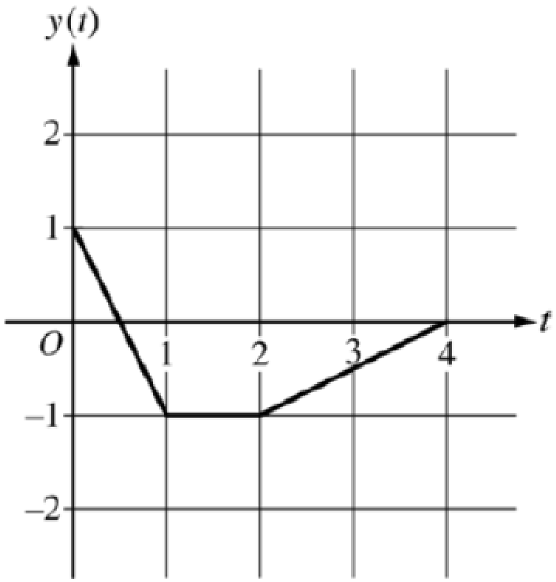
2016 Question 2 (Calc Active)



At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $x(3) = x(0) + \int_0^3 x'(t)\,dt = 5 + 9.377035 = 14.377$ $y(3) = -\frac{1}{2}$ The position of the particle at $t = 3$ is $(14.377, -0.5)$.	$3: \begin{cases} 1: \text{integral} \\ 1: \text{uses initial condition} \\ 1: \text{answer} \end{cases}$
(b) Slope $= \frac{y'(3)}{x'(3)} = \frac{0.5}{9.956376} = 0.05$	1 : slope
(c) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 9.969$ (or 9.968)	$2: \begin{cases} 1: \text{expression for speed} \\ 1: \text{answer} \end{cases}$
(d) Distance $= \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ $= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\,dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2}\,dt$ $= 2.237871 + 2.112003 = 4.350$ (or 4.349)	$3: \begin{cases} 1: \text{expression for distance} \\ 1: \text{integrals} \\ 1: \text{answer} \end{cases}$

2015 Question 2 (Calc Active)

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

(a) $x(2) = 3 + \int_1^2 \cos(t^2)\,dt = 2.557$ (or 2.556)	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$
(b) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{0.5t}}{\cos(t^2)}$ $\frac{e^{0.5t}}{\cos(t^2)} = 2$ $t = 0.840$	$2 : \begin{cases} 1 : \text{slope in terms of } t \\ 1 : \text{answer} \end{cases}$
(c) Speed $= \sqrt{\cos^2(t^2) + e^t}$ $\sqrt{\cos^2(t^2) + e^t} = 3$ $t = 2.196$ (or 2.195)	$2 : \begin{cases} 1 : \text{speed in terms of } t \\ 1 : \text{answer} \end{cases}$
(d) Distance $= \int_0^1 \sqrt{\cos^2(t^2) + e^t}\,dt = 1.595$ (or 1.594)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2012 Question 2 (Calc Active)

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\frac{dx}{dt}\right\|_{t=2} = \frac{2}{e^2}$ Because $\left.\frac{dx}{dt}\right\|_{t=2} > 0$, the particle is moving to the right at time $t = 2$. $\left.\frac{dy}{dx}\right\|_{t=2} = \frac{dy/dt\|_{t=2}}{dx/dt\|_{t=2}} = 3.055$ (or 3.054)	$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$
(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574) Acceleration $= \langle x''(4), y''(4) \rangle$ $= \langle -0.041, 0.989 \rangle$	$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$
(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ $= 0.651$ (or 0.650)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

2010 Question 3 (Calc Active)

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

(i) The two values of t when that occurs

(ii) The slopes of the lines tangent to the particle's path at that point

(iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second	1 : answer
(b) $x'(t) = 2t - 4$ Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\, dt = 11.587$ or 11.588 meters	2 : { 1 : integral 1 : answer
(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \neq 0$ This occurs at $t = 2.20794$. Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.	3 : { 1 : considers $\frac{dy}{dx} = 0$ 1 : $t = 2.207$ or 2.208 1 : direction of motion with reason
(d) $x(t) = 5$ at $t = 1$ and $t = 3$ At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right\|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=1} = 0.432$. At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right\|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=3} = 1$. $y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\, dt = 4$	3 : { 1 : $t = 1$ and $t = 3$ 1 : slopes 1 : y-coordinate

2006 Question 3 (Calc Active)

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1 + t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t \to \infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740 \rangle$ Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208	$2 : \begin{cases} 1 : \text{acceleration} \\ 1 : \text{speed} \end{cases}$
(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$ $1 - 2e^{-t} = 0$ $t = \ln 2 = 0.693$ and $\dfrac{dy}{dt} \neq 0$ when $t = \ln 2$	$2 : \begin{cases} 1 : x'(t) = 0 \\ 1 : \text{answer} \end{cases}$
(c) $m(t) = \dfrac{4t}{1+t^3} \cdot \dfrac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)}$ $\lim\limits_{t \to \infty} m(t) = \lim\limits_{t \to \infty} \left(\dfrac{4t}{1+t^3} \cdot \dfrac{1}{\sin^{-1}\left(1 - 2e^{-t}\right)} \right)$ $= 0\left(\dfrac{1}{\sin^{-1}(1)}\right) = 0$	$2 : \begin{cases} 1 : m(t) \\ 1 : \text{limit value} \end{cases}$
(d) Since $\lim\limits_{t \to \infty} x(t) = \infty$, $c = \lim\limits_{t \to \infty} y(t) = -3 + \int_2^{\infty} \dfrac{4t}{1+t^3}\, dt$	$3 : \begin{cases} 1\text{: integrand} \\ 1\text{: limits} \\ 1\text{: initial value consistent} \\ \quad \text{with lower limit} \end{cases}$