Polar Coordinates and Polar Graphs

So far we have been working with functions and curve defined in the rectangular coordinate system. We will now study the **polar coordinate system**.

origin

- The polar coordinate system is formed from a fixed point, called the **pole,** and an initial ray called the **polar axis.** Each point in the plane can be assigned **polar coordinates** of the form:

  RADIAL Line

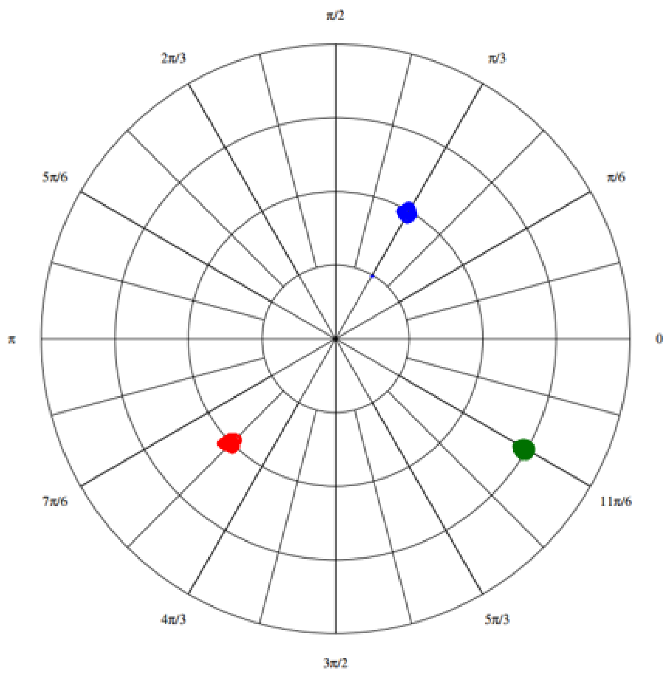
  (r, θ) where **r= directed distance from the pole to the point** and
  θ =directed angle, counterclockwise from the polar axis

Example 1: On the polar grid below, graph the following polar coordinates. $[0, 2\pi]$

A. $(2, \frac{\pi}{3})$ → $(-2, \frac{4\pi}{3})$

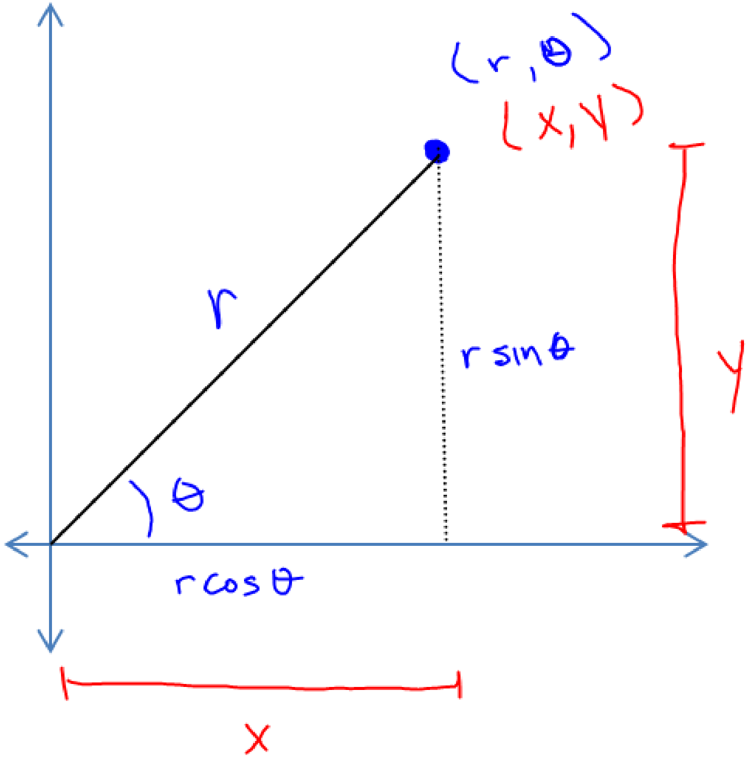
B. $(3, -\frac{\pi}{6})$ → $(-3, \frac{5\pi}{6})$

C. $(-2, \frac{\pi}{4})$ → $(2, \frac{5\pi}{4})$



Coordinate Conversion

Because we operate mainly in the rectangular system, we need to be able to work back and forth between the polar and rectangular coordinate system.



The polar coordinates (r, θ) of a point are related to the rectangular coordinates (x, y) of the point as follows:

1. $x = r\cos\theta$ $\quad y = r\sin\theta$
2. $\tan\theta = \dfrac{y}{x}$ $\quad r^2 = x^2 + y^2$

Example 2: Convert the following points from polar to rectangular form.

a. $(2, \pi)$

$x = 2\cos(\pi)$

$x = -2$

$y = 2\sin(\pi)$

$y = 0$

$(-2, 0)$

b. $(\sqrt{3}, \pi/6)$

$x = \sqrt{3}\cos(\pi/6)$

$x = \frac{3}{2}$

$y = \sqrt{3}\sin(\pi/6)$

$y = \frac{\sqrt{3}}{2}$

$\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$

Example 3: Convert the following points from rectangular to polar.

a. $(\overset{x}{-1}, \overset{y}{1}) \rightarrow$ Quad II

$r^2 = (-1)^2 + (1)^2$

$r^2 = 2$

$r = \pm\sqrt{2}$

$\tan\theta = \frac{1}{-1} = -1$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\left(\sqrt{2}, \frac{3\pi}{4}\right)$ OR $\left(-\sqrt{2}, \frac{7\pi}{4}\right)$

b. $(0, 2) \rightarrow$ Y-AXIS

$r^2 = 0^2 + 2^2$

$r = \pm 2$

$\tan\theta = \frac{2}{0}$ = UNDEFINED

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\left(2, \frac{\pi}{2}\right)$ OR $\left(-2, \frac{3\pi}{2}\right)$

Example 4: Convert the rectangular equation to polar form.

a. $x + y = 2x \longrightarrow y = x$

$y = x$

$r\sin\theta = r\cos\theta$

$r\sin\theta - r\cos\theta = 0$

$r(\sin\theta - \cos\theta) = 0$

$r = 0$ → Point on origin

$\sin\theta - \cos\theta = 0$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

* An equation $\theta = \#$ is the graph of line passing the origin

b. $\underbrace{x^2 + y^2}_{r^2} - 2y = 0 \longrightarrow x^2 + (y^2 - 2y + 1) = 0 + 1$

$x^2 + (y-1)^2 = 1$

$r^2\cos^2\theta + r^2\sin^2\theta - 2r\sin\theta = 0$

$r^2(\cos^2\theta + \sin^2\theta) - 2r\sin\theta = 0$

$r^2 - 2r\sin\theta = 0$

$r(r - 2\sin\theta) = 0$

$r = 0$

$r - 2\sin\theta = 0$

$\boxed{r = 2\sin\theta}$

$r(\theta) = 2\sin\theta$

Example 5: Convert the polar equation to rectangular form.

a. $r = 4\sin\theta$

$r \cdot r = r \cdot 4\sin\theta$

$r^2 = 4r\sin\theta$

$x^2 + y^2 = 4y$

$x^2 + (y^2 - 4y + 4) = 0 + 4$

$x^2 + (y-2)^2 = 4$

b. $r(-2\sin\theta + 3\cos\theta) = 2 \rightarrow \boxed{r = \frac{2}{-2\sin\theta + 3\cos\theta}}$

$-2r\sin\theta + 3r\cos\theta = 2$

$-2y + 3x = 2$

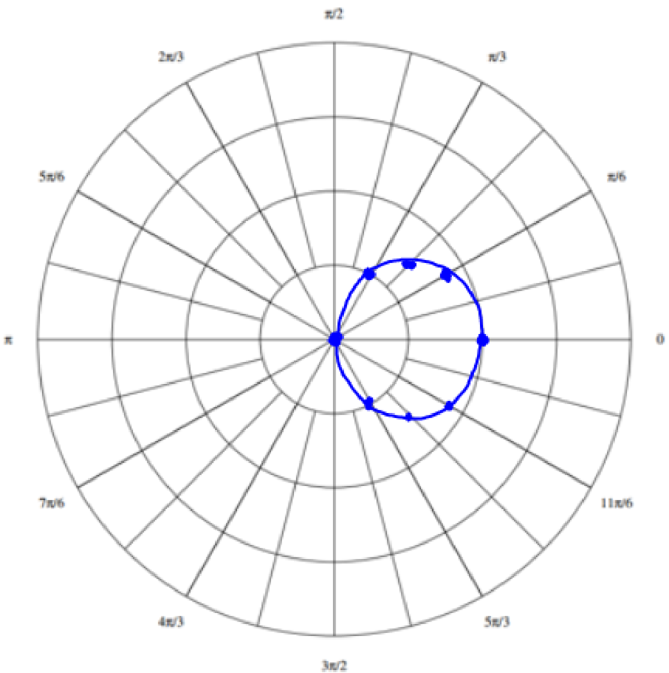
$y = \frac{3}{2}x - 1$

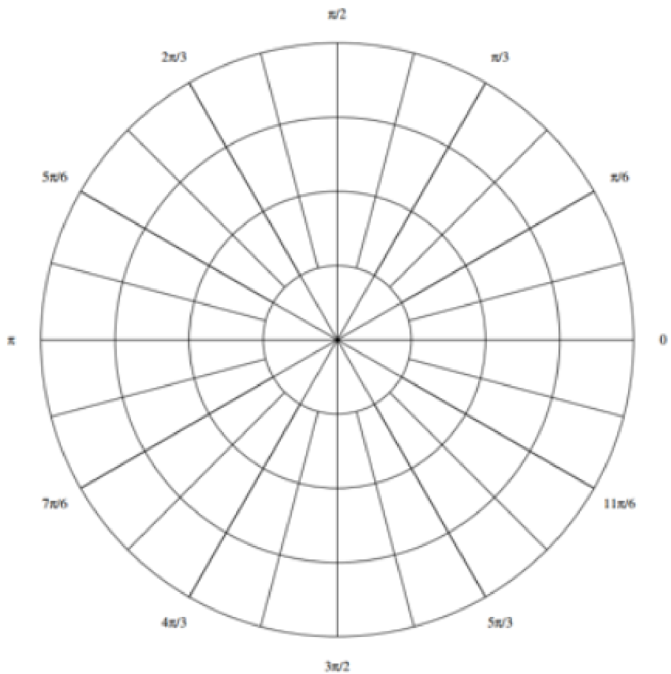
Example 4: Sketch the graph of the following polar curves.

a. $r = 2\cos\theta$ $[0, \pi]$

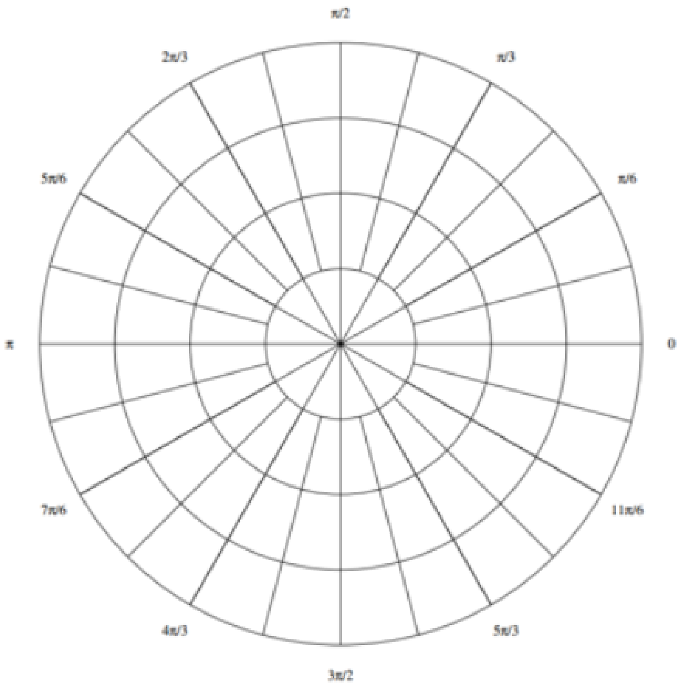
b. $r = 1 - \sin\theta$

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
r	2	$\sqrt{3}$	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2





c. $r = 2\cos 3\theta$



d. $r = 2\sqrt{\sin 2\theta}$