AP Calculus

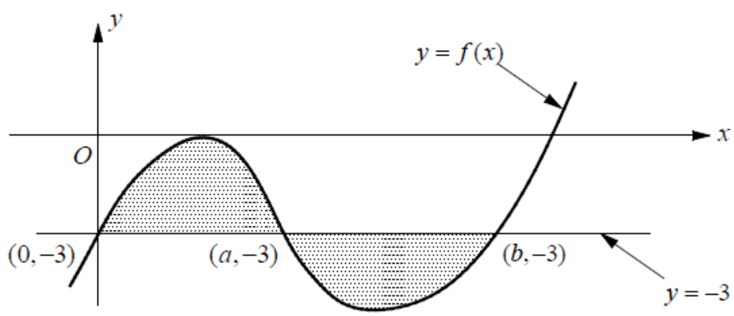
Applications of Integration Review

1. What is the area of the region enclosed by the graphs of $f(x) = x + 2$ and $g(x) = x^3 - 4x^2 + 6$?

(A) $\frac{193}{12}$ (B) $\frac{218}{12}$ (C) $\frac{253}{12}$ (D) $\frac{305}{12}$

2. What is the area of the region in the first quadrant, bounded by the curve $y = \sqrt[3]{x}$ and $y = x$?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$



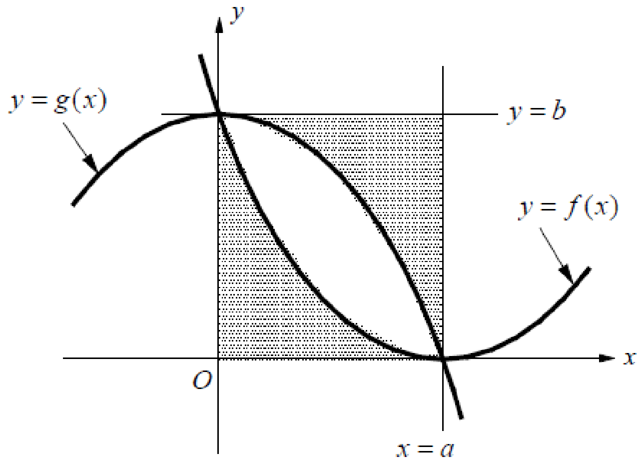
3. The curve $y = f(x)$ and the line $y = -3$, shown in the figure above, intersect at the points $(0, -3)$, $(a, -3)$, and $(b, -3)$. The sum of area of the shaded region enclosed by the curve and the line is given by

(A) $\int_0^a [3 - f(x)]\, dx + \int_a^b [-3 + f(x)]\, dx$

(B) $\int_0^a [-3 + f(x)]\, dx + \int_a^b [3 - f(x)]\, dx$

(C) $\int_0^a [f(x) + 3]\, dx + \int_a^b [-3 - f(x)]\, dx$

(D) $\int_0^a [f(x) - 3]\, dx + \int_a^b [3 - f(x)]\, dx$



4. Which of the following is the area of the shaded region in the figure above?

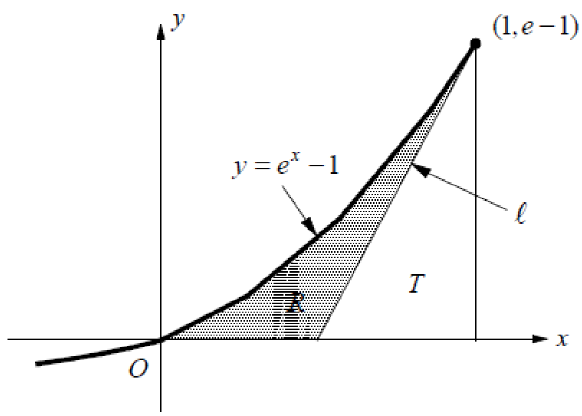
(A) $\int_0^a [g(x) - f(x)]\, dx$

(B) $\int_0^a [b + g(x) - f(x)]\, dx$

(C) $\int_0^a [b - g(x) - f(x)]\, dx$

(D) $\int_0^a [b - g(x) + f(x)]\, dx$

Free Response Questions



5. The figure above shows the graph of $y = e^x - 1$ and the line ℓ tangent to the graph at $(1, e-1)$.

(a) Find the area of the triangular region T, which is bounded by the line $x = 1$, x-axis and ℓ.

(b) Find the area of region R, which is bounded by the graph of $y = e^x - 1$, x-axis and ℓ.

Volumes of Revolution

1. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{3}$, and the coordinate axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\sqrt{3}\pi$ (D) 3π

2. The region enclosed by the graphs of $y = e^{(x/2)}$ and $y = (x-1)^2$ from $x = 0$ to $x = 1$ is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{11}{4}\pi$ (B) $2(e-1)\pi$ (C) $(e-\frac{3}{2})\pi$ (D) $(e-\frac{6}{5})\pi$

3. Let R be the region between the graphs of $y = 1+\sin(\pi x)$ and $y = x^3$ from $x = 0$ to $x = 1$. The volume of the solid obtained by revolving R about the x-axis is given by

(A) $\pi\int_0^1 \left[1+\sin(\pi x)-x^3\right] dx$
(B) $\pi\int_0^1 \left[(1+\sin(\pi x))^2-x^6\right] dx$
(C) $\pi\int_0^1 \left[1+\sin(\pi x)-x^3\right]^2 dx$
(D) $2\pi\int_0^1 \left[1+\sin(\pi x)-x^3\right] dx$

4. The region R is enclosed by the graph of $y = \sqrt{x+1}$, the line $y = x-1$, and the y-axis. The volume of the solid generated when is R rotated about the line $y = 2$ is

(A) $\frac{13}{2}\pi$ (B) $\frac{20}{3}\pi$ (C) $\frac{49}{6}\pi$ (D) 9π

5. The region R is enclosed by the graph of $y = 3x - x^2$ and the line $y = x$. If the region R is rotated about the line $y = -1$, the volume of the solid that is generated is represented by which of the following integrals?

(A) $\pi \int_0^2 \left[3x - x^2 - x + 1\right]^2 \, dx$

(B) $\pi \int_0^2 \left[\left(3x - x^2 + 1\right)^2 - (x+1)^2\right] dx$

(C) $\pi \int_0^2 \left[\left(3x - x^2 + 1\right) - (x+1)\right]^2 \, dx$

(D) $\pi \int_0^2 \left[\left(3x - x^2 - 1\right)^2 - (x-1)^2\right] dx$

6. The region R is enclosed by the graph of $y = x + \frac{3}{x}$ and the line $y = 4$. The volume of the solid generated when is R rotated about the x-axis is

(A) $\frac{16}{3}\pi$ (B) 4π (C) 6π (D) $\frac{15\pi}{2}$

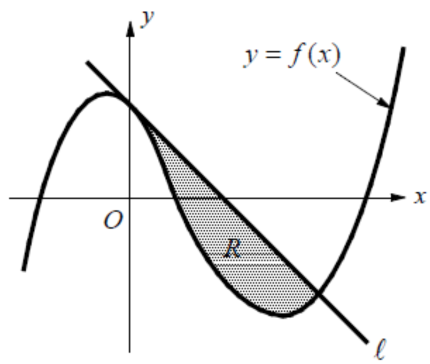
7. The volume of the solid generated by revolving the region enclosed by the ellipse $x^2 + 9y^2 = 36$ about the x-axis is

(A) 14π (B) 16π (C) 24π (D) 32π

8. The volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x}$, $y = 2$, and y-axis about the y-axis is

(A) $\frac{32}{5}\pi$ (B) $\frac{16}{3}\pi$ (C) $\frac{10}{3}\pi$ (D) $\frac{8}{3}\pi$

Free Response Questions



9. Let f be the function given by $f(x) = x^3 - 2x^2 - x + \cos x$. Let R be the shaded region bounded by the graph of f and the line ℓ, which is the line tangent to the graph of f at $x = 0$, as shown above.

(a) Find the equation of the line ℓ.

(b) Find the area of R.

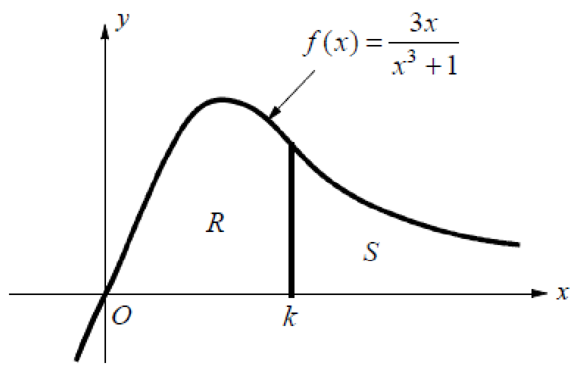
(c) Set up, but do not evaluate, an integral expression for the volume of the solid generated when R is revolved about the line $y = 2$.

10. Let R be the region between the graphs of $y = e^x$, $y = 2$ and $x = -1$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the line $x = -1$.

(c) Find the volume of the solid generated when R is revolved about the line $y = -1$.



11. Let f be the function given by $f(x) = \dfrac{3x}{x^3+1}$. Let R be the region bounded by the graph of f, the x-axis, and the vertical line $x = k$, where $k > 0$. BC

(a) Find the volume of the solid generated when R is revolved about the x-axis in terms of k.

(b) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f, as shown in the figure above. Find the value of k such that the volume of the solid generated when S is revolved about the x-axis is equal to the volume of the solid found in part (a).

Volumes of Known Cross Sections

1. The base of a solid is the region enclosed by the graph of $y = e^x$, the coordinate axes, and the line $x = 1$. If the cross sections of the solid perpendicular to the x-axis are squares, what is the volume of the solid?

(A) $\dfrac{e^2}{4}$ (B) $\dfrac{e^2-1}{2}$ (C) $\dfrac{e^2+1}{2}$ (D) $e^2 - \dfrac{1}{2}$

2. The base of a solid is the region enclosed by the graph of $y = \sqrt{x}$, the x-axis, and the line $x = 2$. If each cross section perpendicular to the x-axis is an equilateral triangle, what is the volume of the solid?

(A) $\dfrac{\sqrt{3}}{8}$ (B) $\dfrac{\sqrt{3}}{6}$ (C) $\dfrac{\sqrt{3}}{4}$ (D) $\dfrac{\sqrt{3}}{2}$

3. The base of a solid is the region in the first quadrant bounded by the coordinate axes, and the line $2x+3y=6$. If the cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) $\frac{\pi}{2}$ (B) $\frac{3\pi}{4}$ (C) π (D) $\frac{3\pi}{2}$

4. The base of a solid S is the semicircular region enclosed by the graph of $y=\sqrt{9-x^2}$ and the x-axis. If the cross sections of S perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) $\frac{20\pi}{3}$ (B) 6π (C) $\frac{9\pi}{2}$ (D) $\frac{7\pi}{2}$

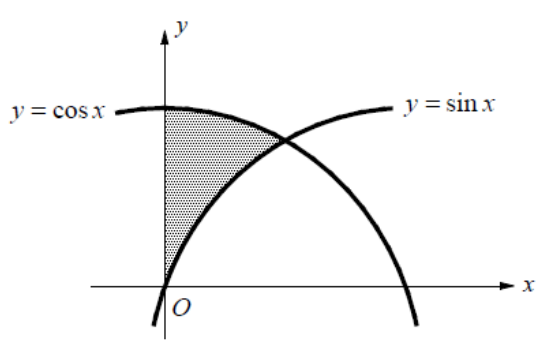
5. The base of a solid is the region bounded by the graph of $y=\sqrt{x}$, the x-axis and the line $x=4$. If the cross sections of the solid perpendicular to the y-axis are squares, the volume of the solid is given by

(A) $\int_0^2 (4-y^2)^2\,dy$

(B) $\int_0^2 (4-y)^2\,dy$

(C) $\int_0^2 \left[(2-y)^2\right]^2\,dy$

(D) $\int_0^4 \left[(2-y)^2\right]^2\,dy$



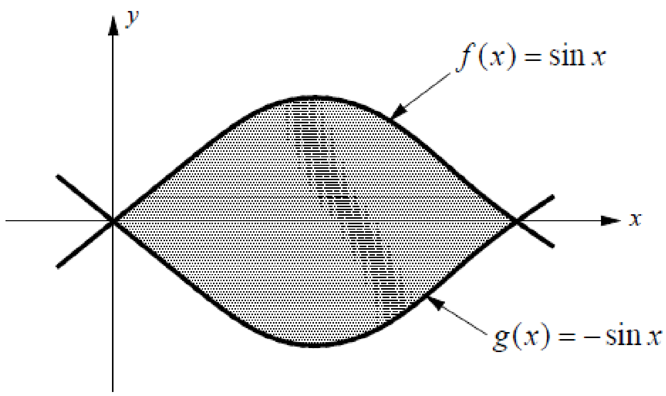
6. The base of a solid is the region in the first quadrant bounded by the y-axis and the graphs of $y=\cos x$ and $y=\sin x$, as shown in the figure above. If the cross sections of the solid perpendicular to the x-axis are squares, what is the volume of the solid?

(A) $\pi-1$ (B) $\pi+1$ (C) $\frac{\pi-2}{4}$ (D) $\frac{\pi+2}{4}$

7. Let R be the region enclosed by the graph of $y = 3\sqrt{x} - x$ and the x-axis. The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $g(x) = \dfrac{1}{\sqrt{x}}$. What is the volume of the water in the pond?

(A) $2\sqrt{3}$ (B) 6 (C) $4\sqrt{3}$ (D) 9

Free Response Questions



8. Let $f(x) = \sin x$ and $g(x) = -\sin x$ for $0 \le x \le \pi$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 3$.

(c) Let h be the function given by $h(x) = k\sin x$ for $0 \le x \le \pi$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. If the volume of the solid is equal to 8π, what is the value of k?

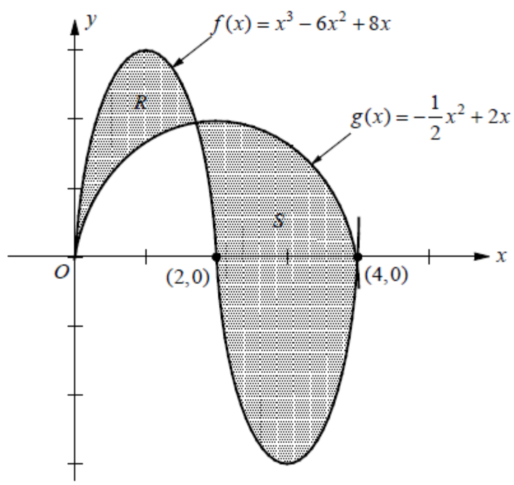
x (meters)	0	2	4	6	8	10	12
$D(x)$ (meters)	1.7	1.5	1.46	1.42	1.5	1.38	1.21

9. A 12 meter long tree trunk with circular cross sections of varying diameter is represented in the table above. The distance, x, of the tree trunk is measured from the ground and $D(x)$ represents the diameter at that point.

 (a) Write an integral expression in terms of $D(x)$ that represents the volume of the tree trunk between $x = 0$ and $x = 12$.

 (b) Approximate the volume of the tree trunk between $x = 0$ and $x = 12$ using the data from the table and a midpoint Riemann sum with three subintervals of equal length.

 (c) Explain why there must be a value x for $0 < x < 12$ such that $D'(x) = 0$?



10. Let R and S be the region bounded by the graphs of $f(x) = x^3 - 6x^2 + 8x$ and $g(x) = -\frac{1}{2}x^2 + 2x$ as shown in the figure above.

 (a) Write, but do not evaluate, an integral expression that can be used to find the area of R.

 (b) Write, but do not evaluate, an integral expression that can be used to find the area of S.

 (c) The region R is the base of a solid. At all points in R at a distance x from the y-axis, the height of the solid is given by $g(x) = 4e^{-x}$. Find the volume of this solid.

 (d) The region S models the surface of a small pond. At all points in S at a distance x from the y-axis, the depth of the water is given by $h(x) = 4 - \sqrt{x}$. Find the volume of water in the pond.

Total Change in a Functions (Application of FTC)

1. Oil is pumped out from a tank at the rate of $\frac{20e^{(-0.1t)}}{1+e^{-t}}$ gallons per minute, where t is measured in minutes. To the nearest gallon, how many gallons of oil are pumped out from a tank during the time interval $0 \le t \le 6$?

(A) 62 (B) 78 (C) 85 (D) 93

2. Pollutant is released into a lake at the rate of $\frac{50e^{-t/2}}{\sqrt{t+1}}$ gallons per hour. To the nearest gallon, how many gallons of pollutant are released during the time interval $0 \le t \le 12$?

(A) 53 (B) 58 (C) 66 (D) 75

3. Oil is pumped into an oil tank at the rate of $S(t)$ gallons per hour during the time interval $0 \le t \le 8$ hours. During the same time interval, oil is removed from the tank at the rate of $R(t)$ gallons per hour. If the oil tank contained 200 gallons of oil at time $t = 0$, which of the following expressions shows the amount of oil in the tank at time $t = 6$ hours?

(A) $200 + S(6) - R(6)$

(B) $200 + S'(6) - R'(6)$

(C) $200 + \int_0^6 (S(t) - R(t))\, dt$

(D) $200 + \int_0^6 (S'(t) - R'(t))\, dt$

4. The rate at which people enter a supermarket, measured in people per hour on a given day, is modeled by the function S defined by $S(t) = \frac{720}{t^2 - 28t + 205}$, for $6 \le t \le 22$. To the nearest whole number, how many people entered the supermarket from time $t = 6$ to $t = 22$?

(A) 426 (B) 475 (C) 524 (D) 582

5. The height of the water in a cylindrical storage tank is modeled by a differential function $h(t)$, where h is measured in meters and t is measured in hours. At time $t=0$ the height of the water in the tank is 8 meters. During the time interval $0 \le t \le 20$ hours, the height is changing at the rate $h'(t) = 0.01t^3 - 0.3t^2 + 2.2t - 1.5$ meters per hour. What is the maximum height of the water in meters during the time period $0 \le t \le 20$?

(A) 28.156 (B) 30.108 (C) 32.654 (D) 33.975

Free Response Questions

6. Water is pumped into a tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\frac{1}{2}t^{2/3}$ gallons per minute, for $0 \le t \le 90$ minutes. At time $t=0$, the tank contains 50 gallons of water.

(a) How many gallons of water leak out of the tank from time $t=0$ to $t=10$ minutes?

(b) How many gallons of water are in the tank at time $t=10$ minutes?

(c) Write an expression for $f(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 90$, is the amount of water in the tank a maximum?

7. The rate at which the amount of granules of plastic at a toy factory is changing during a workday is modeled by $P(t) = 5 - 2\sqrt{x} - 4\sin(\frac{x^2}{12})$ tons per hour, where $0 \le t \le 8$. At the beginning of the workday $(t=0)$, the factory has 6 tons of granules of plastic.

(a) Find $P'(3)$. Using correct unit, interpret your answer in the context of the problem.

(b) At what time during the 8 hours was the amount of granules of plastic decreasing most rapidly?

(c) What was the maximum amount of granules of plastic at the factory during the 8 working hours?

Particle Motion

1. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 2t - 6$.

   If at $t = 1$, the velocity of the particle is 3 and its position is $\frac{1}{3}$, then the position $x(t) =$

   (A) $\frac{t^3}{3} - 6t^2 + 5t + \frac{1}{3}$

   (B) $\frac{t^3}{3} - 3t^2 + 8t - 5$

   (C) $\frac{t^3}{3} - 6t + 9$

   (D) $\frac{t^3}{3} - 3t^2 + 8t - \frac{7}{3}$

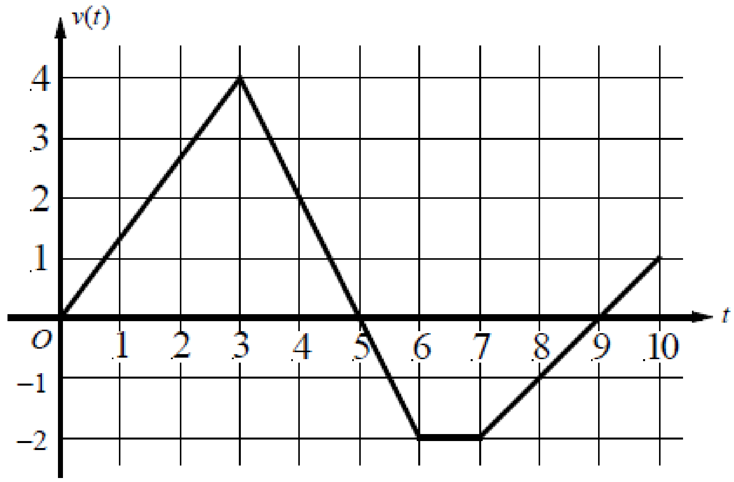
2. The velocity of a particle moving along the x-axis at any time t is given by $v(t) = 3e^{-t} - t$.
   What is the average speed of the particle over the time interval $0 \le t \le 3$?

   (A) 0.873 (B) 1.096 (C) 1.273 (D) 1.482

3. A particle travels along a straight line with a velocity of $v(t) = e^{t}(t^2 - 5t + 6)$ meters per second.
   What is the average velocity of the particle over the time interval $0 \le t \le 5$?

   (A) 58.602 (B) 64.206 (C) 79.351 (D) 86.448

Questions 4-8 refer to the following situation.



A particle is moving along the x-axis. The velocity v of the particle at time t, $0 \le t \le 10$, is given by the function whose graph is shown above.

4. At what value(s) of t does the particle change direction?

(A) 3 only (B) 3 and 6 (C) 5 and 9 (D) 6 and 7

5. What is the total distance traveled by the particle over the time interval $0 \le t \le 10$?

(A) 15.5 (B) 12 (C) 9.5 (D) 8

6. At what time t during the time interval $0 \le t \le 10$ is the particle farthest to the right?

(A) 3 (B) 5 (C) 7 (D) 9

7. What is the velocity of the particle at time $t = 4$?

(A) −2 (B) 2 (C) 5 (D) 7

8. What is the acceleration of the particle at time $t = 4$?

(A) −2 (B) 2 (C) 5 (D) 7

9. A car is traveling on a straight road with position function given by $s(t)=(4t^2-3)e^{-0.5t}$, where s is measured in meters and t is measured in seconds. At time $t=0$ seconds the brakes are applied to stop the car. To the nearest meters, how far does the car travel from time $t=0$ to the moment the car stops?

(A) 9 (B) 10 (C) 11 (D) 12

Free Response Questions

10. A particle moves along the x-axis with a velocity given by $v(t)=t\cos(t^2-1)$ for $t\geq 0$.

(a) In which direction (left or right) is the particle moving at time $t=2$?

(b) Find the acceleration of the particle at time $t=2$. Is the velocity of the particle increasing at time $t=2$?Justify your answer.

(c) Is the speed of the particle increasing at time $t=2$? Justify your answer.

(d) Given that $x(t)$ is the position of the particle at time t and that $x(0)=4$, find $x(2.5)$.

(e) During the time interval $0\leq t\leq 2.5$, what is the greatest distance between the particle and the origin?

(f) Find the total distance traveled by the particle from $t=0$ to $t=2.5$.

11. A particle moves along the y-axis with a velocity given by $v(t)=3t^2-14t+8$ for $t\geq 0$. At time $t=0$, the position of the particle is $y(0)=2$.

(a) Find the minimum acceleration of the particle.

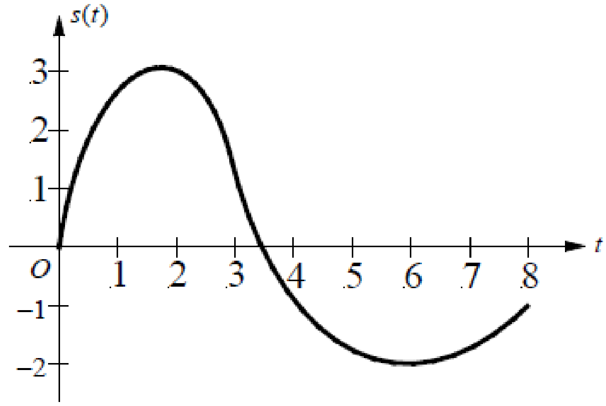
(b) For what values of t is the particle moving downward?

(c) What is the average velocity of the particle on the closed interval $[0,3]$?

(d) What is the average acceleration of the particle on the closed interval $[0,3]$?

(e) Find the position of the particle at time $t=3$.

(f) Find the total distance traveled by the particle from $t=0$ to $t=3$.



12. A particle is moves along a horizontal line. The graph of the particle's position $s(t)$ at time t is shown above for $0 < t < 8$. The graph has horizontal tangents at $t = 2$ and $t = 6$ and has a point of inflection at $t = 3$.

(a) What is the velocity of the particle at time $t = 6$?

(b) The slope of tangent to the graph(not shown) at $t = 4$ is -1. What is the speed of the particle at time $t = 4$?

(c) For what values of t is the particle moving to the left?

(d) For what values of t is the velocity of the particle decreasing?

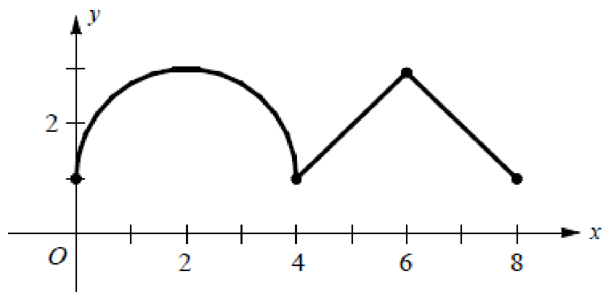
(e) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(f) During what time intervals, if any, is the acceleration of the particle positive? Justify your answer.

Average Value of a Function

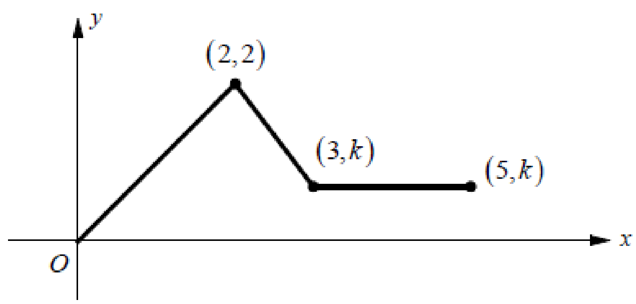
1. What is the average value of $f(x) = \sqrt{x}(4 - x)$ on the closed interval $[0, 4]$?

(A) $\frac{7}{3}$ (B) $\frac{21}{5}$ (C) $\frac{32}{15}$ (D) $\frac{35}{4}$



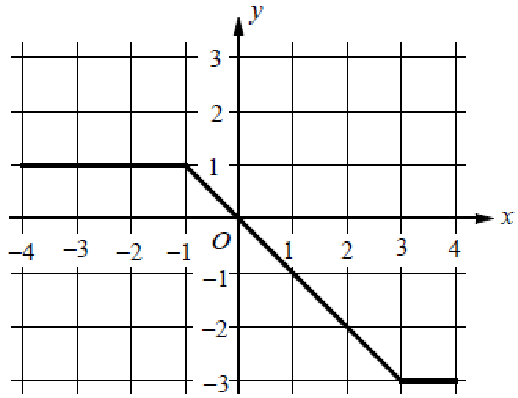
2. The graph of $y = f(x)$ consists of a semicircle and two line segments. What is the average value of f on the interval $[0,8]$?

(A) $\dfrac{\pi+2}{4}$ (B) $\dfrac{\pi+3}{4}$ (C) $\pi+1$ (D) $\dfrac{\pi+6}{4}$



3. The graph of $y = f(x)$ consists of three line segments as shown above. If the average value of f on the interval $[0,5]$ is 1 what is the value of k?

(A) $\dfrac{3}{5}$ (B) $\dfrac{7}{10}$ (C) $\dfrac{4}{5}$ (D) $\dfrac{9}{10}$



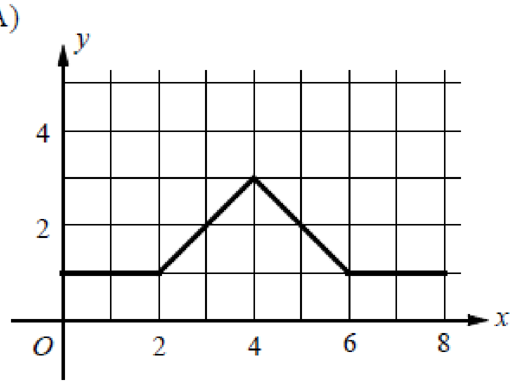
Graph of f

4. The function f is continuous for $-4 \le x \le 4$. The graph of f shown above consists of three line segments. What is the average value of f on the interval $-4 \le x \le 4$?

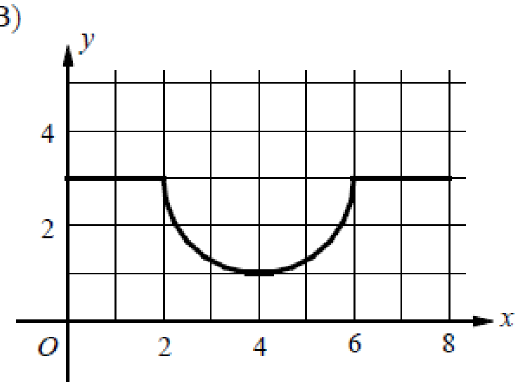
(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1

5. On the closed interval $[0, 8]$, which of the following could be the graph of a function f with the property that $\frac{1}{8-0}\int_0^8 f(t)\,dt > 2$?

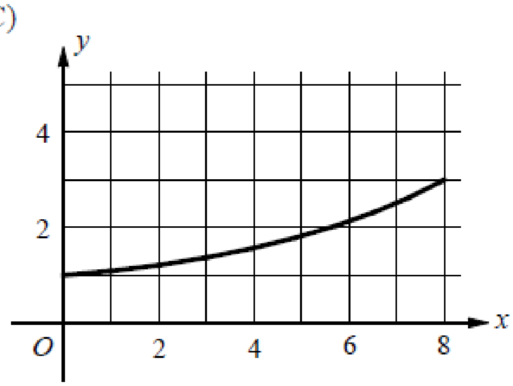
(A)



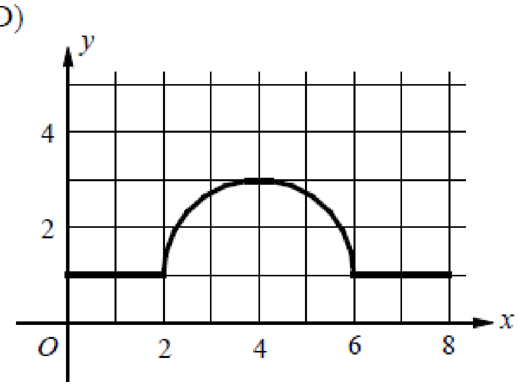
(B)



(C)



(D)



6. Let f be the function defined by

$$f(x)=\begin{cases}\frac{1}{16}x^2+1 & \text{for } 0\le x\le 4\\ 3\sqrt{x}-x & \text{for } 4<x\le 9.\end{cases}$$

What is the average value of f on the closed interval $0\le x\le 9$?

(A) $\frac{65}{54}$ (B) $\frac{35}{27}$ (C) $\frac{85}{27}$ (D) $\frac{55}{9}$

Free Response Questions

7. Let f be the function given by $f(x)=x\cos(x^2)$.

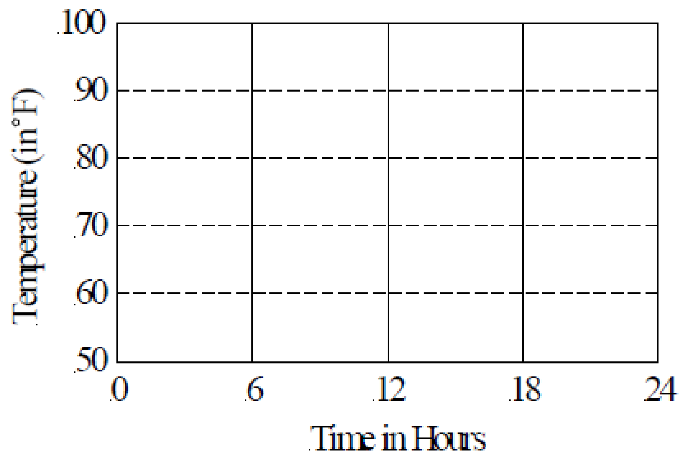
(a) Find the average rate of change of f on the closed interval $\left[0,\sqrt{\pi}\right]$.

(b) Find the average value of f on the closed interval $\left[0,\sqrt{\pi}\right]$.

(c) Find the average value of f' on the closed interval $\left[0,\sqrt{\pi}\right]$.

8. The temperature outside a house during a 24-hour period is given by $F(t) = 75 + 15\sin\left[\frac{\pi(t-6)}{12}\right]$, for $0 \le t \le 24$, where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

(a) Sketch the graph of F on the grid below.



(b) Find the average temperature, to the nearest degree, between $t = 4$ and $t = 10$.

(c) An air conditioner cooled the house whenever the outside temperature was 80 degrees or above. For what values of t was the air conditioner cooling the house?

(d) The hourly cost of cooling the house is \$0.12 for each degree the outside temperature exceeds 80 degrees. What is the total cost, to the nearest cent, to cool the house for the 24 hour period?

Arc Length

1. What is the length of the curve of $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x = 1$ to $x = 2$?

(A) $\frac{8}{3}$ (B) $\frac{10}{3}$ (C) 4 (D) $\frac{14}{3}$

2. Which of the following integrals gives the length of the graph of $y = \ln(\sin x)$ between $x = \frac{\pi}{3}$ to $x = \frac{2\pi}{3}$?

(A) $\int_{\pi/3}^{2\pi/3} \csc^2 x \, dx$

(B) $\int_{\pi/3}^{2\pi/3} \sqrt{1+\cot x} \, dx$

(C) $\int_{\pi/3}^{2\pi/3} \csc x \, dx$

(D) $\int_{\pi/3}^{2\pi/3} \sqrt{1+\csc^2 x} \, dx$

3. Which of the following integrals gives the length of the graph of $y = \frac{1}{3}x^{3/2} - x^{1/2}$ between $x = 1$ to $x = 4$?

(A) $\frac{1}{2}\int_1^4 (\sqrt{x} + \frac{1}{\sqrt{x}}) \, dx$

(B) $\frac{1}{2}\int_1^4 (\sqrt{x} - \frac{1}{\sqrt{x}}) \, dx$

(C) $\frac{1}{2}\int_1^4 (1+\sqrt{x} + \frac{1}{\sqrt{x}}) \, dx$

(D) $\frac{1}{2}\int_1^4 (1+\sqrt{x} - \frac{1}{\sqrt{x}}) \, dx$

4. What is the length of the curve of $y = \ln(x^2+1) - x$ from $x = 0$ to $x = 3$?

(A) 1.026 (B) 1.826 (C) 2.227 (D) 3.135

5. If the length of a curve from $(0,-3)$ to $(3,3)$ is given by $\int_0^3 \sqrt{1+(x^2-1)^2}\,dx$, which of the following could be an equation for this curve?

(A) $y=\frac{x^3}{3}-\frac{x}{3}-3$

(B) $y=\frac{x^3}{3}-3x-3$

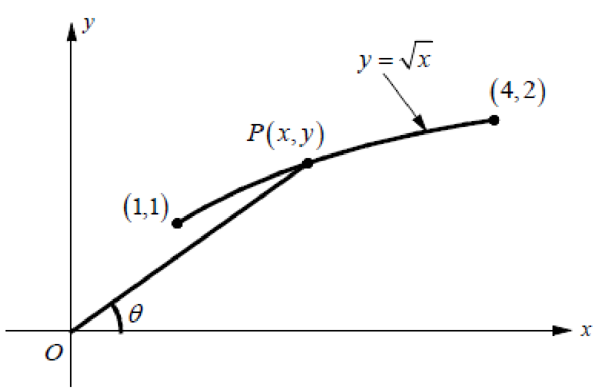
(C) $y=\frac{x^3}{3}-x-3$

(D) $y=\frac{x^3}{3}+x-3$

6. If $F(x)=\int_1^{x^2}\sqrt{t+1}\,dt$, what is the length of the curve of from $x=1$ to $x=2$?

(A) $\frac{8}{3}$ (B) $\frac{10}{3}$ (C) $\frac{15}{3}$ (D) $\frac{17}{3}$

Free Response Questions



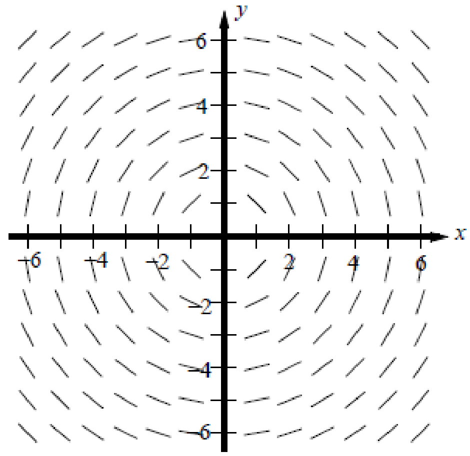
7. The figure above shows a point, $P(x,y)$, moving on the curve of $y=\sqrt{x}$, from the point $(1,1)$ to the point $(4,2)$. Let θ be the angle between $\overline{OP}$ and the positive x-axis.

(a) Find the x- and y-coordinates of point P in terms of $\cot\theta$.

(b) Find the length of the curve from the point $(1,1)$ to the point $(4,2)$.

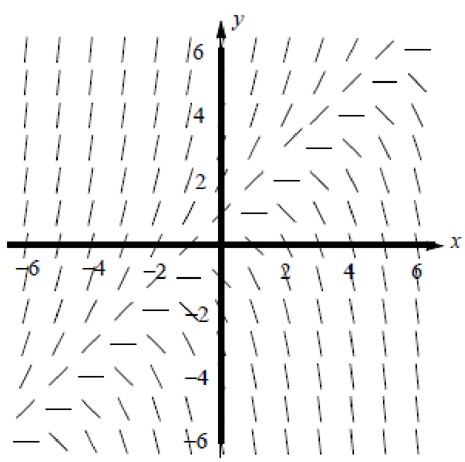
(c) If the angle θ is changing at the rate of -0.1 radian per minute, how fast is the point P moving along the curve at the instant it is at the point $(3,\sqrt{3})$?

Slope Fields



1. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = -\frac{x}{y}$ (C) $\frac{dy}{dx} = \frac{x^2}{y}$ (D) $\frac{dy}{dx} = -\frac{x^2}{y}$

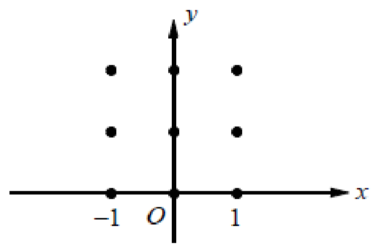


2. Shown above is a slope field for which of the following differential equations?

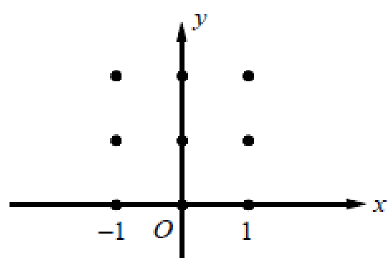
(A) $\frac{dy}{dx} = x + y$ (B) $\frac{dy}{dx} = x - y$ (C) $\frac{dy}{dx} = -x + y$ (D) $\frac{dy}{dx} = x^2 - y$

Free Response Questions

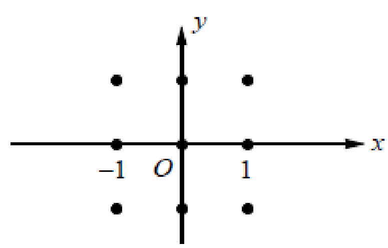
3. On the axis provided, sketch a slope field for the differential equation $\frac{dy}{dx} = y - x^2$.



4. On the axis provided, sketch a slope field for the differential equation $\frac{dy}{dx} = x^2 + y^2$.



5. On the axis provided, sketch a slope field for the differential equation $\frac{dy}{dx} = (x+1)(y-2)$.



Differentials Equations

1. The solution to the differential equation $\frac{dy}{dx} = \frac{3x^2}{2y}$, where $y(3) = 4$, is

(A) $y = \sqrt{\frac{x^3}{3} + 1}$ (B) $y = 7 - \sqrt{\frac{x^3}{3}}$ (C) $y = \sqrt{x^3 - 9}$ (D) $y = \sqrt{x^3 - 11}$

2. If $\frac{dy}{dx} = \frac{x + \sec^2 x}{y}$ and $y(0) = 2$, then $y =$

(A) $\sqrt{x^2 + 2\sec x + 2}$

(B) $\sqrt{x^2 + 2\tan x + 4}$

(C) $\sqrt{x^2 + \sec^2 x + 2}$

(D) $\sqrt{x^2 + \tan^2 x + 4}$

3. At each point (x, y) on a certain curve, the slope of the curve is xy. If the curve contains the point $(0, -1)$, which of the following is the equation for the curve?

(A) $y = x^2 - 2$ (B) $y = 3x^2 - 4$ (C) $y = -e^{\frac{x^2}{2}}$ (D) $y = -e^{(x^2 - 1)}$

4. If $\frac{dy}{dx} = (y - 4)\sec^2 x$ and $y(0) = 5$, then $y =$

(A) $e^{\tan x} + 4$ (B) $6e^{\tan x} - 1$ (C) $2e^{\tan x} + 2$ (D) $4\sec x + 1$

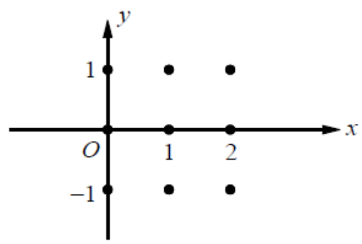
5. What is the value of $m + b$, if $y = mx + b$ is a solution to the differential equation $\frac{dy}{dx} = \frac{1}{4}x - y + 1$?

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) 1 (D) $\frac{5}{4}$

Free Response Questions

6. Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axis provided sketch a slope field for the given differential equation at the nine points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $y(1) = \sqrt{3}$. Write an equation for the line tangent to the graph of f at $(1, \sqrt{3})$ and use it to approximate $f(1.2)$.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $y(1) = \sqrt{3}$.

(d) Use your solution from part (c) to find $f(1.2)$.

7. Consider the differential equation $\frac{dy}{dx} = \frac{2x+3}{e^y}$.

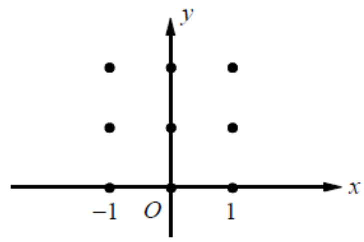
(a) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $y(0) = 2$. Write an equation for the line tangent to the graph of f at $(0, 2)$.

(b) Find $f''(0)$ with the initial condition $y(0) = 2$.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = \frac{2x+3}{e^y}$ with the initial condition $y(0) = 2$.

8. Consider the differential equation $\frac{dy}{dx} = \frac{y^2(1-2x)}{3}$.

(a) On the axis provided sketch a slope field for the given differential equation at the nine points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

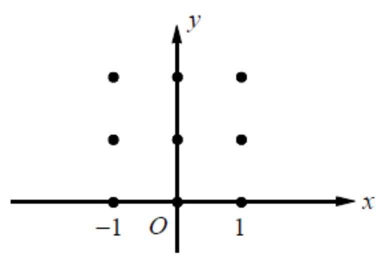
(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $y(\frac{1}{2}) = 4$.

Does f have a relative minimum, a relative maximum, or neither at $x = \frac{1}{2}$? Justify your answer.

(d) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $y(\frac{1}{2}) = 4$.

9. Consider the differential equation $\frac{dy}{dx} = -2x + y + 1$.

(a) On the axis provided sketch a slope field for the given differential equation at the nine points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all the solution curves to the differential equation are concave down.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = -1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the value of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

Exponential Growth and Decay

1. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles every four hours, in how many hours will the number of bacteria triple?

(A) $\ln(\frac{27}{2})$ (B) $\ln(\frac{81}{2})$ (C) $\frac{4\ln 2}{\ln 3}$ (D) $\frac{4\ln 3}{\ln 2}$

2. Population y grows according to the equation $\frac{dy}{dt}=ky$, where k is a constant and t is measured in years. If the population doubles every 15 years what is the value of k?

(A) 0.035 (B) 0.046 (C) 0 .069 (D) 0.078

3. A baby weighs 6 pounds at birth and 9 pounds three months later. If the weight of baby increasing at a rate proportional to its weight, then how much will the baby weigh when she is 6 months old?

(A) 11.9 (B) 12.8 (C) 13.5 (D) 14.6

4. Temperature F changes according to the differential equation $\frac{dF}{dt}=kF$, where k is a constant and t is measured in minutes. If at time $t=0$, $F=180$ and at time $t=16$, $F=120$, what is the value of k?

(A) -0.025 (B) -0.032 (C) -0.045 (D) -0.058

Free Response Questions

5. The rate at which the amount of coffee in a coffeepot changes with time is given by the differential equation $\frac{dV}{dt}=kV$, where V is the amount of coffee left in the coffeepot at any time t seconds. At time $t=0$ there were 16 ounces of coffee in the coffeepot and at time $t=80$ there were 8 ounces of coffee remaining in the pot.

(a) Write an equation for V, the amount of coffee remaining in the pot at any time t.

(b) At what rate is the amount of coffee in the pot decreasing when there are 4 ounces of coffee remaining?

(c) At what time t will the pot have 2 ounces of coffee remaining?

Logistic Equations

1. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = 3P - 0.0006P^2$, where the initial population is $P(0) = 1000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 1000 (B) 2000 (C) 3000 (D) 5000

2. A healthy population $P(t)$ of animals satisfies the logistic differential equation $\frac{dP}{dt} = 5P(1 - \frac{P}{240})$, where the initial population is $P(0) = 150$ and t is the time in years. For what value of P is the population growing the fastest?

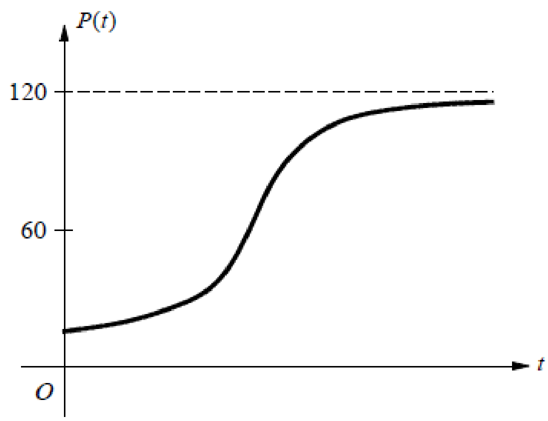
(A) 48 (B) 60 (C) 120 (D) 240

3. A population is modeled by a function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{150}\right)$, where the initial population is $P(0) = 800$ and t is the time in years. What is the slope of the graph of P at the point of inflection?

(A) 5 (B) 7.5 (C) 10 (D) 12.5

4. A certain rumor spreads in a small town at the rate $\frac{dy}{dt} = y(1 - 3y)$, where y is the fraction of the population that has heard the rumor at any time t. What fraction of the population has heard the rumor when it is spreading the fastest?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$



5. Which of the following differential equations for population P could model the logistic growth shown in the figure above

(A) $\frac{dP}{dt} = 0.03P^2 - 0.0005P$

(B) $\frac{dP}{dt} = 0.03P^2 - 0.000125P$

(C) $\frac{dP}{dt} = 0.03P - 0.001P^2$

(D) $\frac{dP}{dt} = 0.03P - 0.00025P^2$

Free Response Questions

6. Let f be a function with $f(2) = 1$, such that all points (t, y) on the graph of f satisfy the differential equation $\frac{dy}{dt} = 2y\left(1 - \frac{t}{4}\right)$.

Let g be a function with $g(2) = 2$, such that all points (t, y) on the graph of g satisfy the logistic differential equation $\frac{dy}{dt} = y\left(1 - \frac{y}{5}\right)$.

(a) Find $y = f(t)$.

(b) For the function found in part (a), what is $\lim_{t\to\infty} f(t)$?

(c) Given that $g(2) = 2$, find $\lim_{t\to\infty} g(t)$ and $\lim_{t\to\infty} g'(t)$.

(d) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection.

Euler's Method

1. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = 1 + 2x - y$ with the initial condition $f(1) = 2$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 1$ with a step size of 0.5?

(A) 2.5 (B) 2.75 (C) 3.25 (D) 3.75

2. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - xy$ with the initial condition $f(0.5) = 0$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 0.5$ with a step size of 0.5?

(A) 0.825 (B) 0.906 (C) 1.064 (D) 1.178

3. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = \arctan(xy)$ with the initial condition $f(0) = 1$. What is the approximation for $f(2)$ if Euler's method is used, starting at $x = 0$ with a step size of 1?

(A) $\frac{\pi}{2}$ (B) $1 + \frac{\pi}{4}$ (C) $1 + \frac{\pi}{2}$ (D) π

x	-1	-0.6	-0.2	0.2	0.6
$f'(x)$	1	2	-0.5	-1.5	1.2

4. The table above gives selected values for the derivative of a function f on the interval $-1 \le x \le 0.6$. If $f(-1) = 1.5$ and Euler's method is used to approximate $f(0.6)$ with step size of 0.8, what is the resulting approximation?

(A) 1.9 (B) 2.1 (C) 2.3 (D) 2.5

$x_0 = 0$	$f(x_0) = 1$
$x_1 = 0.5$	$f(x_1) \approx 1.5$
$x_2 = 1$	$f(x_2) \approx 3$

5. Consider the differential equation $\frac{dy}{dx} = kx + y - 2x^2$, where k is a constant. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. Euler's method, starting at $x = 0$ with step size of 0.5, is used to approximate $f(1)$. Steps from this approximation are shown in the table above. What is the value of k?

(A) 2.5 (B) 3 (C) 3.5 (D) 4

Free Response Questions

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x - y - \frac{1}{2}$.

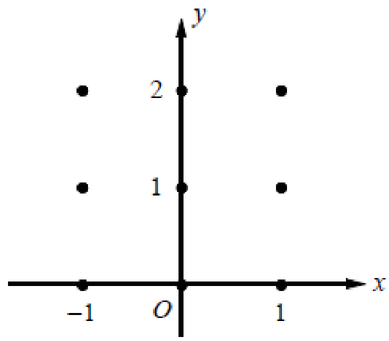
(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Let $y = f(x)$ be the particular solution to the given differential equation whose graph passes through the point $(0, -\frac{1}{2})$. Does the graph of f have relative minimum, a relative maximum, or neither at the point $(0, -\frac{1}{2})$? Justify your answer.

(c) Let $y = g(x)$ be another solution to the given differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 0.5, gives the approximation $g(1) \approx 1$. Find the value of k.

7. Consider the differential equation $\frac{dy}{dx} = 2x + y$.

(a) On the axis provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(1,1)$.



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(1) = 1$. Use Euler's method, starting at $x = 1$ with a step size of 0.1, to approximate $f(1.2)$. Show the work that leads to your answer.

(c) Find the value of b for which $y = -2x + b$ is a solution to the given differential equation. Show the work that leads to your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(1) = -2$. Does the graph of g have a local extremum at the point $(1, -2)$? If so, is the point a local maximum or a local minimum? Justify your answer.