1.

(a) $\int_0^4 E(t)\,dt \approx 3981$ gallons	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Let $S(t)$ be the amount of sewage in the treatment tank at time t. Then $S'(t) = E(t) - 645$ and $S'(t) = 0$ when $E(t) = 645$. On the interval $0 \le t \le 4$, $E(t) = 645$ when $t = 2.309$ and $t = 3.559$.	$4: \begin{cases} 1: \text{sets } E(t) = 645 \\ 1: \text{identifies } t = 2.309 \text{ as a candidate} \\ 1: \text{amount of sewage at } t = 2.309 \\ 1: \text{conclusion} \end{cases}$

t (hours)	amount of sewage in treatment tank
0	0
2.309	$\int_0^{2.309} E(t)\,dt - 645(2.309) = 1637.178$
3.559	$\int_0^{3.559} E(t)\,dt - 645(3.559) = 1228.520$
4	$3981.022 - 645(4) = 1401.022$

The amount of sewage in the treatment tank is greatest at $t = 2.309$ hours. At that time, the amount of sewage in the tank, rounded to the nearest gallon, is 1637 gallons.

(c) Total cost $= \int_0^4 (0.15 - 0.02t)E(t)\,dt = 474.320$

The total cost of treating the sewage that enters the tank during the time interval $0 \le t \le 4$, to the nearest dollar, is \$474.

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

2.

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$

$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{interpretation with units} \end{cases}$

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

$2: \begin{cases} 1: \text{value} \\ 1: \text{interpretation with units} \end{cases}$

(c) $\dfrac{1}{20}\int_0^{20} W(t)\,dt \approx \dfrac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$

$= \dfrac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$

$= \dfrac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$3: \begin{cases} 1: \text{left Riemann sum} \\ 1: \text{approximation} \\ 1: \text{underestimate with reason} \end{cases}$

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\,dt$

$= 71.0 + 2.043155 = 73.043$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

3. (a) $g(-6) = \int_{-2}^{-6} f(t)\, dt = -\int_{-6}^{-2} f(t)\, dt = -\frac{1}{2}\cdot 4\cdot 5 = -10$

$g(3) = \int_{-2}^{3} f(t)\, dt = \frac{1}{2}\pi\cdot 2^2 - \frac{1}{2}\cdot 1\cdot 2 = 2\pi - 1$

$2 : \begin{cases} 1 : g(-6) \\ 1 : g(3) \end{cases}$

(b) $g'(0) = f(0) = 2$

$1 : g'(0)$

(c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

$3 : \begin{cases} 1 : \text{horizontal tangent at } x = -2 \text{ and } x = 2 \\ 2 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.
$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

$3 : \begin{cases} 2 : \text{values of } x \\ 1 : \text{explanation} \end{cases}$

4.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \;\Rightarrow\; t = 3,\ 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

$2 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 1 : \text{interval} \end{cases}$

(b) $\int_0^6 |v(t)|\, dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

$3 : \begin{cases} 1 : a(t) \\ 2 : \text{conclusion with reason} \end{cases}$

(d) $$\begin{aligned} x(4) &= -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt \\ &= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4 \\ &= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right] \\ &= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$