AP Calculus

**Applications of Derivatives**

Related Rates

1. The radius of a circle is changing at the rate of $1/\pi$ inches per second. At what rate, in square inches per second, is the circle's area changing when $r = 5$ in ?

(A) $\dfrac{5}{\pi}$ (B) 10   (C) $\dfrac{10}{\pi}$ (D) 15

2. The volume of a cube is increasing at the rate of 12 in³/min . How fast is the surface area increasing, in square inches per minute, when the length of an edge is 20 in?

(A) 1   (B) $\dfrac{6}{5}$ (C) $\dfrac{4}{3}$ (D) $\dfrac{12}{5}$
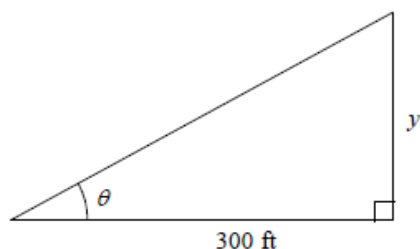
300 ft

3. In the figure shown above, a hot air balloon rising straight up from the ground is tracked by a television camera 300 ft from the liftoff point. At the moment the camera's elevation angle is $\pi/6$, the balloon is rising at the rate of 80 ft/min . At what rate is the angle of elevation changing at that moment?

(A) 0.12 radian per minute

(B) 0.16 radian per minute

(C) 0.2 radian per minute

(D) 0.4 radian per minute

4. A car is approaching a right-angled intersection from the north at 70 mph and a truck is traveling to the east at 60 mph. When the car is 1.5 miles north of the intersection and the truck is 2 miles to the east, at what rate, in miles per hour, is the distance between the car and truck is changing?

(A) Decreasing 15 miles per hour

(B) Decreasing 9 miles per hour

(C) Increasing 6 miles per hour

(D) Increasing 12 miles per hour

5. The radius r of a sphere is increasing at a constant rate. At the time when the surface area and the radius of sphere are increasing at the same numerical rate, what is the radius of the sphere?

   (The surface area of a sphere is $S = 4\pi r^2$.)

   (A) $\dfrac{1}{8\pi}$ (B) $\dfrac{1}{4\pi}$ (C) $\dfrac{1}{3\pi}$ (D) $\dfrac{\pi}{8}$

6. If the radius r of a cone is decreasing at a rate of 2 centimeters per minute while its height h is increasing at a rate of 4 centimeters per minute, which of the following must be true about the volume V of the cone?
   $(V = \dfrac{1}{3}\pi r^2 h)$

   (A) V is always decreasing.

   (B) V is always increasing.

   (C) V is increasing only when $r > h$.

   (D) V is increasing only when $r < h$.

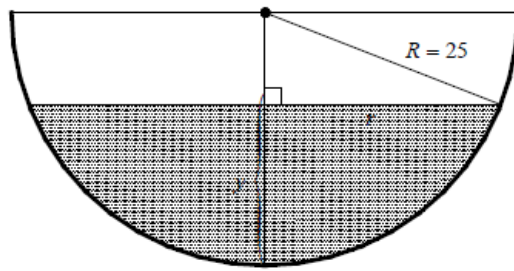## Free Response Questions
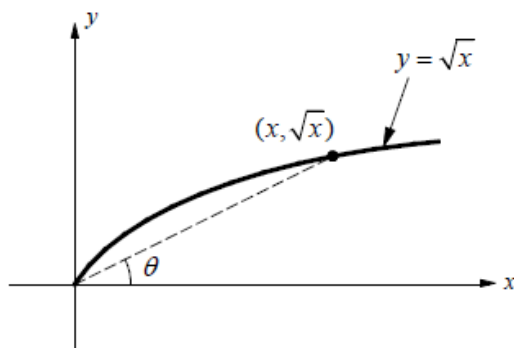
7. A particle moves along the curve $y = \sqrt{x}$. When $y = 2$ the x-component of its position is increasing at the rate of 4 units per second.

   (a) What is the value of $\dfrac{dy}{dt}$ when $y = 2$?

   (b) How fast is the distance from the particle to the origin changing when $y = 2$?

   (c) What is the value of $\dfrac{d\theta}{dt}$ when $y = 2$?

8. As shown in the figure above, water is draining at the rate of 12 ft^3/min from a hemispherical bowl of radius 25 feet. The volume of water in a hemispherical bowl of radius R when the depth of the water is y meters is given as $V = \dfrac{\pi}{3} y^2 (3R - y)$.

   (a) Find the rate at which the depth of water is decreasing when the water is 18 meters deep. Indicate units of measure.

   (b) Find the radius r of the water's surface when the water is y feet deep.

   (c) At what rate is the radius r changing when the water is 18 meters deep. Indicate units of measure.
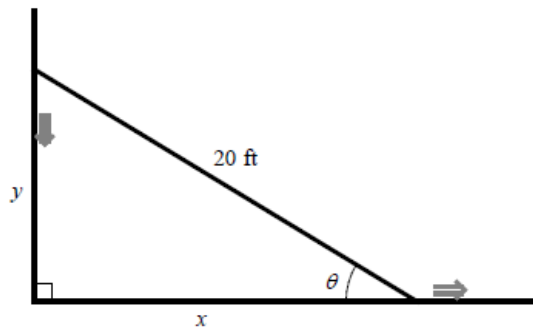
9. In the figure shown above, the top of a 20-foot ladder is sliding down a vertical wall at a constant rate of 2 feet per second.

   (a) When the top of the ladder is 12 feet from the ground, how fast is the bottom of the ladder moving away from the wall?

   (b) The triangle is formed by the wall, the ladder and the ground. At what rate is the area of the triangle is changing when the top of the ladder is 12 feet from the ground?

   (c) At what rate is the angle θ between the ladder and the ground is changing when the top of the ladder is 12 feet from the ground?

10. Consider the curve given by $2y^2 + 3xy = 1$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has a slope of $-\dfrac{3}{4}$.

(c) Let x and y be functions of time t that are related by the equation $2y^2 + 3xy = 1$. At time $t = 3$, the value of y is 2 and $\dfrac{dy}{dt} = -2$. Find the value of $\dfrac{dx}{dt}$ at time $t = 3$.

Particle Motion

1. A particle moves along the x-axis so that at any time $t \geq 0$, its position is given by $x(t) = -\dfrac{1}{2}\cos t - 3t$.

   What is the acceleration of the particle when $t = \dfrac{\pi}{3}$?

   (A) $-\dfrac{\sqrt{3}}{4}$ (B) $-\dfrac{1}{4}$ (C) $\dfrac{1}{4}$ (D) $\dfrac{\sqrt{3}}{4}$

2. A point moves along the x-axis so that at any time t, its position is given by $x(t) = \sqrt{x}\ln x$. For what values of t is the particle at rest?

   (A) No values (B) $\dfrac{1}{e^2}$ (C) $\dfrac{1}{e}$ (D) e

3. A particle moves along the x-axis so that at any time t, its position is given by $x(t) = 3\sin t + t^2 + 7$. What is velocity of the particle when its acceleration is zero?

   (A) 1.504 (B) 1.847 (C) 2.965 (D) 3.696

4. Two particles start at the origin and move along the x-axis. For $0 \leq t \leq 8$, their respective position functions are given by $x_1(t) = \sin^2 t$ and $x_2(t) = e^{-t}$. For how many values of t do the particles have the same velocity?

   (A) 3 (B) 4 (C) 5 (D) 6

5. A particle moves along a line so that at time t, where $0 \le t \le 5$, its velocity is given by $v(t) = -t^3 + 6t^2 - 15t + 10$. What is the minimum acceleration of the particle on the interval?

(A) −30 (B) −15 (C) −3 (D) 0

6. A particle moves along the x-axis so that at any time $t \ge 0$, its velocity is given by $v(t) = -t^3 e^{-t}$. At what value of t does v attain its minimum?

(A) $3\sqrt[3]{e}$ (B) 3 (C) 0 (D) $\sqrt[3]{e}$

7. The position of a particle moving along a line is given by $s(t) = t^3 - 12t^2 + 21t + 10$ for $t \ge 0$. For what value of t is the speed of the particle increasing?

(A) $1 < t < 7$ only

(B) $4 < t < 7$ only

(C) $0 < t < 1$ and $4 < t < 7$

(D) $1 < t < 4$ and $t > 7$

Free Response Questions

8. A particle moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = (t-2)^3(t-6)$.

(a) Find the velocity and acceleration of the particle at any time $t \ge 0$.

(b) Find the value of t when the particle is moving and the acceleration is zero.

(c) When is the particle moving to the right?

(d) When is the velocity of the particle decreasing?

(e) When is the speed of the particle increasing?

Rolle's and Mean Value Theorem

1. Let f be the function given by $f(x) = \sin(\pi x)$. What are the values of c that satisfy Roll's Theorem on the closed interval $[0, 2]$?

   (A) $\dfrac{1}{4}$ only   (B) $\dfrac{1}{2}$ only   (C) $\dfrac{1}{4}$ and $\dfrac{1}{2}$ (D) $\dfrac{1}{2}$ and $\dfrac{3}{2}$

2. Let f be the function given by $f(x) = -x^3 + 3x + 2$. What are the values of c that satisfy Mean Value Theorem on the closed interval $[0, 3]$?

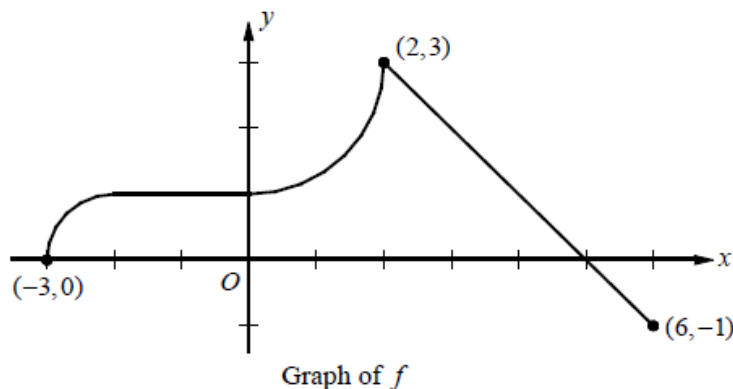   (A) $-\sqrt{3}$ only   (B) $-\sqrt{3}$ and $\sqrt{3}$ (C) $\sqrt{3}$ only   (D) 1.5 and $\sqrt{3}$
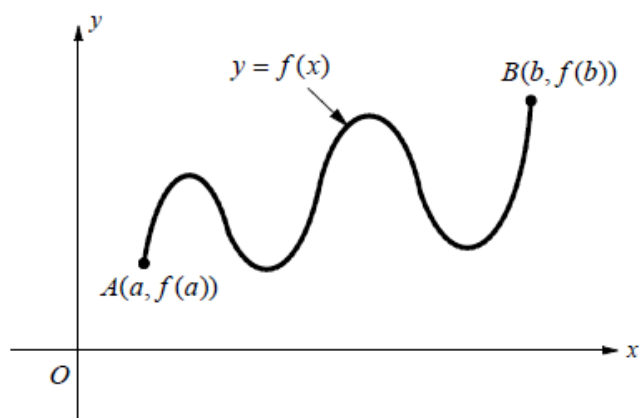
3. The figure above shows the graph of f. On the closed interval $[a, b]$, how many values of c satisfy the conclusion of the Mean Value Theorem?

   (A) 2   (B) 3   (C) 4   (D) 5

4. Let f be the function given by $f(x) = \dfrac{x}{x+2}$. What are the values of c that satisfy the Mean Value Theorem on the closed interval $[-1, 2]$?

   (A) -4 only   (B) 0 only   (C) 0 and $\dfrac{3}{2}$ (D) -4 and 0

Graph of f

5. The continuous function f is defined on the interval $-3 \le x \le 6$. The graph of f consists of two quarter circles and two line segments, as shown in the figure above. Which of the following statements must be true?

I. The average rate of change of f on the interval $-3 \le x \le 6$ is $-\dfrac{1}{9}$.

II. There is a point c on the interval $-3 < x < 6$, for which $f'(c)$ is equal to the average rate of change of f on the interval $-3 \le x \le 6$.

III. If h is the function given by $h(x) = f(\tfrac{1}{2}x)$, then $h'(6) = -\dfrac{1}{2}$.

(A) I and II only

(B) I and III only

(C) II and III only

(D) I, II, and III

Free Response Questions

t (min)	0	5	10	15	20	25	30	35	40	45	50
$v(t)$ (km/min)	1.5	1.8	2.3	2.4	1.8	1.3	0.8	0.3	0	−0.4	−1.2

6. A car drives on a straight road with positive velocity $v(t)$, in kilometers per minute at time t minutes. The table above gives selected values of $v(t)$ for $0 \le t \le 50$. The function $v(t)$ is a twice-differentiable function of t.

(a) For $0 < t < 50$, must there be a time t when $v(t) = -1$? Justify your answer.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the car could equal zero in the open interval $0 < t < 50$? Justify your answer.

First Derivative Test and Extreme Value Theorem

1. At what values of x does $f(x) = (x-1)^3(3-x)$ have the absolute maximum?

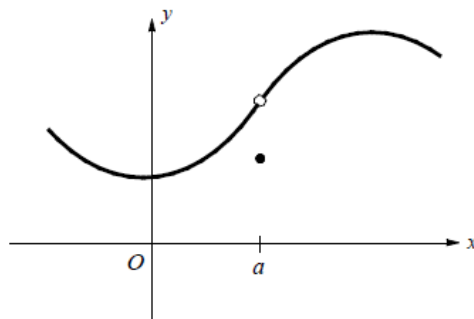(A) 1   (B) $\dfrac{3}{2}$ (C) 2   (D) $\dfrac{5}{2}$

2. At what values of x does $f(x) = x - 2x^{2/3}$ have a relative minimum?

(A) $\dfrac{64}{27}$ (B) $\dfrac{16}{9}$ (C) $\dfrac{4}{3}$ (D) 2

3. What is the minimum value of $f(x) = x^2 \ln x$?

(A) $-e$

(B) $-\dfrac{1}{2e}$

(C) $-\dfrac{1}{e}$

(D) $-\dfrac{1}{\sqrt{e}}$

4. The graph of a function f is shown above. Which of the following statements about f are true?

I. $\lim\limits_{x \to a} f(x)$ exists.

II. $x = a$ is the domain of f.

III. f has a relative minimum at $x = a$.

(A) I only

(B) I and II only

(C) I and III only

(D) I, II, and III

5. A polynomial $f(x)$ has a relative minimum at $(-4, 2)$, a relative maximum at $(-1, 5)$, a relative minimum at $(3, -3)$ and no other critical points. How many zeros does $f(x)$ have?

(A) one   (B) two   (C) three   (D) four

6. At $x = 2$, which of the following is true of the function f defined by $f(x) = x^2 e^{-x}$?

(A) f has a relative maximum.

(B) f has a relative minimum.

(C) f is increasing.

(D) f is decreasing.
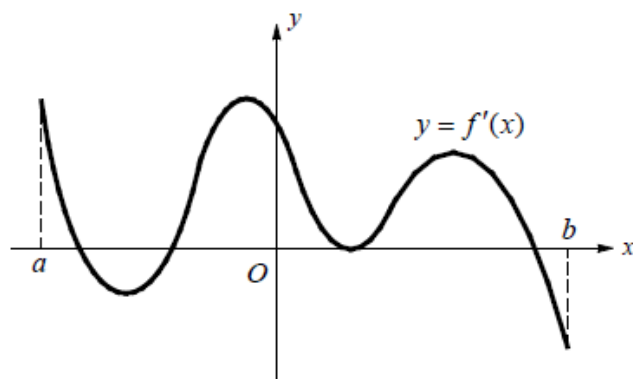
7. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima

(B) Two relative maxima and one relative minimum

(C) Two relative maxima and two relative minima

(D) Three relative maxima and two relative minim

8. The first derivative of a function f is given by $f'(x) = \dfrac{3\sin(2x)}{x^2}$. How many critical values does f have on the open interval $(0,10)$?

(A) four   (B) five   (C) six   (D) seven

9. The function f is continuous on the closed interval $[-1,5]$ and differentiable on the open interval $(-1,5)$. If $f(-1)=4$ and $f(5)=-2$, which of the following statements could be false?

(A) There exist c, on $[-1,5]$, such that $f(c) \le f(x)$ for all x on the closed interval $[-1,5]$.

(B) There exist c, on $(-1,5)$, such that $f(c)=0$.

(C) There exist c, on $(-1,5)$, such that $f'(c)=0$.

(D) There exist c, on $(-1,5)$, such that $f(c)=2$.

x	−4	−3	−2	−1	0	1	2	3	4	5
$f'(x)$	−1	−2	0	1	2	1	0	−2	−3	−1

10. The derivative, f', of a function f is continuous and has exactly two zeros on $[-4,5]$. Selected values of $f'(x)$ are given in the table above. On which of the following intervals is f increasing?

(A) $-3 \le x \le 0$ or $4 \le x \le 5$

(B) $-2 \le x \le 0$ or $4 \le x \le 5$

(C) $-3 \le x \le 2$ only

(D) $-2 \le x \le 2$ only

11. The height h, in meters, of an object at time t is given by $h(t) = t^3 - 6t^2 + 20t$. What is the height of the object, in meters, at the instant it reaches its ~~maximum~~ upward velocity?

minimum

(A) 24   (B) 28   (C) 33   (D) 42

12. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = x^2 + c$, where c is a constant?
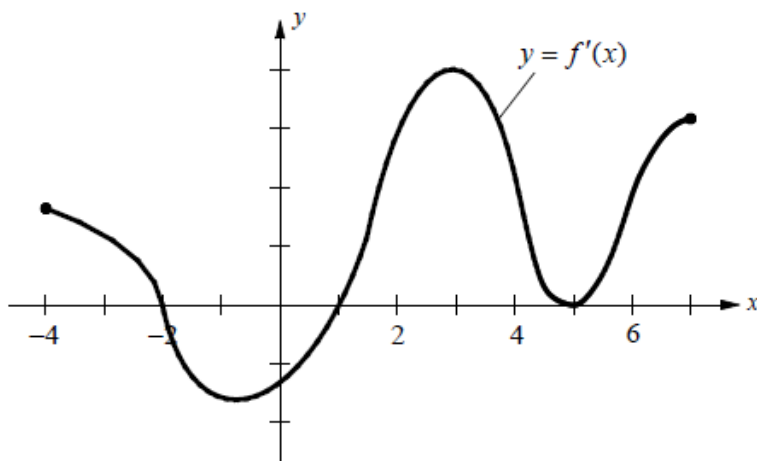
(A) $y = -\dfrac{1}{x}$ (B) $y = -x^2$ (C) $y = \dfrac{1}{\ln x}$ (D) $y = -\dfrac{1}{2}\ln x$

Free Response Questions

13. The figure above shows the graph of f', the derivative of the function f, for $-4 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -1$, $x = 3$, and $x = 5$.

(a) Find all values of x, for $-4 \le x \le 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-4 \le x \le 7$, at which f attains a relative maximum. Justify your answer.

(c) At what value of x, for $-4 \le x \le 7$, does f attain its absolute maximum. Justify your answer.

Second Derivative Test

1. The graph of $y = x^4 - 2x^3$ has a point of inflection at

   (A) $(0,0)$ only

   (B) $(0,0)$ and $(1,-1)$

   (C) $(1,-1)$ only

   (D) $(0,0)$ and $(\frac{3}{2}, -\frac{27}{16})$

2. If the graph of $y = ax^3 - 6x^2 + bx - 4$ has a point of inflection at $(2,-2)$, what is the value of $a+b$?

   (A) –2   (B) 3   (C) 6   (D) 10

3. At what value of x does the graph of $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}}$ have a point of inflection?

   (A) $\dfrac{1}{2}$ (B) 1   (C) 3   (D) $\dfrac{7}{2}$

4. The graph of $y = 3x^5 - 40x^3 - 21x$ is concave up for

   (A) $x < 0$

   (B) $x > 2$

   (C) $x < 0$ or $0 < x < 2$

   (D) $-2 < x < 0$ or $x > 2$

5. Let f be a twice differentiable function such that $f(1) = 7$ and $f(3) = 12$. If $f'(x) > 0$ and $f''(x) < 0$ for all real numbers x, which of the following is a possible value for $f(5)$?

(A) 16   (B) 17   (C) 18   (D) 19
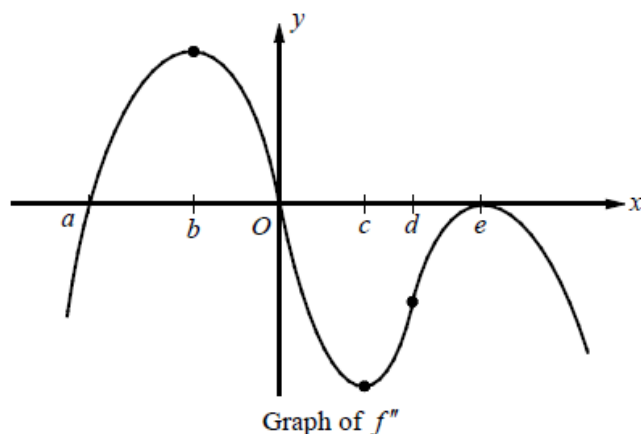
Graph of f''

6. The second derivative of the function f is given by $f''(x) = x(x+a)(x-e)^2$ and the graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

(A) b and c (B) b, c and e (C) b, c and d (D) a and 0

7. The first derivative of the function f is given by $f'(x) = (x^3 + 2) e^x$. What is the x-coordinate of the inflection point of the graph of f?

(A) -3.196 (B) -1.260 (C) -1 (D) 0

8. Let f be a twice differentiable function with $f'(x) > 0$ and $f''(x) > 0$ for all x, in the closed interval $[2,8]$. Which of the following could be a table of values for f?

(A)

x	$f(x)$
2	-1
4	3
6	6
8	8

(B)

x	$f(x)$
2	-1
4	2
6	5
8	8

(C)

x	$f(x)$
2	-1
4	1
6	4
8	8

(D)

x	$f(x)$
2	8
4	4
6	1
8	-1

9. Let f be the function given by $f(x) = 3\sin(\frac{2x}{3}) - 4\cos(\frac{3x}{4})$. For $0 \le x \le 7$, f is increasing most rapidly when $x =$

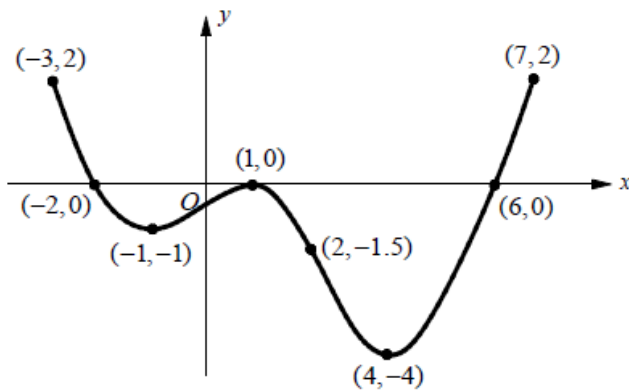(A) 0.823   (B) 1.424   (C) 1.571   (D) 3.206
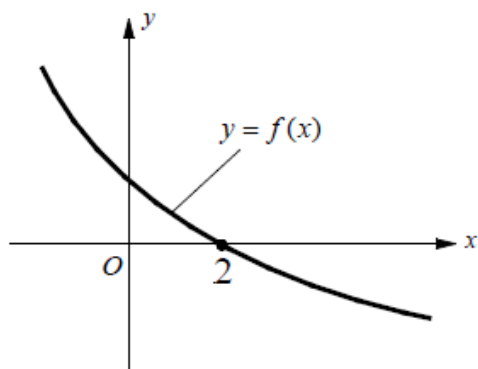
10. The graph of a twice differentiable function f is shown in the figure above. Which of the following is true?

(A) $f''(2) < f(2) < f'(2)$

(B) $f'(2) < f''(2) < f(2)$

(C) $f'(2) < f(2) < f''(2)$

(D) $f(2) < f'(2) < f''(2)$
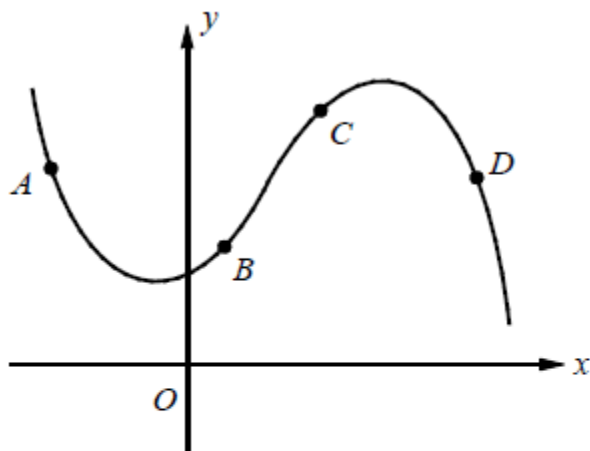
11. At which of the five points on the graph in the figure above is $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} > 0$?

(A) A (B) B (C) C (D) D

Graph of f'

12. The figure above shows the graph of f', the derivative of the function f, on the closed interval $[-3,7]$. The graph of f' has horizontal tangent lines at $x = -1$, $x = 1$, and $x = 4$. The function f is twice differentiable and $f(-2) = \dfrac{1}{2}$.

(a) Find the x-coordinates of each of the points of inflection of the graph of f. Justify your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $[-3,7]$.

(c) Let h be the function defined by $h(x) = x^2 f(x)$. Find an equation for the line tangent to the graph of h at $x = -2$.

13. Let f be a twice differentiable function with $f(1) = -1$, $f'(1) = 2$, and $f''(1) = 0$. Let g be a function whose derivative is given by $g'(x) = x^2 \left[2f(x) + f'(x) \right]$ for all x.

(a) Write an equation for the line tangent to the graph of f at $x = 1$.

(b) Does the graph of f have a point of inflection when $x = 1$? Explain.

(c) Given that $g(1) = 3$, write an equation for the line tangent to the graph of g at $x = 1$.

(d) Show that $g''(x) = 4x f(x) + 2x(x+1)f'(x) + x^2 f''(x)$. Does g have a local maximum or minimum at $x = 1$? Explain your reasoning.

Graphical Analysis

1. If f is a function such that $f' > 0$ for $a < x < c$, $f'' < 0$ for $a < x < b$, and $f'' > 0$ for $b < x < c$ which of the following could be the graph of f ?

(A)

(B)

(C)

(D)
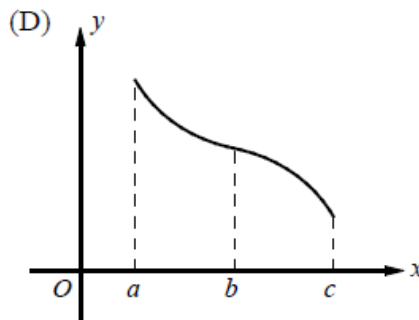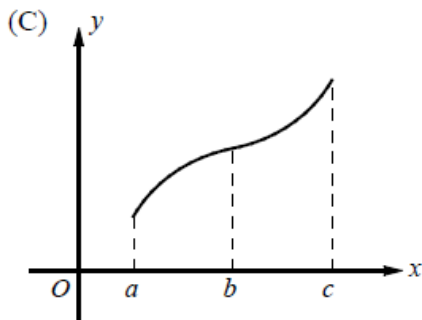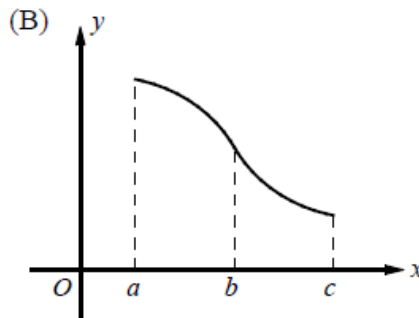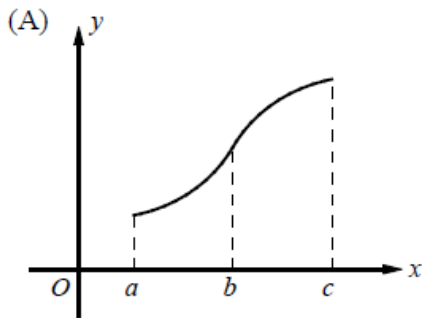
2. The graph of $f(x) = xe^{-x^2}$ is symmetric about which of the following

   I. The x-axis

   II. The y-axis

   III. The origin

(A) I only   (B) II only   (C) III only   (D) II and III only

3. Let f be the function given by $f(x) = \dfrac{-3x^2}{\sqrt{3x^4+1}}$. Which of the following is the equation of horizontal asymptote of the graph of f ?
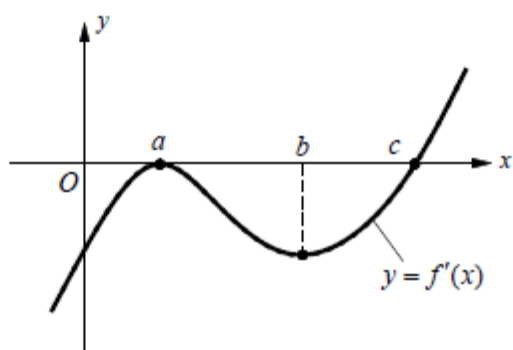
(A) $y = -3$ (B) $y = -\sqrt{3}$ (C) $y = \sqrt{3}$ (D) $y = 3$

4. Let f be a function that is continuous on the closed interval $[a,c]$, such that the derivative of function f has the properties indicated on the table below.

x	$a<x<b$	b	$b<x<0$	0	$0<x<c$
$f'(x)$	$-$	0	$+$	3	$+$
$f''(x)$	$+$	$+$	$+$	0	$-$

Which of the following could be the graph of f ?
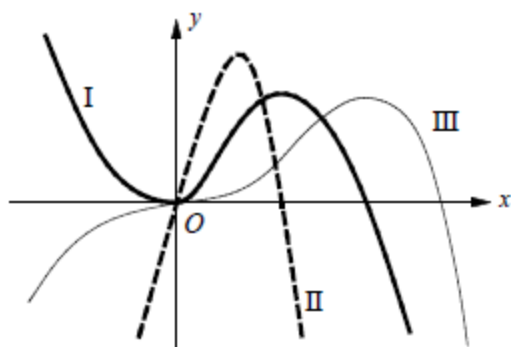
(A)

(B)

(C)

(D)

$$y = f'(x)$$
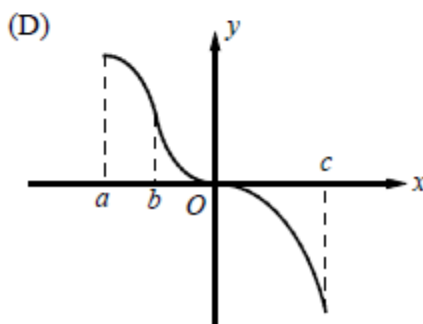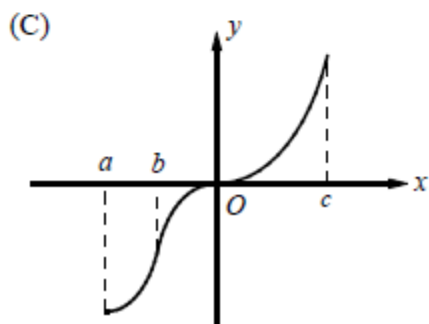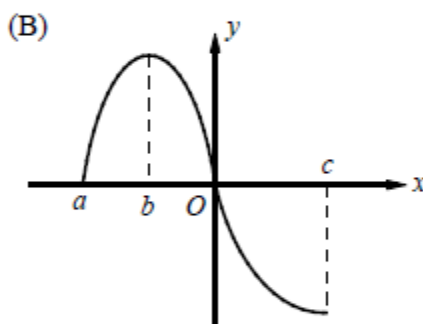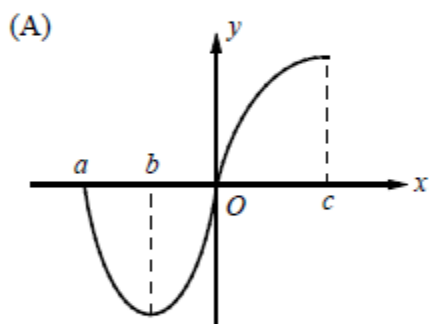
5. The graph of f', the derivative of function f, is shown above. If f is a twice differentiable function, which of the following statements must be true?
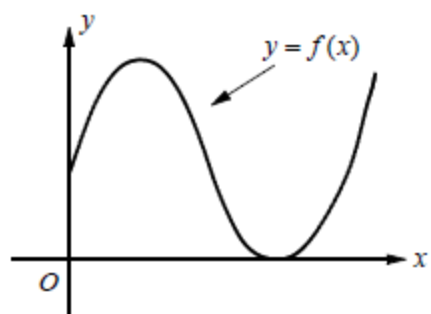
   I. $f(c) > f(a)$

   II. The graph of f is concave up on the interval $b < x < c$.

   III. f has a relative minimum at $x = c$.

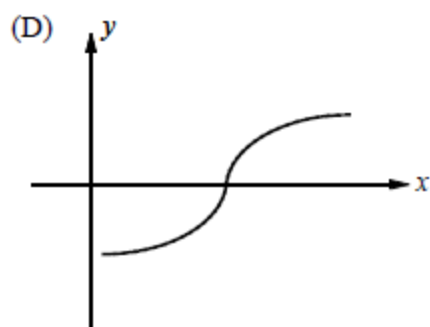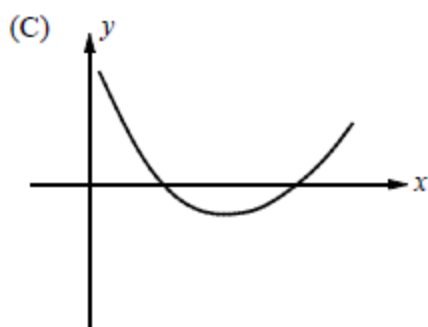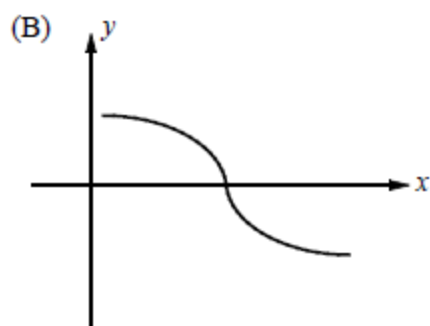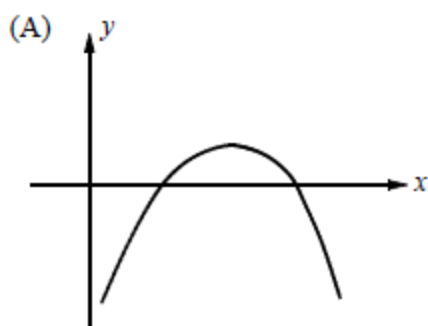(A) I only   (B) II only   (C) III only   (D) II and III only

6. Three graphs labeled I, II, and III are shown above. They are the graphs of f, f', and f''. Which of the following correctly identifies each of the three graphs?
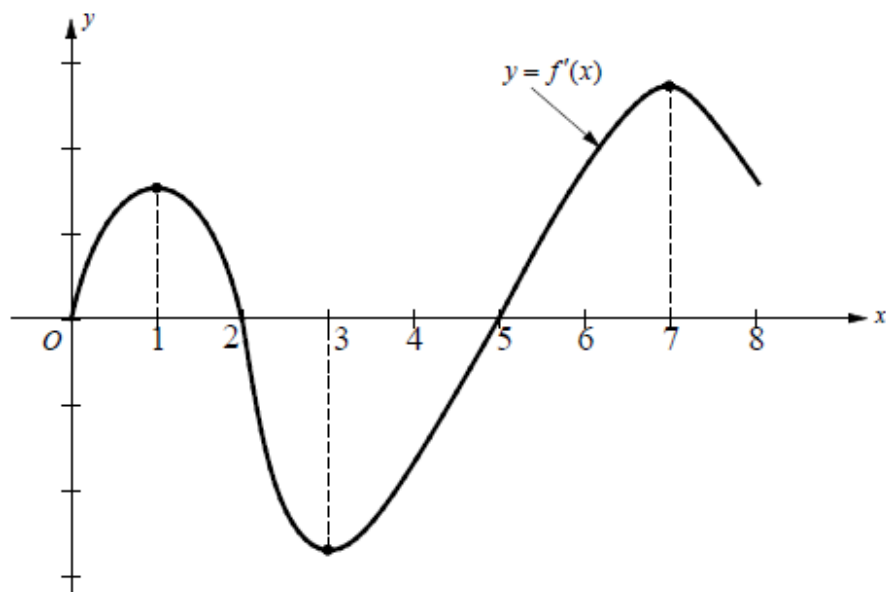
$f \quad f' \quad f''$

(A) I   II   III
(B) II   I   III
(C) III   I   II
(D) I   III   II

7. The graph of f is shown in the figure above. Which of the following could be the graph of f'?

(A)

(B)

(C)

(D)

8. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $0 \le x \le 8$.

    (a) For what values of x does the graph of f have a horizontal tangent?

    (b) On what intervals is f increasing?

    (c) On what intervals is f concave upward?

    (d) For what values of x does the graph of f have a relative maximum?

    (e) Find the x-coordinate of each inflection point on the graph of f.

x	-1	$-1<x<0$	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	+	0	−	−1	−	0	+
$f'(x)$	−4	−	0	−	DNE	+	1	+
$f''(x)$	2	+	0	−	DNE	−	0	+

9. Let f be a function that is continuous on the interval $-1 \le x < 3$. The function is twice differentiable except at $x = 1$. The function f and its derivatives have the properties indicated in the table above.

(a) For $-1 < x < 3$, find all values of x at which f has a relative extrema. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axis provided, sketch the graph of a function that has all the given characteristics of f.
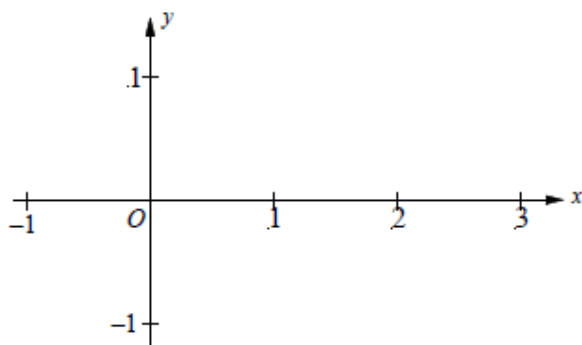
(c) Let h be the function defined by $h'(x) = f(x)$ on the open interval $-1 < x < 3$. For $-1 < x < 3$, find all values of x at which h has a relative extremum. Determine whether h has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function h, find all values of x, for $-1 < x < 3$, at which h has a point of inflection. Justify your answer.

Optimization

1. The point on the curve $y = 2 - x^2$ nearest to $(3,2)$ is

   (A) $(0,2)$ (B) $(\frac{1}{2}, \frac{7}{4})$ (C) $(\frac{3}{4}, \frac{23}{16})$ (D) $(1,1)$
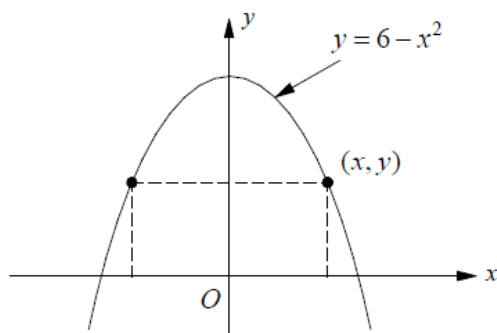
2. What is the area of the largest rectangle that has its base on the x-axis and its other two vertices on the parabola $y = 6 - x^2$?

   (A) $8\sqrt{2}$ (B) $6\sqrt{2}$ (C) $4\sqrt{3}$ (D) $3\sqrt{2}$
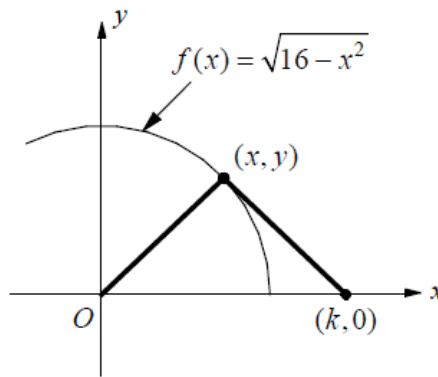
3. If $y = \dfrac{1}{\sqrt{x}} - \sqrt{x}$, what is the maximum value of the product of xy ?

   (A) $\dfrac{1}{9}$ (B) $\dfrac{\sqrt{3}}{9}$ (C) $\dfrac{2\sqrt{3}}{9}$ (D) $\dfrac{2}{3}$
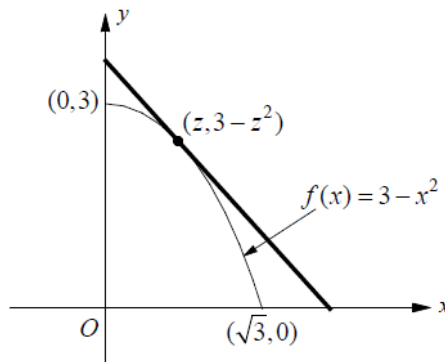
4. If the maximum value of the function $y = \dfrac{\cos x - m}{\sin x}$ is at $x = \dfrac{\pi}{4}$, what the value of m ?

   (A) $-\sqrt{2}$ (B) $\sqrt{2}$ (C) -1 (D) 1

5. Let $f(x) = \sqrt{16 - x^2}$. An isosceles triangle, whose base is the line segment from $(0,0)$ to $(k,0)$, where $k > 0$, has its vertex on the graph of f as shown in the figure above.

(a) Find the area of the triangle in terms of k .

(b) For what values of k does the triangle have a maximum area?

6. The figure above shows the graph of the function $f(x) = 3 - x^2$. For $0 < z < \sqrt{3}$, let $A(z)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(z, 3 - z^2)$.

(a) Find the equation of the line tangent to the graph of f at the point $(z, 3 - z^2)$.

(b) For what values of z does the triangle bounded by the coordinate axis and tangent line have a minimum area?

Tangent Line Approximation

1. For small values of h, the function $h(x) = \sqrt[3]{8+h}$ is best approximated by which of the following?

   (A) $\dfrac{h}{12}$ (B) $2 - \dfrac{h}{12}$ (C) $2 + \dfrac{h}{12}$ (D) $3 + \dfrac{h}{12}$

2. The approximate value of $y = \sqrt{1 - \sin x}$ at $x = -0.1$, obtained from the line tangent to the graph at $x = 0$, is

   (A) 0.9   (B) 0.95   (C) 1.01   (D) 1.05

4. Let f be a differentiable function such that $f(2) = \dfrac{5}{2}$ and $f'(2) = \dfrac{1}{2}$. If the line tangent to the graph of f at $x = 2$ is used to find an approximation of a zero of f, that approximation is

   (A) −3   (B) −2.4   (C) −1.8   (D) −1.2

5. The approximate value of $y = \dfrac{1}{\sqrt{x}}$ at $x = 4.1$, obtained from the line tangent to the graph at $x = 4$ is

   (A) $\dfrac{39}{80}$ (B) $\dfrac{79}{160}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{81}{160}$

Free Response Questions

8. Let f be the function given by $f(x) = \dfrac{2}{e^{\sin x} + 1}$.

   (a) Write an equation for the line tangent to the graph of f at $x = 0$.

   (b) Using the tangent line to the graph of f at $x = 0$, approximate $f(0.1)$.

   (c) Find $f^{-1}(x)$.

L'Hospital's Rule

1. $\lim\limits_{x \to 0} \dfrac{e^x - 1 - x}{x^2} =$

    (A) 0   (B) $\dfrac{1}{2}$ (C) 1   (D) ∞

2. $\lim\limits_{x \to 0} \dfrac{\sin^{-1} x}{x} =$

    (A) $-\infty$ (B) 0   (C) $\dfrac{\pi}{2}$ (D) 1

3. $\lim\limits_{\theta \to \pi} \dfrac{\sin \theta}{\theta - \pi} =$

    (A) -1 (B) $-\dfrac{1}{2}$ (C) 0   (D) $\dfrac{1}{2}$

4. $\lim\limits_{x \to 0^+} (\tan x)^x =$

    (A) $-\infty$ (B) 0   (C) $\dfrac{\pi}{4}$ (D) 1

5. $\lim\limits_{x \to \infty} (x)^{1/x} =$

    (A) $-\infty$ (B) e (C) 1   (D) ∞

6. $\lim\limits_{x \to 0} \left(\dfrac{1}{\tan x} - \dfrac{1}{x} \right) =$

    (A) $-\infty$ (B) $-\dfrac{1}{2}$ (C) 0   (D) ∞

7. $\lim\limits_{x \to 1^-} \left(\dfrac{2}{x^2-1} - \dfrac{x}{x-1} \right) =$

(A) $-\infty$ (B) $-\dfrac{3}{2}$ (C) -1 (D) ∞

Free Response Questions

8. Use L'Hospital's Rule to find the exact value of $\lim\limits_{x \to \infty} x \left[\ln(x+3) - \ln x \right]$. Show the work that leads to your answer.

9. Use L'Hospital's Rule to find the exact value of $\lim\limits_{x \to \infty} \left[x - \sqrt{x^2+x} \right]$. Show the work that leads to your answer.