Applications of Integration Review:

1. $x+2 = x^3-4x^2+6$

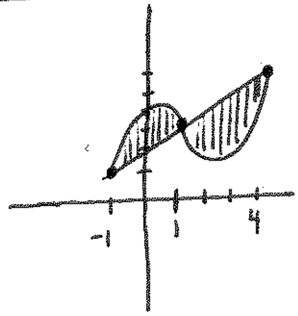
$0 = x^3-4x^2-x+4$

$0 = x^2(x-4)-1(x-4)$

$0 = (x^2-1)(x-4)$

$0 = (x+1)(x-1)(x-4)$

$x = \pm 1, 4.$



$\boxed{C}$

$$\int_{-1}^{1}\left[x^3-4x^2+6-(x+2)\right]dx + \int_{1}^{4}\left[x+2-(x^3-4x^2+6)\right]dx$$

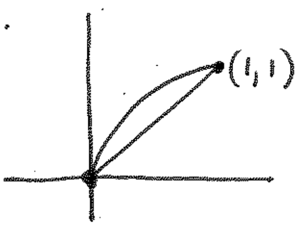
$$\int_{-1}^{1}\left[x^3-4x^2-x+4\right]dx + \int_{1}^{4}\left[-x^3+4x^2+x-4\right]dx$$

$$\left[\frac{1}{4}x^4-\frac{4}{3}x^3-\frac{1}{2}x^2+4x\right]_{-1}^{1} + \left[-\frac{1}{4}x^4+\frac{4}{3}x^3+\frac{1}{2}x^2-4x\right]_{1}^{4}$$

$$\left(\cancel{\tfrac{1}{4}}-\tfrac{4}{3}-\cancel{\tfrac{1}{2}}+4\right)-\left(\cancel{\tfrac{1}{4}}+\tfrac{4}{3}-\cancel{\tfrac{1}{2}}-4\right)+\left(-64+\tfrac{256}{3}+8-16\right)-\left(-\tfrac{1}{4}+\tfrac{4}{3}+\tfrac{1}{2}-4\right)=\left(\tfrac{16}{3}\right)+\left(\tfrac{63}{4}\right)$$

$$= \frac{253}{12}$$

2.



$$\int_{0}^{1}\left[\sqrt[3]{x}-x\right]dx$$

$$\left[\frac{3}{4}x^{4/3}-\frac{1}{2}x^2\right]_0^1 = \left(\frac{3}{4}-\frac{1}{2}\right)-\left(0\right) = \frac{1}{4}$$

$\boxed{B}$

3. $\int_0^a \left[f(x)+3\right]dx + \int_a^b \left[-3-f(x)\right]dx$ $\boxed{C}$

4. $a \cdot b - \int_0^a \left[g(x)-f(x)\right]dx = \int_0^a b - \left[g(x)-f(x)\right]dx$ $\boxed{D}$

5. (a) $T = \frac{1}{2}\left(1-\frac{1}{e}\right)(e-1) = \frac{1}{2}\left(e-1-1+\frac{1}{e}\right) = \frac{e}{2} - 1 + \frac{1}{2e}$

$y'(1) = e$

$\ell: \; y-(e-1) = e(x-1)$

$y - e + 1 = ex - e$

$y = ex - 1$

$x \text{ int}: \; x = 1/e$

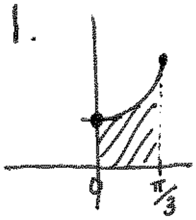
(b) $\int_0^1 [e^x - 1]\,dx - T = \left[e^x - x\right]_0^1 - \left[\frac{e}{2} - 1 + \frac{1}{2e}\right]$

$= (e-1) - (1) - \frac{e}{2} + 1 - \frac{1}{2e}$

$= e - 2 - \frac{e}{2} + 1 - \frac{1}{2e} \quad = \frac{e}{2} - \frac{1}{2e} - 1$

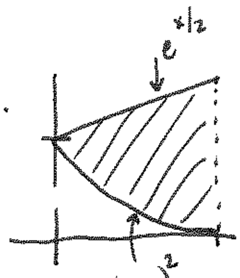
Volumes of Revolution :

1.



$\pi \int_0^{\pi/3} [\sec x]^2\,dx = \pi\left[\tan x\right]_0^{\pi/3} = \pi\left[\sqrt{3} - 0\right] = \pi\sqrt{3}$ $\boxed{C}$

2.

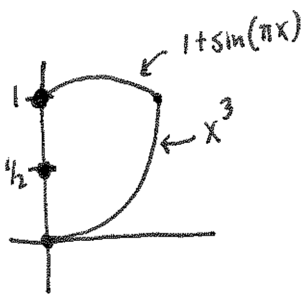


$\pi \int_0^1 \left[e^{x/2}\right]^2 - \left[(x-1)^2\right]^2 dx = \pi \int_0^1 e^x - (x-1)^4\,dx$ $\boxed{D}$

$= \pi\left[e^x - \frac{1}{5}(x-1)^5\right]_0^1$

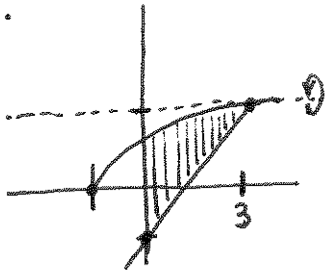
$= \pi\left[e - \left(1 + \frac{1}{5}\right)\right] = \pi\left[e - \frac{6}{5}\right]$

3.



$\pi \int_0^1 [1 + \sin(\pi x)]^2 - [x^3]^2\,dx$ $\boxed{B}$

4.



$\sqrt{x+1}^2 = (x-1)^2$

$x+1 = x^2-2x+1$

$0 = x^2-3x$

$0 = x(x-3)$

$x=0 \quad x=3$

$$V = \pi\int_0^3 [2-(x-1)]^2 - [2-\sqrt{x+1}]^2 dx$$

$$= \pi\int_0^3 [3-x]^2 - [2-\sqrt{x+1}]^2 dx$$

$$= \pi\int_0^3 9-6x+x^2 - [4-4\sqrt{x+1}+(x+1)] dx$$

$$= \pi\int_0^3 9-6x+x^2 - 5 - x + 4\sqrt{x+1}\, dx$$

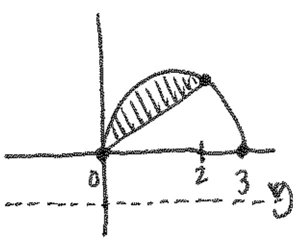
$$= \pi\int_0^3 4-7x+x^2+4\sqrt{x+1}\, dx$$

$$= \pi\left[4x - \tfrac{7}{2}x^2 + \tfrac{1}{3}x^3 + \tfrac{8}{3}(x+1)^{3/2}\right]_0^3$$

$$= \pi\left[12 - \tfrac{63}{2} + 9 + \tfrac{64}{3} - \tfrac{8}{3}\right] = \frac{49}{6}\pi$$

$\boxed{C}$

5.



$3x - x^2 = x$

$0 = x^2-2x$

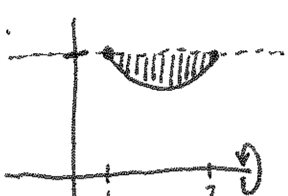
$= x(x-2)$

$$\pi\int_0^2 [3x-x^2--1]^2 - [x--1]^2 dx$$

$$\pi\int_0^2 [3x-x^2+1]^2 - [x+1]^2 dx$$

$\boxed{B}$

6.



$x+\frac{3}{x} = 4$

$\frac{x^2+3}{x} = 4$

$x^2+3 = 4x$

$x^2-4x+3 = 0$

$(x-3)(x-1) = 0$

$x = 1, 3$

$$\pi\int_1^3 [4]^2 - \left[x+\frac{3}{x}\right]^2 dx$$

$$\pi\int_1^3 16 - \left[x^2+6+\frac{9}{x^2}\right] dx$$

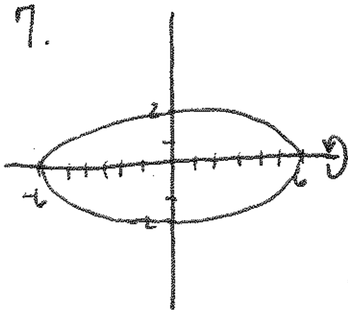
$$\pi\int_1^3 10 - x^2 - 9x^{-2}\, dx$$

$$\pi\left[10x - \tfrac{1}{3}x^3 + 9x^{-1}\right]_1^3$$

$$\pi\left[(30-9+3)-(10-\tfrac{1}{3}+9)\right] = \pi\left[24-\frac{56}{3}\right] = \frac{16\pi}{3}$$

$\boxed{B}$

7.



$x^2+9y^2=36$

$\frac{x^2}{36}+\frac{y^2}{4}=1$

$y=\sqrt{\frac{36-x^2}{9}}=\sqrt{4-\frac{1}{9}x^2}$

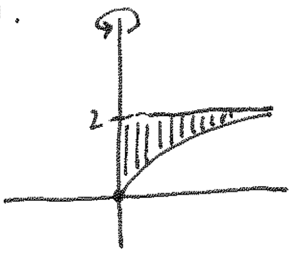
$\pi\int_{-6}^{6}\left[\sqrt{4-\frac{1}{9}x^2}\right]^2dx$

$\pi\int_{-6}^{6}4-\frac{1}{9}x^2\,dx$

$\pi\left[4x-\frac{1}{27}x^3\right]_{-6}^{6}=\pi\left[(24-8)-(-24+8)\right]=32\pi$

$\boxed{D}$

8.



$y=\sqrt{x}$

$x=y^2$

$\pi\int_0^2\left[y^2\right]^2dy$

$\pi\left[\frac{1}{5}y^5\right]_0^2=\pi\left[\frac{32}{5}-0\right]=\frac{32}{5}\pi$

$\boxed{A}$

9. (a) $f(0)=1$ $\quad f'(x)=3x^2-4x-1-\sin x$

$f'(0)=-1$

$y-1=-x$

$y=-x+1$

(b) $\int_0^{2.313}\left[-x+1-f(x)\right]dx=2.670$

$x^3-2x^2-x+\cos x=-x+1$

$x=2.313$

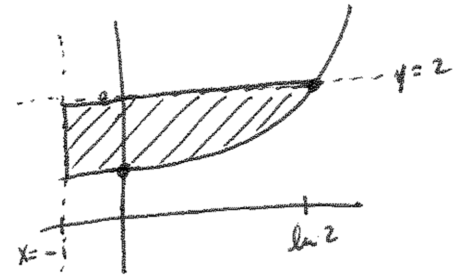
(c) $\pi\int_0^{2.313}\left[\left(2-f(x)\right)^2-\left(2-(-x+1)\right)^2\right]dx$

10. (a) $\int_{-1}^{\ln 2} [2 - e^x]\, dx$

$[2x - e^x]_{-1}^{\ln 2}$

$(\ln 4 - 2) - (-2 - \frac{1}{e})$

$\ln 4 + \frac{1}{e}$



$e^x = 2$ $\quad$ $y = e^x$

$x = \ln 2$ $\quad$ $x = \ln y$

(b) $\pi \int_{e^{-1}}^{2} [\ln y + 1]^2\, dy = 6.991$

(c) $\pi \int_{-1}^{\ln 2} [2 - -1]^2 - [e^x - -1]^2\, dx$

$\pi \int_{-1}^{\ln 2} 9 - [e^x + 1]^2\, dx = 26.228$

11. (a) $\pi \int_0^k \left[\frac{3x}{x^3+1}\right]^2 dx = \pi \int_0^k \frac{9x^2}{(x^3+1)^2}\, dx = 3\pi \int_1^{k^3+1} u^{-2}\, du = 3\pi \left[-\frac{1}{u}\right]_1^{k^3+1}$

$u = x^3 + 1$

$du = 3x^2\, dx$

$= 3\pi \left[-\frac{1}{k^3+1} + 1\right]$

$= \pi \left[3 - \frac{3}{k^3+1}\right]$

(b) $S = \pi \int_k^{\infty} \left[\frac{3x}{x^3+1}\right]^2 dx = \pi \cdot \lim_{b \to \infty} \int_k^b \left[\frac{9x^2}{x^3+1}\right] dx = 3\pi \cdot \lim_{b \to \infty} \left[-\frac{1}{u}\right]_{k^3+1}^{b^3+1}$

$= 3\pi \cdot \lim_{b \to \infty} \left[-\frac{1}{b^3+1} + \frac{1}{k^3+1}\right]$

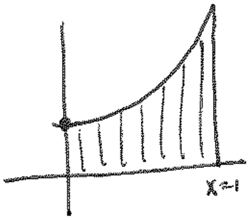
$= 3\pi \left[\frac{1}{k^3+1}\right]$

$\frac{3\pi}{k^3+1} = \pi \left[3 - \frac{3}{k^3+1}\right]$

$\frac{3}{k^3+1} = 3 - \frac{3}{k^3+1} \Rightarrow \frac{6}{k^3+1} = 3 \Rightarrow 6 = 3k^3 + 3 \Rightarrow 3k^3 = 3 \Rightarrow k^3 = 1 \Rightarrow k = 1$

Volumes of Known Cross Sections :

1.



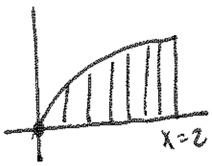
$$\int_0^1 [e^x]^2 dx = \int_0^1 e^{2x}\,dx = \left[\tfrac{1}{2}e^{2x}\right]_0^1 = \tfrac{1}{2}[e^2-1] \qquad \boxed{B}$$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

2.



$$\frac{\sqrt{3}}{4}\int_0^2 [\sqrt{x}]^2 dx = \frac{\sqrt{3}}{4}\int_0^2 x\,dx = \frac{\sqrt{3}}{4}\left[\frac{x^2}{2}\right]_0^2 \qquad \boxed{D}$$

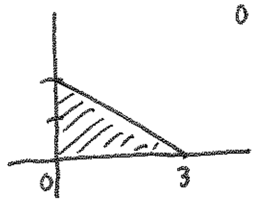
$$= \frac{\sqrt{3}}{8}[4-0] = \frac{\sqrt{3}}{2}$$

3. $2x+3y=6 \Rightarrow y = -\frac{2}{3}x+2$

$0 = -\frac{2}{3}x+2$

$\frac{2}{3}x = 2$

$x = 3$

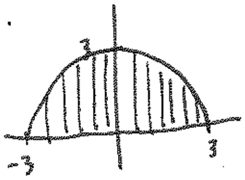


$$\frac{\pi}{8}\int_0^3 \left[-\tfrac{2}{3}x+2\right]^2 dx \qquad \boxed{A}$$

$$\frac{\pi}{8}\int_0^3 \left[\tfrac{4}{9}x^2-\tfrac{8}{3}x+4\right]dx$$

$$\frac{\pi}{8}\left[\tfrac{4}{27}x^3-\tfrac{4}{3}x^2+4x\right]_0^3 = \frac{\pi}{8}\left[(4-12+12)-0\right] = \frac{\pi}{2}$$

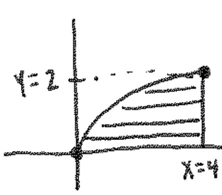
4.



$$\frac{\pi}{8}\int_{-3}^3 \left[\sqrt{9-x^2}\right]^2 dx = \frac{\pi}{8}\int_{-3}^3 9-x^2\,dx \qquad \boxed{C}$$

$$= \frac{\pi}{8}\left[9x-\tfrac{1}{3}x^3\right]_{-3}^3 = \frac{\pi}{8}\left[(27-9)-(-27+9)\right] = \frac{9\pi}{2}$$

5.



$y = \sqrt{x}$

$x = y^2$

$$\int_0^2 [4-y^2]^2 dy \qquad \boxed{A}$$

6. $\int_0^{\pi/4} [\cos x - \sin x]^2 dx = \int_0^{\pi/4} [\cos^2 x - 2\cos x \sin x + \sin^2 x] dx$

$\cos x = \sin x$

$x = \pi/4$

$= \int_0^{\pi/4} [1 - \sin 2x] dx$

$u = 2x$

$du = 2dx$

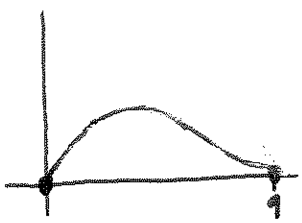
$\frac{1}{2} du = dx$

$\boxed{C}$

$= \left[x + \frac{1}{2}\cos(2x)\right]_0^{\pi/4}$

$= \left(\frac{\pi}{4} + \frac{1}{2}\cos(\pi/2)\right) - \left(0 + \frac{1}{2}\cos(0)\right) = \frac{\pi}{4} - \frac{1}{2} = \frac{\pi - 2}{4}$

7.



$3\sqrt{x} - x = 0$

$3\sqrt{x} = x$

$9x = x^2$

$0 = x^2 - 9x$

$0 = x(x-9)$

$\int_0^9 (3\sqrt{x} - x)\left(\frac{1}{\sqrt{x}}\right) dx$

$\int_0^9 3 - x^{1/2} dx$

$\left[3x - \frac{2}{3}x^{3/2}\right]_0^9 = (27 - 18) - 0 = 9$

$\boxed{D}$

8. (a) $\int_0^{\pi} (\sin x - -\sin x) dx = 2\int_0^{\pi} \sin x \, dx$

$= 2\left[-\cos x\right]_0^{\pi} = 2\left[-\cos\pi + \cos 0\right] = 4$

(b) $\pi \int_0^{\pi} [3 - -\sin x]^2 - [3 - \sin x]^2 dx = \pi \int_0^{\pi} 9 + 6\sin x + \sin^2 x - 9 + 6\sin x - \sin^2 x \, dx$

$= \pi \int_0^{\pi} 12\sin x \, dx$

$= 12\pi\left[-\cos x\right]_0^{\pi} = 12\pi\left[-\cos\pi + \cos 0\right] = 24\pi$

(c) $\int_0^{\pi} [k\sin x + \sin x]^2 dx = 8\pi$

$\int_0^{\pi} [\sin x(k+1)]^2 dx = \int_0^{\pi} \sin^2 x (k+1)^2 dx = (k+1)^2 \cdot \frac{\pi}{2}$

$(k+1)^2 \cdot \frac{\pi}{2} = 8\pi$

$(k+1)^2 = 16$

$k+1 = 4$

$k = 3$

9. (a) $\pi \int_0^{12} \left[\frac{D(x)}{2}\right]^2 dx$

(b) $\pi \int_0^{12} \left[\frac{D(x)}{2}\right]^2 dx = \frac{\pi}{4} \int_0^{12} [D(x)]^2 dx \approx \frac{\pi}{4}\left[4(1.5)^2 + 4(1.42)^2 + 4(1.38)^2\right]$

$= \pi\left[6.171\right] = 19.386$

(c) Since $D(2) = 1.5 = D(8)$ on the continuous and differentiable function $D(x)$,

(not provided but necessary)

Rolle's Theorem guarantees some value such that $D'(x) = 0$

NOTE: This can also be done using MVT.

10. $f(x) = g(x) \Rightarrow x = 1.5, 4$

(a) $\int_0^{1.5} [f(x) - g(x)]\, dx$

(b) $\int_{1.5}^{4} [g(x) - f(x)]\, dx$

(c) $\int_0^{1.5} [f(x) - g(x)] \cdot (4e^{-x})\, dx = 3.777$

(d) $\int_{1.5}^{4} [g(x) - f(x)]\left(4 - \sqrt{x}\right) dx = 16.585$

Total Change in a Function :

1. $\int_0^6 \frac{20e^{(-0.1t)}}{1+e^{-t}}\,dt = 77.881$ $\boxed{B}$

2. $\int_0^{12} \frac{50e^{-t/2}}{\sqrt{t+1}}\,dt = 65.504$ $\boxed{C}$

3. $200 + \int_0^8 S(t) - R(t)\,dt$ $\boxed{C}$

4. $\int_0^{22} \frac{720}{t^2-28t+205}\,dt = 581.772$ $\boxed{D}$

5. $h'(t) = 0.01t^3 - 0.3t^2 + 2.2t - 1.5$ $\boxed{A}$

$h'(t) = 0 \Rightarrow t = .758,\ 10.628,\ 18.614$

t	0	.758	10.628	18.614	20
$h(t)$	8	7.452	28.157	16.391	18

$h(.758) = h(0) + \int_0^{.758} h'(t)\,dt$

$h(10.628) = h(0) + \int_0^{10.628} h'(t)\,dt$

$h(18.614) = h(0) + \int_0^{18.614} h'(t)\,dt$

$h(20) = h(0) + \int_0^{20} h'(t)\,dt$

6. (a) $\int_0^{10} \frac{1}{2} t^{2/3}\, dt = 13.925$

(b) $W(10) = W(0) + \int_0^{10} 8 - \frac{1}{2} t^{2/3}\, dt = 116.075$

(c) $f(t) = f(0) + \int_0^{t} 8 - \frac{1}{2} x^{2/3}\, dx$

(d) $f'(t) = 8 - \frac{1}{2} t^{2/3}$

$f'(t) = 0 \Rightarrow \frac{1}{2} t^{2/3} = 8$

$t^{2/3} = 16$

$t = 64$

t	$f(t)$
0	50
64	254.800
90	227.759

Amount of water is at a maximum when $t = 64$.

7. (a) $P'(x) = -x^{-1/2} - 4\cos\left(\frac{x^2}{12}\right) \cdot \frac{1}{6} x$

$= -\frac{1}{\sqrt{x}} - \frac{2}{3} x \cos\left(\frac{x^2}{12}\right)$

$P'(3) = -2.041 \text{ tons/hour}^2$

The rate at which granules (in tons) of plastic is decreasing by 2.041 tons/hour² at $x = 3$ hours.

(b) Decreasing most rapidly $\Rightarrow$ minimum of $P(x)$

$P'(x) = 0 \Rightarrow x = 4.551, 7.461$

x	0	4.551	7.461	8
$P(x)$	5	-3.219	3.526	2.596

Decreasing most rapidly at $x = 4.551$ hours

(c)

$A(x) \rightarrow$ Amount of granules

$A(x) = A(0) + \int_0^{t} P(x)\, dx$

$P(x) = 0 \Rightarrow x = 2.426, 6.128$

x	$A(x)$
0	6
2.426	11.532
6.128	4.015
8	8.648

Maximum Amount of gravel is 11.532 tons.

Particle Motion:

1. $\int 2t-6 = t^2-6t+C$

$3 = 1-6+C$

$8 = C$

$v(t) = t^2-6t+8$

$\int t^2-6t+8\,dt = \frac{1}{3}t^3-3t^2+8t+C$

$\frac{1}{3} = \frac{1}{3}-3+8+C$

$-5 = C$

$x(t) = \frac{1}{3}t^3-3t^2+8t-5$

$\boxed{B}$

2. Speed $= |3e^{-t}-t|$

Avg Speed $= \frac{1}{3}\int_0^3 |3e^{-t}-t|\,dt = 1.482$

$\boxed{D}$

3. Avg Velocity $= \frac{1}{5}\int_0^5 v(t)\,dt = 86.448$

$\boxed{D}$

4. Change in direction $\Rightarrow$ $v(t)$ changes sign $\Rightarrow t = 5, 9$

$\boxed{C}$

5. $\int_0^{10} |v(t)|\,dt = \frac{1}{2}(5)(4)+\frac{1}{2}(4+1)\cdot 2+\frac{1}{2}(1)(1)$

$= 10+5+\frac{1}{2} = \frac{31}{2}$

$\boxed{A}$

6. Farthest to the right $\Rightarrow$ $\int_0^t v(x)\,dx$ is the largest $\Rightarrow t=5$

$\boxed{B}$

7. $v(4) = 2$

$\boxed{B}$

8. $v'(4) = a(4) = -2$

$\boxed{A}$

9. $v(t) = (4t^2-3)\cdot e^{-0.5t}\cdot -0.5 + e^{-0.5t}(8t)$

$= e^{-0.5t}\left(-2t^2 + \frac{3}{2} + 8t\right)$

$= -\frac{1}{2}e^{-0.5t}\left(4t^2 - 16t - 3\right)$

$v(t) = 0 \Rightarrow 4t^2 - 16t - 3 = 0 \Rightarrow t = 4.179$

$s(0) = -3$

$s(4.179) = 8.273$

$s(4.179) - s(0) = 11.273$

$\boxed{C}$

10. (a) $v(2) = 2\cos(3) = -1.980 < 0$

The particle is moving to the left at $t=2$.

(b) $a(t) = t \cdot -\sin(t^2-1)\cdot 2t + \cos(t^2-1)$

$= -2t^2\sin(t^2-1) + \cos(t^2-1)$

$a(2) = -8\sin(3) + \cos(3) = -2.119$

Velocity is decreasing at $t=2$ since $a(2) < 0$.

(c) Speed is increasing at $t=2$ since $v(2)$ and $a(2)$ are both negative.

(d) $x(2.5) = x(0) + \int_0^{2.5} v(t)\,dt = 3.991$

(e) Greatest distance $\Rightarrow$ max of $x(t)$ on $0 \le t \le 2.5$.

$v(t) = 0 \Rightarrow t = 0, 1.603, 2.390$

$x(t) = x(0) + \int_0^t v(x)\,dx$

t	$x(t)$
0	4
1.603	4.921
2.390	3.921
2.5	3.991

Greatest distance from origin is 4.921 @ $t = 1.603$.

(f) Total Distance $= \int_0^{2.5} |v(t)|\,dt = 1.991$

11. (a) $a(t) = 6t - 14$

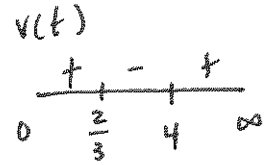
$a'(t) = 6$ $\quad$ $a'(t) \neq 0 \Rightarrow$ minimum accelaration occurs at endpoint $t = 0$

Minimum $a(t)$ is -14.

(b) Downward $\Rightarrow$ $v(t) < 0$

$v(t) = (3t-2)(t-4) = 0$

$t = \frac{2}{3}, 4$



Downward when $\frac{2}{3} < t < 4$

(c) $\text{Avg } V(t) = \dfrac{1}{3-0} \displaystyle\int_0^3 v(t)\,dt = \frac{1}{3}\left[t^3 - 7t^2 + 8t\right]_0^3$

$= \frac{1}{3}\left[27 - 63 + 24 - 0\right] = -4$

(d) $\text{Avg } a(t) = \dfrac{v(3) - v(0)}{3-0} = \dfrac{-7-8}{3} = \dfrac{-15}{3} = -5$

(e) $y(3) = y(0) + \displaystyle\int_0^3 v(t)\,dt = 2 + -12 = -10$

(f) Total Dist $= \displaystyle\int_0^3 |v(t)|\,dt = \int_0^{2/3} v(t)\,dt - \int_{2/3}^3 v(t)\,dt = 17.037$

12. (a) $v(6)=0$ since there is a horizontal tangent at $t=6$.

(b) Speed $= |v(4)| = |-1| = 1$

(c) Moving left $\Rightarrow v(t)<0 \Rightarrow s(t)$ is decreasing $\Rightarrow 2<t<6$

(e) On $2<t<3$, $v(t)<0$ and $a(t)<0$, therefore speed is increasing.

(d) $v(t)$ decreasing $\Rightarrow a(t)<0 \Rightarrow s(t)$ concave down $\Rightarrow 0<t<3$

(f) $a(t)$ positive $\Rightarrow s(t)$ concave up $\Rightarrow 3<t<8$

Average Value of a Function:

1. $\frac{1}{4-0}\int_0^4 \sqrt{x}(4-x)\,dx = \frac{1}{4}\int_0^4 4x^{1/2}-x^{3/2}\,dx = \frac{1}{4}\left[\frac{8}{3}x^{3/2}-\frac{2}{5}x^{5/2}\right]_0^4$ $\boxed{C}$

$= \frac{1}{4}\left[\frac{64}{3}-\frac{64}{5}\right] = \frac{32}{15}$

2. $\frac{1}{8-0}\int_0^8 f(x)\,dx = \frac{1}{8}\left[8+\frac{1}{2}\pi(2)^2+\frac{1}{2}(4)(2)\right] = \frac{1}{8}\left[8+2\pi+4\right]$ $\boxed{D}$

$= \frac{12+2\pi}{8} = \frac{6+\pi}{4}$

3. $\frac{1}{5-0}\int_0^k f(x)\,dx = 1$ $\boxed{C}$

$\frac{1}{5}\left[\frac{1}{2}(2)(2)+\frac{1}{2}(2+k)+2k\right]=1$

$\frac{1}{5}\left[2+1+\frac{1}{2}k+2k\right]=1$

$3+\frac{5}{2}k=5$

$\frac{5}{2}k=2$

$k=\frac{4}{5}$

4. $\frac{1}{4--4}\int_{-4}^{4} f(x)\,dx = \frac{1}{8}\left[3+\frac{1}{2}-\frac{1}{2}(3)(3)-3\right]$ $\boxed{B}$

$= \frac{1}{8}\left[-4\right] = -\frac{1}{2}$

$\boxed{B}$

5. $\int_0^8 f(t)\,dt > 16$

6. $\frac{1}{9}\int_0^9 f(x)\,dx = \frac{1}{9}\left[\int_0^4 \frac{1}{16}x^2+1\,dx + \int_4^9 3\sqrt{x}-x\,dx\right]$ $\boxed{A}$

$= \frac{1}{9}\left[\left[\frac{x^3}{48}+x\right]_0^4 + \left[2x^{3/2}-\frac{x^2}{2}\right]_4^9\right]$

$= \frac{1}{9}\left[\left(\frac{64}{48}+4\right) + \left(54-\frac{81}{2}\right)-\left(16-8\right)\right] = \frac{1}{9}\left[\frac{65}{6}\right] = \frac{65}{54}$

7.

(a) $\frac{f(\sqrt{\pi})-f(0)}{\sqrt{\pi}-0} = \frac{\sqrt{\pi}\cos(\pi)-\sqrt{0}\cos(0)}{\sqrt{\pi}} = \frac{-\sqrt{\pi}}{\sqrt{\pi}} = -1$

(b) $\frac{1}{\sqrt{\pi}}\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \frac{1}{2\sqrt{\pi}}\int_0^{\pi}\cos(u)\,du = \frac{1}{2\sqrt{\pi}}\left[\sin u\right]_0^{\pi} = 0$

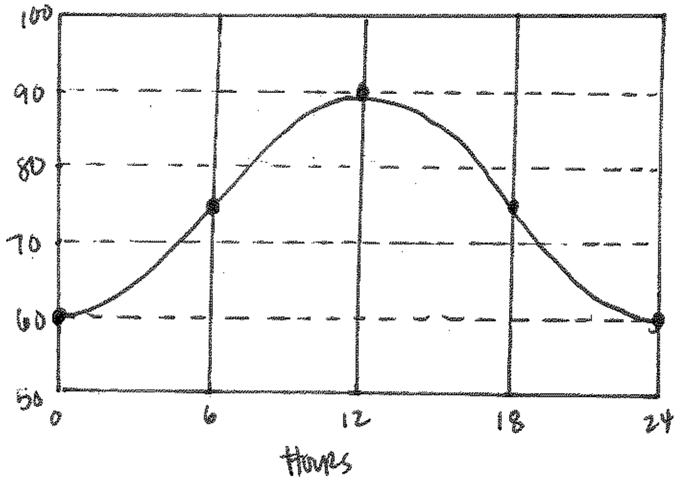
$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

(c) $f'(x) = x\cdot -\sin(x^2)\cdot 2x + \cos(x^2)$

$= -2x^2\sin(x^2) + \cos(x^2)$

$\frac{1}{\sqrt{\pi}}\int_0^{\sqrt{\pi}} -2x^2\sin(x^2)+\cos(x^2)\,dx = -1$

8. (a)



$F(t) = 75 + 15\sin\left[\frac{\pi(t-6)}{12}\right]$

(b) $\frac{1}{6}\int_4^{10} F(t)\,dt = \frac{1}{6}\int_4^{10} 75 + 15\sin\left(\frac{\pi(t-6)}{12}\right)dt$

$u = \frac{\pi(t-6)}{12}$

$du = \frac{\pi}{12}dt \qquad \frac{12}{\pi}du = dt$

$= \frac{1}{6}\left[\int_4^{10} 75\,dt + \int_{-\pi/6}^{\pi/3} \frac{180}{\pi}\sin(u)\,du\right]$

$= \frac{1}{6}\left[(75t)\Big|_4^{10} - \frac{180}{\pi}(\cos u)\Big|_{-\pi/6}^{\pi/3}\right]$

$= \frac{1}{6}\left[(750-300) - \frac{180}{\pi}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\right] = 78°F$

(c) $75 + 15\sin\left[\frac{\pi(t-6)}{12}\right] > 80$

$15\sin\left[\frac{\pi(t-6)}{12}\right] > 5$

$\sin\left[\frac{\pi(t-6)}{12}\right] > \frac{1}{3} \implies 7.298 \le t \le 16.702$

(d) Avg Temp when incurring charges $= \frac{1}{16.702 - 7.298}\int_{7.298}^{16.702} F(t)\,dt = 86.489$

Avg temp exceeding 80°F $= 6.489$

Cost $= 6.489(16.702 - 7.298) \cdot .12 = \$7.32$

Arc Length (BC only):

1. $y' = \frac{1}{2}(x^2+2)^{1/2} \cdot 2x = x(x^2+2)^{1/2}$

$\boxed{B}$

$$\int_1^2 \sqrt{1+\left[x(x^2+2)^{1/2}\right]^2}\,dx = \int_1^2 \sqrt{1+x^2(x^2+2)}\,dt = \int_1^2 \sqrt{1+x^4+2x^2}\,dx$$

$$= \int_1^2 \sqrt{(x^2+1)^2}\,dx$$

$$= \int_1^2 x^2+1\,dx$$

$$= \left[\frac{x^3}{3}+x\right]_1^2 = \left(\frac{8}{3}+2\right)-\left(\frac{1}{3}+1\right)$$

$$= \frac{10}{3}$$

2. $y' = \frac{1}{\sin x} \cdot \cos x = \cot x$

Identity: $1+\cot^2 x = \csc^2 x$

$\boxed{C}$

$$\int_{\pi/3}^{2\pi/3} \sqrt{1+(\cot^2 x)}\,dx$$

$$\int_{\pi/3}^{2\pi/3} \sqrt{1+(\csc^2 x - 1)}\,dx = \int_{\pi/3}^{2\pi/3} \sqrt{\csc^2 x}\,dx$$

3. $y' = \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$

$\boxed{A}$

$$\int_1^4 \sqrt{1+\left[\tfrac{1}{2}x^{1/2}-\tfrac{1}{2}x^{-1/2}\right]^2}\,dx$$

$$\int_1^4 \sqrt{1+\tfrac{1}{4}x-\tfrac{1}{2}+\tfrac{1}{4}x^{-1}}\,dx$$

$$\int_1^4 \sqrt{\tfrac{1}{2}+\tfrac{1}{4}x+\tfrac{1}{4}x^{-1}}\,dx$$

$$\int_1^4 \sqrt{\frac{2x+x^2+1}{4x}}\,dx$$

$$\frac{1}{2}\int_1^4 \sqrt{\frac{(x+1)^2}{x}}\,dx = \frac{1}{2}\int_1^4 \frac{x+1}{\sqrt{x}}\,dx = \frac{1}{2}\int_1^4 x^{1/2}+x^{-1/2}\,dx$$

4. $y' = \frac{1}{x^2+1} \cdot 2x - 1 = \frac{2x}{x^2+1} - 1 = \frac{2x - x^2 - 1}{x^2+1} = \frac{-(x^2 - 2x + 1)}{x^2+1} = \frac{-(x-1)^2}{x^2+1}$

$$\int_0^3 \sqrt{1 + \left[\frac{(x-1)^2}{x^2+1}\right]^2}\, dx = 3.135$$

$\boxed{D}$

5. $y' = x^2 - 1$

$\int x^2 - 1\, dx = \frac{1}{3}x^3 - x + C$

$-3 = \frac{1}{3}(0)^3 - 0 + c$

$-3 = c$

$y = \frac{1}{3}x^3 - x - 3$

$\boxed{C}$

6. $F'(x) = \sqrt{x^2+1} \cdot 2x = 2x\sqrt{x^2+1}$

$\boxed{D}$

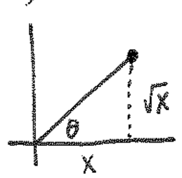
$$\int_1^2 \sqrt{1 + \left[2x\sqrt{x^2+1}\right]^2}\, dx$$

$$\int_1^2 \sqrt{1 + 4x^2(x^2+1)}\, dx$$

$$\int_1^2 \sqrt{4x^4 + 4x^2 + 1}\, dx = \int_1^2 \sqrt{(2x^2+1)^2}\, dx = \int_1^2 2x^2 + 1\, dx = \left[\frac{2}{3}x^3 + x\right]_1^2$$

$$= \left(\frac{16}{3} + 2\right) - \left(\frac{2}{3} + 1\right) = \frac{17}{3}$$

7. (a)



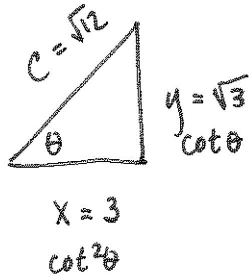
$\cot\theta = \frac{x}{\sqrt{x}} = \sqrt{x}$

$(x, y) \longrightarrow (\cot^2\theta, \cot\theta)$

(b) $\int_1^4 \sqrt{1 + \left[\frac{1}{2\sqrt{x}}\right]^2}\, dx = 3.168$

$y' = \frac{1}{2\sqrt{x}}$

7. (c)



$$c = \sqrt{\cot^4\theta + \cot^2\theta}$$

$$c = \cot\theta \cdot \sqrt{\cot^2\theta + 1}$$

$$c = \cot\theta \cdot \sqrt{\csc^2\theta}$$

$$c = \cot\theta \cdot \csc\theta$$

$$\frac{dc}{dt} = \cot\theta \cdot -\csc\theta\cot\theta \cdot \frac{d\theta}{dt} + \csc\theta \cdot -\csc^2\theta \cdot \frac{d\theta}{dt}$$

$$= -\frac{d\theta}{dt}\left[\cot^2\theta\csc\theta + \csc^3\theta\right]$$

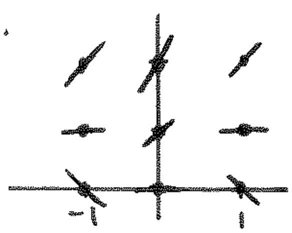
$$= -\left(-\frac{1}{10}\right)\left[\left(\frac{3}{\sqrt{3}}\right)^2(2) + (2)^3\right] = \frac{1}{10}\left[14\right] = 1.4 \text{ units/min.}$$

Slope fields:

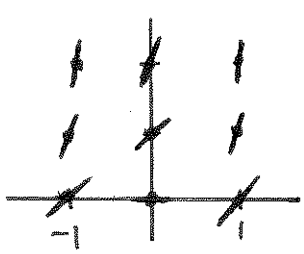
1. B

2. C

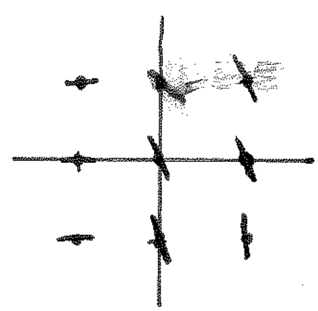
3.



4.



5.



Differential Equations:

1. $\frac{dy}{dx} = \frac{3x^2}{2y} \Rightarrow \int 2y\,dy = \int 3x^2\,dx$

$y^2 = x^3 + c$

$y = \pm\sqrt{x^3+c} \rightarrow y = \sqrt{x^3-11}$

$4 = \pm\sqrt{3^3+c}$

$4 = \pm\sqrt{27+c}$

$16 = 27+c$

$-11 = c$

$\boxed{D}$

2. $\frac{dy}{dx} = \frac{x+\sec^2 x}{y} \Rightarrow \int y\,dy = \int x + \sec^2 x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \tan x + c \searrow$

$\frac{1}{2}(4) = \frac{1}{2}(0)^2 + \tan(0) + c$

$2 = c$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \tan x + 2$

$y^2 = x^2 + 2\tan x + 4$

$y = \pm\sqrt{x^2+2\tan x+4}$

$\boxed{B}$

3. $\frac{dy}{dx} = xy \Rightarrow \frac{1}{y}\,dy = x\,dx$

$\ln|y| = \frac{1}{2}x^2 + c \rightarrow \ln|y| = \frac{1}{2}x^2$

$\ln|-1| = \frac{1}{2}(0)^2 + c$

$0 = c$

$|y| = e^{\frac{1}{2}x^2}$

$y = \pm e^{\frac{1}{2}x^2}$

$\boxed{C}$

4. $\frac{dy}{dx} = (y-4)\cdot \sec^2 x \Rightarrow \frac{1}{y-4}\,dy = \sec^2 x\,dx$

$\ln|y-4| = \tan x + c$

$\ln|1| = \tan(0) + c$

$0 = c$

$\ln|y-4| = \tan x$

$y-4 = \pm e^{\tan x}$

$y = \pm e^{\tan x} + 4$

$\boxed{A}$

5. $\frac{dy}{dx} = \frac{1}{4}x - y + 1 = m$

$\boxed{C}$

$y = \left(\frac{1}{4}x - y + 1\right)x + b$

$y = \frac{1}{4}x^2 - xy + x + b \Rightarrow b = y - \frac{1}{4}x^2 + xy - x$

$m+b = \left(\frac{1}{4}x - y + 1\right) + \left(y - \frac{1}{4}x^2 + xy - x\right)$

$m+b = -\frac{1}{4}x^2 + xy - \frac{3}{4}x + 1$

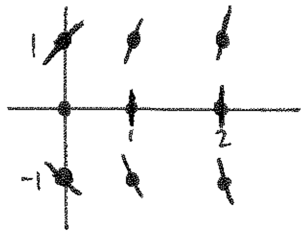
$m+b = -\frac{1}{4}x^2 + x(mx+b) - \frac{3}{4}x + 1$

$m+b = -\frac{1}{4}x^2 + mx^2 + bx - \frac{3}{4}x + 1$

$m+b = -\frac{1}{4}x^2(1-4m) + x\left(b - \frac{3}{4}\right) + 1$

Since $m+b =$ constant $\Rightarrow m+b = 1$

6. (a)



(b) $(1, \sqrt{3})$

$$\left.\frac{dy}{dx}\right|_{(1,\sqrt{3})} = \frac{1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$y - \sqrt{3} = \frac{2}{\sqrt{3}}(x-1)$$

$$y = \frac{2}{\sqrt{3}}(x-1) + \sqrt{3}$$

$$y(1.2) = \frac{2}{\sqrt{3}}(0.2) + \sqrt{3} \approx 1.963$$

(c)

$$y\ dy = x+1\ dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C \rightarrow \frac{1}{2}y^2 = \frac{1}{2}x^2 + x$$

$$y^2 = x^2 + 2x$$

$y(1) = \sqrt{3}$

$$\frac{1}{2}(\sqrt{3})^2 = \frac{1}{2}(1)^2 + 1 + C$$

$$\frac{3}{2} = \frac{3}{2} + C$$

$$0 = C$$

$$y = \pm\sqrt{x^2+2x}$$

since $y(1) = \sqrt{3}$, $y = \sqrt{x^2+2x}$.

(d) $f(1.2) = \sqrt{(1.2)^2 + 2(1.2)} = 1.959$

7.(a) $(0,2)$

$$\left.\frac{dy}{dx}\right|_{(0,2)} = \frac{2(0)+3}{e^2} = \frac{3}{e^2} \qquad y-2 = \frac{3}{e^2}x$$

(b) $$f''(x) = \frac{d^2y}{dx^2} = (2x+3)\cdot -e^{-y}\cdot\frac{dy}{dx} + e^{-y}(2) = \frac{-(2x+3)}{e^y}\cdot\frac{dy}{dx} + \frac{2}{e^y}$$

$$\frac{dy}{dx} = (2x+3)e^{-y} \qquad f''(0) = \left.\frac{d^2y}{dx^2}\right|_{(0,2)} = \frac{-3}{e^2}\cdot\frac{3}{e^2} + \frac{2}{e^2} = \frac{2e^2-9}{e^4}$$

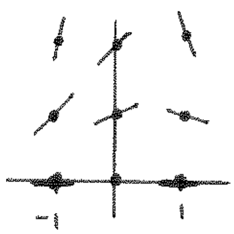
$$\left.\frac{dy}{dx}\right|_{(0,2)} = \frac{3}{e^2}$$

(c) $$e^y\,dy = (2x+3)\,dx$$

$$e^y = x^2+3x+C \rightarrow e^y = x^2+3x+e^2$$

$$e^2 = c \qquad y = \ln|x^2+3x+e^2|$$

8. (a)



(b) $$\frac{d^2y}{dx^2} = \frac{1}{3}\left[y^2(-2) + (1-2x)\cdot 2y\cdot\frac{dy}{dx}\right] = \frac{1}{3}\left[-2y^2 + 2y(1-2x)\cdot\left(\frac{y^2(1-2x)}{3}\right)\right]$$

$$= \frac{1}{3}\left[-2y^2 + \frac{2y^3}{3}(1-2x)^2\right]$$

(c) $$\left.\frac{dy}{dx}\right|_{(\frac{1}{2},4)} = \frac{16(1-1)}{3} = 0 \qquad \left.\frac{d^2y}{dx^2}\right|_{(\frac{1}{2},4)} = \frac{1}{3}\left[-2(16) + \frac{2(64)}{3}(1-1)^2\right] = \frac{-32}{3} < 0$$

Therefore f has a relative maxima at $\left(\frac{1}{2}, 4\right)$.

8. (d) $\frac{3}{y^2}\,dy = 1-2x\,dx$

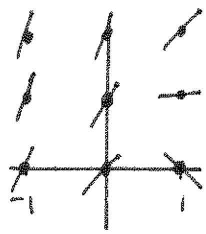
$-\frac{3}{y} = x - x^2 + C \rightarrow -\frac{3}{y} = -x^2 + x - 1$

$-\frac{3}{4} = \frac{1}{2} - \frac{1}{4} + C \qquad \frac{3}{y} = x^2 - x + 1$

$-1 = C \qquad \frac{y}{3} = \frac{1}{x^2 - x + 1}$

$y = \frac{3}{x^2-x+1}$

9. (a)



(b) $\frac{d^2y}{dx^2} = -2 + \frac{dy}{dx} = -1 - 2x + y$

Concave down $\Rightarrow \frac{d^2y}{dx} = -1-2x+y < 0$

$y < 2x+1$

The solution curves that are concave down will be in the plane below the line $y = 2x+1$.

(c) $\left.\frac{dy}{dx}\right|_{(0,-1)} = -2(0) + -1 + 1 = 0 \qquad \left.\frac{d^2y}{dx^2}\right|_{(0,-1)} = -1 - 2(0) + -1 = -2 < 0$

Since $\left.\frac{dy}{dx}\right|_{(0,-1)} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(0,-1)} < 0$, there is a relative maximum at $(0,-1)$.

(d). $y = mx+b$

$m = -2x + y + 1$

$m = -2x + mx + b + 1$

$m = -2x(1 - \frac{1}{2}m) + (b+1)$

$-2(1-\frac{1}{2}m) = 0 \qquad b+1 = m$

$1 - \frac{1}{2}m = 0 \qquad b+1 = 2$

$\boxed{m = 2} \qquad \boxed{b = 1}$

Exponential Growth and Decay:

1. $\frac{dP}{dt} = kP$ $(0, A)$ $(4, 2A)$ $P = Ce^{kt}$ $A = C$

$2A = A \cdot e^{4k}$
$2 = e^{4k}$
$\ln 2 = 4k$
$\frac{1}{4}\ln 2 = k$

$3A = A \cdot e^{\frac{1}{4}\ln 2 \cdot t}$
$3 = e^{1/4 \ln 2 \cdot t}$
$\ln 3 = 1/4 \ln 2 \cdot t$
$\frac{4\ln 3}{\ln 2} = t$

$\boxed{D}$

P = population of Bacteria

2. $\frac{dy}{dt} = ky$ $(0, A)$ $(15, 2A)$ $y = Ce^{kt}$ $A = C$

$2A = A \cdot e^{15k}$
$2 = e^{15k}$
$\ln 2 = 15k$
$\frac{\ln 2}{15} = k = 0.046$

$\boxed{B}$

3. $(0, 6)$ $(3, 9)$ $\frac{dw}{dt} = kw \Rightarrow w = Ce^{kt}$ $6 = C$

$9 = 6e^{3k}$
$\frac{3}{2} = e^{3k}$
$\ln(3/2) = 3k$
$\frac{\ln(3/2)}{3} = k$

$w = 6e^{\frac{1}{3}\ln(3/2) \cdot t}$
$w(6) = 6e^{1/3 \ln(3/2) \cdot 6} = 13.5$

$\boxed{C}$

4. $\frac{dF}{dt} = kF \Rightarrow F = Ce^{kt}$ $180 = C$

$(0, 180)$ $(16, 120)$

$120 = 180e^{16k}$
$\frac{2}{3} = e^{16k}$
$\ln(2/3) = 16k$
$\frac{\ln(2/3)}{16} = k = -.025$

$\boxed{A}$

5. (a) $\frac{dV}{dt} = kV \Rightarrow V = Ce^{kt}$ $\quad 16 = C \quad 8 = 16e^{80k}$

$(0,16) \quad (80,8)$

$\frac{1}{2} = e^{80k}$

$\frac{\ln(1/2)}{80} = k$

$V = 16e^{\frac{1}{80}\ln(1/2)\cdot t}$

$= 16e^{-.00866t}$

(b) $\frac{dV}{dt} = \frac{1}{80}\ln(1/2)\cdot V$

$\left.\frac{dV}{dt}\right|_{V=4} = \frac{1}{80}\ln(1/2)\cdot 4 = -.035 \text{ oz/sec}$

Amount of coffee is decreasing at a rate of 0.035 ounces per second when V = 4 ounces.

(c) $2 = 16e^{\frac{1}{80}\ln(1/2)\cdot t}$

$\frac{1}{8} = e^{\frac{1}{80}\ln(1/2)\cdot t}$

$\ln(1/8) = \frac{1}{80}\ln(1/2)\cdot t \rightarrow t = \frac{80\ln(1/8)}{\ln(1/2)} = 240 \text{ seconds}$

<u>Logistic Equations</u> (BC only)

1. $\frac{dp}{dt} = 3p - 0.0006p^2 = 3p - \frac{3}{5000}p^2 = 3p\left(1 - \frac{p}{5000}\right)$ $\boxed{D}$

$\lim_{t\to\infty} p(t) = 5000$

2. P growing fastest when $p = \frac{1}{2}(240) = 120$ $\boxed{C}$

3. Inflection point $\Rightarrow$ $p = \frac{1}{2}(150) = 75$

$$\left.\frac{dp}{dt}\right|_{p=75} = \frac{75}{5}\left(1 - \frac{75}{150}\right) = 7.5$$

$\boxed{B}$

4. $$\frac{dy}{dt} = y(1-3y) = y\left(1 - \frac{y}{1/3}\right)$$

Spreading fastest when $y = \frac{1}{2}\left(\frac{1}{3}\right) = \frac{1}{6}$

$\boxed{A}$

5. $$\frac{dP}{dt} = kp\left(1 - \frac{P}{120}\right) = kp(1 - .008\bar{3}p)$$
$$= kp - .008\bar{3}kp^2$$
$$= k\left(p - \frac{p^2}{120}\right)$$

$\boxed{D}$

6. (a) $$\frac{dy}{dt} = 2y\left(1 - \frac{1}{4}t\right)$$
$$\frac{1}{y}\,dy = \left(2 - \frac{1}{2}t\right)dt$$
$$\ln|y| = 2t - \frac{1}{4}t^2 + C \rightarrow \ln|y| = -\frac{1}{4}t^2 + 2t - 3$$
$$0 = 4 - 1 + C \qquad y = \pm e^{-1/4 t^2 + 2t - 3}$$
$$-3 = C \qquad y = e^{-1/4 t^2 + 2t - 3}$$

6. (b) $\lim_{t \to \infty} e^{2t - \frac{1}{4}t^2 - 3} = e^{-\infty} = 0$

(c) $\lim_{t \to \infty} g(t) = 5$

$\lim_{t \to \infty} g'(t) = 0$

(d) POI when $y = \frac{1}{2}(5) = 5/2$

$$\left.\frac{dy}{dt}\right|_{y=5/2} = \frac{5}{2}\left(1 - \frac{1}{2}\right) = 5/4$$

Euler's Method (BC only):

1. $x_0 = 1$ $\quad y_0 = 2$

$x_1 = 1.5$ $\quad y_1 = 2 + \frac{1}{2}\left[1 + 2(1) - 2\right] = 5/2$

$x_2 = 2$ $\quad y_2 = \frac{5}{2} + \frac{1}{2}\left[1 + 2\left(\frac{3}{2}\right) - \frac{5}{2}\right] = 3.25$

$\boxed{C}$

2. $x_0 = 0.5$ $\quad y_0 = 0$

$x_1 = 1$ $\quad y_1 = 0 + 0.5\left(\frac{1}{2} - \frac{1}{2}(0)\right) = 1/4$

$x_2 = 1.5$ $\quad y_2 = 1/4 + 0.5\left(1 - 1(1/4)\right) = 5/8$

$x_3 = 2$ $\quad y_3 = 5/8 + 0.5\left(1.5 - 1.5(5/8)\right) = 0.906$

$\boxed{B}$

3. $x_0 = 0$ $\quad y_0 = 1$ $\qquad \boxed{B}$

$x_1 = 1$ $\quad y_1 = 1 + 1\left(\arctan(0)\right) = 1$

$x_2 = 2$ $\quad y_2 = 1 + 1\left(\arctan(1)\right) = 1 + \frac{\pi}{4}$

4. $x_0 = -1$ $\quad y_0 = 1.5$ $\qquad \boxed{A}$

$x_1 = -0.2$ $\quad y_1 = 1.5 + 0.8(1) = 2.3$

$x_2 = 0.6$ $\quad y_2 = 2.3 + 0.8(-0.5) = 1.9$

5. $x_0 = 0$ $\quad y_0 = 1$

$x_1 = 0.5$ $\quad y_1 = 1 + \frac{1}{2}(1) = 3/2$ $\qquad \boxed{D}$

$x_2 = 1$ $\quad y_2 = \frac{3}{2} + \frac{1}{2}\left(\frac{1}{2}k + 3/2 - 2(1/2)^2\right) = 3$

$$\frac{3}{2} + \frac{1}{2}\left(\frac{1}{2}k + \frac{3}{2} - \frac{1}{2}\right) = 3$$

$$\frac{1}{2}\left(\frac{1}{2}k + 1\right) = \frac{3}{2}$$

$$\frac{1}{2}k + 1 = 3$$

$$\frac{1}{2}k = 2 \Rightarrow k = 4$$

6. (a) $\dfrac{d^2y}{dx^2} = \dfrac{1}{2} - \dfrac{dy}{dx} = 1 - \dfrac{1}{2}x + y$

(b) $\left.\dfrac{dy}{dx}\right|_{(0,-1/2)} = 0$

$\left.\dfrac{d^2y}{dx^2}\right|_{(0,-1/2)} = 1/2 > 0$

f has a relative minimum at $(0, -1/2)$ since $\left.\dfrac{dy}{dx}\right|_{(0,-1/2)} = 0$ and $\left.\dfrac{d^2y}{dx^2}\right|_{(0,-1/2)} > 0$.

(c). $x_0 = 0 \qquad y_0 = k$

$x_1 = 1/2 \qquad y_1 = k + \frac{1}{2}\left(-k - \frac{1}{2}\right) = \frac{1}{2}k - \frac{1}{4}$

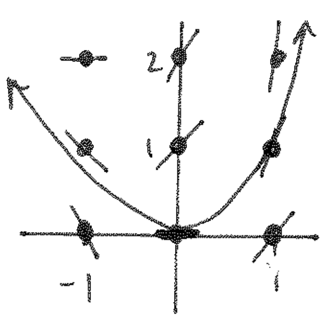
$x_2 = 1 \qquad y_2 = \left(\frac{1}{2}k - \frac{1}{4}\right) + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{2}k + \frac{1}{4} - \frac{1}{2}\right)$

$= \frac{1}{2}k - \frac{1}{4} + \frac{1}{2}\left(-\frac{1}{2}k\right)$

$= \frac{1}{4}k - \frac{1}{4}$

$\frac{1}{4}k - \frac{1}{4} = 1 \Rightarrow k = 5$

7. (a)



7. (b) $x_0 = 1 \qquad y_0 = 1$

$x_1 = 1.1 \qquad y_1 = 1 + 0.1\left(2(1)+1\right) = 1.3$

$x_2 = 1.2 \qquad y_2 = 1.3 + 0.1\left(2(1.1)+1.3\right) = 1.65$

(c)

$-2 = 2x + y$

$-2 = 2x + (-2x + b)$

$-2 = b$

(d) $\left.\frac{dy}{dx}\right|_{(1,-2)} = 2(1) - 2 = 0$

$\frac{d^2y}{dx^2} = 2 + \frac{dy}{dx} = 2 + 2x + y$

$\left.\frac{d^2y}{dx^2}\right|_{(1,-2)} = 2 + 2(1) - 2 = 2 > 0$

g has a local minimum at $(1,-2)$ since $\left.\frac{dy}{dx}\right|_{(1,-2)} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(1,-2)} > 0$.