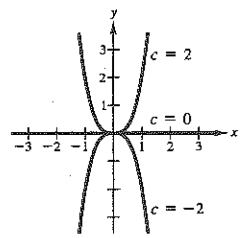
(b) $f(x) = (x - c)^3, c = -2, 0, 2$



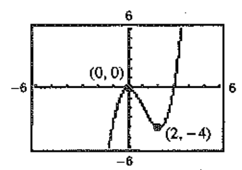
(c) $f(x) = (x - 2)^3 + c, c = -2, 0, 2$



(d) $f(x) = cx^3, c = -2, 0, 2$



42. $f(x) = x^3 - 3x^2$



(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$$g(x) = -[f(x) - 1]$$

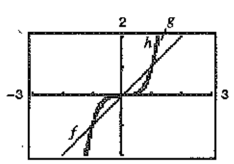
$$= -x^3 + 3x^2 + 1$$

(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

$$g(x) = f(x - 2) + 1$$

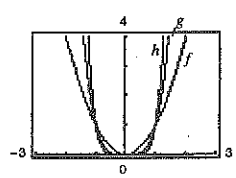
$$= (x - 2)^3 - 3(x - 2)^2 + 1$$

43. (a) Odd powers: $f(x) = x, g(x) = x^3, h(x) = x^5$



The graphs of f, g, and h all rise to the right and fall to the left. As the degree increases, the graph rises and falls more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, -1)$.

Even powers: $f(x) = x^2, g(x) = x^4, h(x) = x^6$

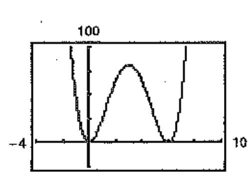


The graphs of f, g, and h all rise to the left and to the right. As the degree increases, the graph rises more steeply. All three graphs pass through the points $(0, 0)$, $(1, 1)$, and $(-1, 1)$.

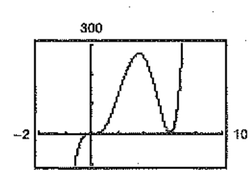
(b) $y = x^7$ will look like $h(x) = x^5$, but rise and fall even more steeply.

$y = x^8$ will look like $h(x) = x^6$, but rise even more steeply.

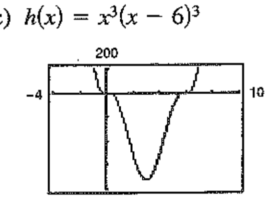
44. (a) $f(x) = x^2(x - 6)^2$



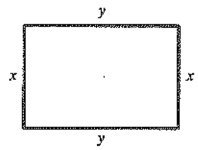
(b) $g(x) = x^3(x - 6)^2$



(c) $h(x) = x^3(x - 6)^3$

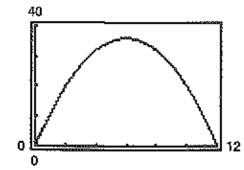


45. (a)



$$2x + 2y = 24$$
$$y = 12 - x$$
$$A = xy = x(12 - x) = 12x - x^2$$

(b) Domain: $0 < x < 12$



(c) Maximum area is $A = 36$. In general, the maximum area is attained when the rectangle is a square. In this case, $x = 6$.

46. For company (a) the profit rose rapidly for the first year, and then leveled off. For the second company (b), the profit dropped, and then rose again later.

47. (a) 3 (cubic), negative leading coefficient

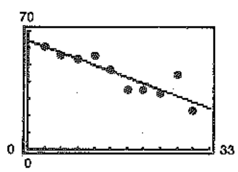
(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

48. (a) $y = -1.204x + 64.2667$

(b)



(c) The data point (27, 44) is probably an error. Without this point, the new model is

$$y = -1.4344x + 66.4387.$$

49. (a) Yes, y is a function of t. At each time t, there is one and only one displacement y.

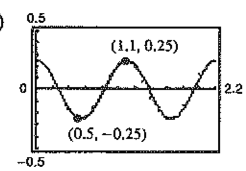
(b) The amplitude is approximately

$$(0.25 - (-0.25))/2 = 0.25.$$

The period is approximately 1.1.

(c) One model is $y = \frac{1}{4}\cos\left(\frac{2\pi}{1.1}t\right) \approx \frac{1}{4}\cos(5.7t)$

(d)



Problem Solving for Chapter P

1. (a)

$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: (3, 4) Radius: 5

(b) Slope of line from (0, 0) to (3, 4) is $\frac{4}{3}$. Slope of tangent line is $-\frac{3}{4}$. Hence,

$$y - 0 = -\frac{3}{4}(x - 0) \implies y = -\frac{3}{4}x \quad \text{Tangent line}$$

(c) Slope of line from (6, 0) to (3, 4) is $\frac{4-0}{3-6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. Hence,

$$y - 0 = \frac{3}{4}(x - 6) \implies y = \frac{3}{4}x - \frac{9}{2} \quad \text{Tangent line}$$

(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

2. Let $y = mx + 1$ be a tangent line to the circle from the point $(0, 1)$. Then

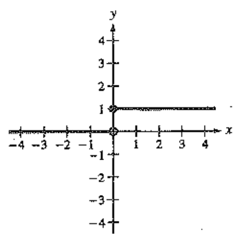
$$x^2 + (y + 1)^2 = 1$$
$$x^2 + (mx + 1 + 1)^2 = 1$$
$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

Setting the discriminant $b^2 - 4ac$ equal to zero,

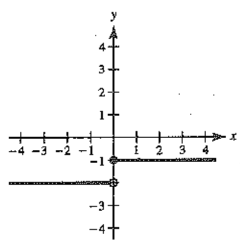
$$16m^2 - 4(m^2 + 1)(3) = 0$$
$$16m^2 - 12m^2 = 12$$
$$4m^2 = 12$$
$$m = \pm\sqrt{3}$$

Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

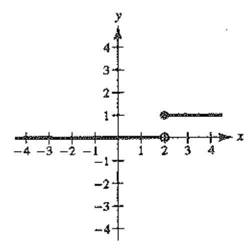
3. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



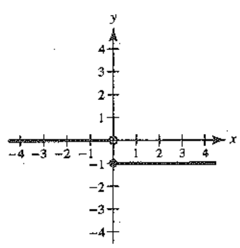
(a) $H(x) - 2$



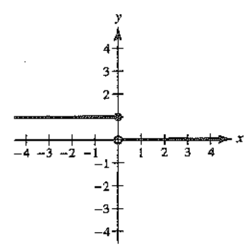
(b) $H(x - 2)$



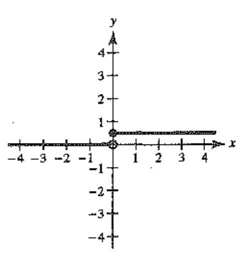
(c) $-H(x)$



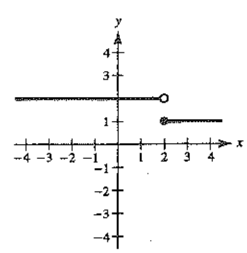
(d) $H(-x)$



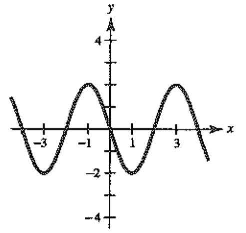
(e) $\frac{1}{2}H(x)$



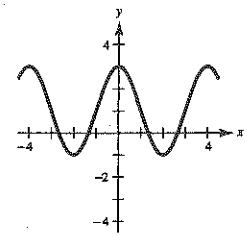
(f) $-H(x - 2) + 2$



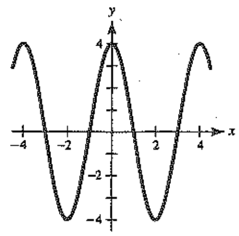
4. (a) $f(x+1)$



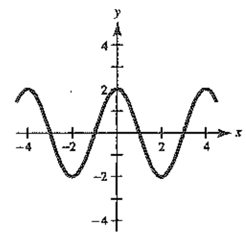
(b) $f(x) + 1$



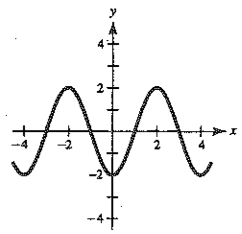
(c) $2f(x)$



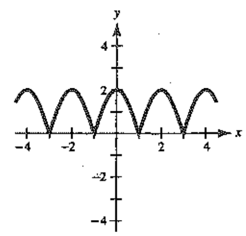
(d) $f(-x)$



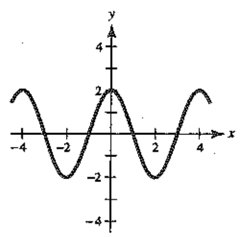
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$

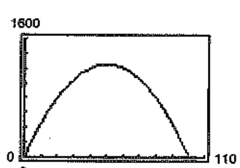


5. (a) $x + 2y = 100 \implies y = \dfrac{100 - x}{2}$

$$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$$

Domain: $0 < x < 100$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c)
$$\begin{aligned} A(x) &= -\frac{1}{2}(x^2 - 100x) \\ &= -\frac{1}{2}(x^2 - 100x + 2500) + 1250 \\ &= -\frac{1}{2}(x - 50)^2 + 1250 \end{aligned}$$

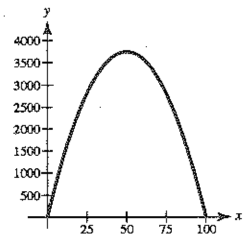
$A(50) = 1250$ m^2 is the maximum.
$x = 50$ m, $y = 25$ m

6. (a) $4y + 3x = 300 \Rightarrow y = \dfrac{300 - 3x}{4}$

$$A(x) = x(2y) = x\left(\frac{300 - 3x}{2}\right) = \frac{-3x^2 + 300x}{2}$$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

(c)
$$A(x) = -\frac{3}{2}(x^2 - 100x)$$
$$= -\frac{3}{2}(x^2 - 100x + 2500) + 3750$$
$$= -\frac{3}{2}(x - 50)^2 + 3750$$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. Hence, the total time is

$$T = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4} \text{ hours.}$$

8. Let d be the distance from the starting point to the beach.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}}$$
$$= \frac{2d}{\dfrac{d}{120} + \dfrac{d}{60}}$$
$$= \frac{2}{\dfrac{1}{120} + \dfrac{1}{60}}$$
$$= 80 \text{ km/hr}$$

9. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$. Slope of tangent line is less than 5.

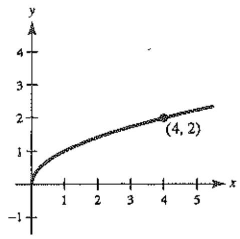
(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$. Slope of tangent line is less than 4.1.

(d)
$$\text{Slope} = \frac{f(2 + h) - f(2)}{(2 + h) - 2}$$
$$= \frac{(2 + h)^2 - 4}{h}$$
$$= \frac{4h + h^2}{h}$$
$$= 4 + h, h \neq 0$$

(e) Letting h get closer and closer to 0, the slope approaches 4. Hence, the slope at $(2, 4)$ is 4.

10.



(a) Slope $= \frac{3-2}{9-4} = \frac{1}{5}$. Slope of tangent line is greater than $\frac{1}{5}$.

(b) Slope $= \frac{2-1}{4-1} = \frac{1}{3}$. Slope of tangent line is less than $\frac{1}{3}$.

(c) Slope $= \frac{2.1-2}{4.41-4} = \frac{10}{41}$. Slope of tangent line is greater than $\frac{10}{41}$.

(d) Slope $= \frac{f(4+h)-f(4)}{(4+h)-4}$

$= \frac{\sqrt{4+h}-2}{h}$

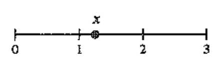
(e) $\frac{\sqrt{4+h}-2}{h} = \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \frac{(4+h)-4}{h\left(\sqrt{4+h}+2\right)}$

$= \frac{1}{\sqrt{4+h}+2}, h \neq 0$

As h gets closer to 0, the slope gets closer to $\frac{1}{4}$. The slope is $\frac{1}{4}$ at the point (4, 2).

11. (a) $\frac{I}{x^2} = \frac{2I}{(x-3)^2}$

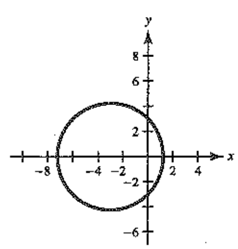


$x^2 - 6x + 9 = 2x^2$

$x^2 + 6x - 9 = 0$

$$x = \frac{-6 \pm \sqrt{36+36}}{2} = -3 \pm \sqrt{18} \approx 1.2426, -7.2426$$

(b) $\frac{I}{x^2+y^2} = \frac{2I}{(x-3)^2+y^2}$



$(x-3)^2 + y^2 = 2(x^2+y^2)$

$x^2 - 6x + 9 + y^2 = 2x^2 + 2y^2$

$x^2 + y^2 + 6x - 9 = 0$

$(x+3)^2 + y^2 = 18$

Circle of radius $\sqrt{18}$ and center $(-3, 0)$.

12. (a) $\dfrac{I}{x^2 + y^2} = \dfrac{kI}{(x-4)^2 + y^2}$

$(x-4)^2 + y^2 = k(x^2 + y^2)$

$(k-1)x^2 + 8x + (k-1)y^2 = 16$

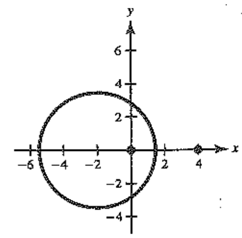
If $k = 1$, then $x = 2$ is a vertical line. Assume $k \neq 1$.

$$x^2 + \frac{8x}{k-1} + y^2 = \frac{16}{k-1}$$

$$x^2 + \frac{8x}{k-1} + \frac{16}{(k-1)^2} + y^2 = \frac{16}{k-1} + \frac{16}{(k-1)^2}$$

$$\left(x + \frac{4}{k-1}\right)^2 + y^2 = \frac{16k}{(k-1)^2}, \qquad \text{Circle}$$

(b) If $k = 3$, $(x+2)^2 + y^2 = 12$



(c) As k becomes very large, $\dfrac{4}{k-1} \to 0$ and $\dfrac{16k}{(k-1)^2} \to 0$.

The center of the circle gets closer to $(0, 0)$, and its radius approaches 0.

13.

$$d_1 d_2 = 1$$

$$[(x+1)^2 + y^2][(x-1)^2 + y^2] = 1$$

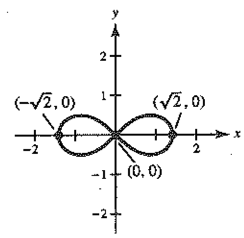
$$(x+1)^2(x-1)^2 + y^2[(x+1)^2 + (x-1)^2] + y^4 = 1$$

$$(x^2 - 1)^2 + y^2[2x^2 + 2] + y^4 = 1$$

$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$

$$(x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 = 0$$

$$(x^2 + y^2)^2 = 2(x^2 - y^2)$$



Let $y = 0$. Then $x^4 = 2x^2 \implies x = 0$ or $x^2 = 2$.

Thus, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.

14. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all $x \neq 1$

Range: all $y \neq 0$

(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1 - \left(\dfrac{1}{1-x}\right)} = \dfrac{1}{\dfrac{1-x-1}{1-x}} = \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

Domain: all $x \neq 0, 1$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1 - \left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

Domain: all $x \neq 0, 1$

(d) The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.