Limit of a Continuous Function

If *f(x)* is a continuous function for all real numbers, then $\lim_{x \to c} f(x) = f(c)$

Limits of Rational Functions

A. If *f(x)* is a rational function given by $f(x) = \frac{p(x)}{q(x)}$,such that $p(x)$ and $q(x)$ have no common factors, and *c* is a real number such that $q(c) = 0$, then

I. $\lim_{x \to c} f(x)$ does not exist

II. $\lim_{x \to c} f(x) = \pm\infty$ → x = c is a vertical asymptote

B. If *f(x)* is a rational function given by $f(x) = \frac{p(x)}{q(x)}$, such that reducing a common factor between $p(x)$ and $q(x)$ results in the agreeable function *k(x)*, then

$\lim_{x \to c} f(x) = \lim_{x \to c} \frac{p(x)}{q(x)} = \lim_{x \to c} k(x) = k(c)$ → Hole at the point $(c, k(c))$

Limits of a Function as x Approaches Infinity

If *f(x)* is a rational function given by $(x) = \frac{p(x)}{q(x)}$, such that $p(x)$ and $q(x)$ are both polynomial functions, then

A. If the degree of *p(x)* > *q(x)*, $\lim_{x \to \infty} f(x) = \infty$

B. If the degree of *p(x)* < *q(x)*, $\lim_{x \to \infty} f(x) = 0$ → y = 0 is a horizontal asymptote

C. If the degree of *p(x)* = *q(x)*, $\lim_{x \to \infty} f(x) = c$, where *c* is the ratio of the leading coefficients.

→ y = c is a horizontal asymptote

Special Trig Limits

A. $\lim_{x \to 0} \frac{\sin ax}{ax} = 1$ **B.** $\lim_{x \to 0} \frac{ax}{\sin ax} = 1$ **C.** $\lim_{x \to 0} \frac{1 - \cos ax}{ax} = 0$

L'Hospital's Rule

If results $\lim_{x \to c} f(x)$ or $\lim_{x \to \infty} f(x)$ results in an indeterminate form $\left(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \cdot \infty, 0^0, 1^\infty, \infty^0\right)$, and $f(x) = \frac{p(x)}{q(x)}$, then

$$\lim_{x \to c} f(x) = \lim_{x \to c} \frac{p(x)}{q(x)} = \lim_{x \to c} \frac{p'(x)}{q'(x)} \quad \text{and} \quad \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{p(x)}{q(x)} = \lim_{x \to \infty} \frac{p'(x)}{q'(x)}$$

The Definition of Continuity

A function $f(x)$ is continuous at *c* if

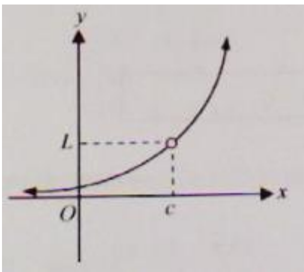
I. $\lim_{x \to c} f(x)$ exists

II. $f(c)$ exists

III. $\lim_{x \to c} f(x) = f(c)$

Types of Discontinuities

Removable Discontinuities (Holes)

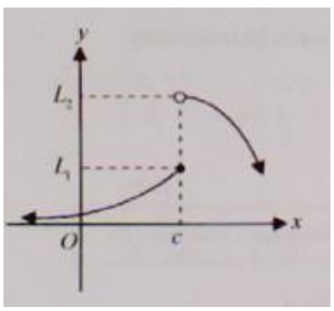


I. $\lim_{x \to c} f(x) = L$ (the limit exists)

II. $f(c)$ is undefined

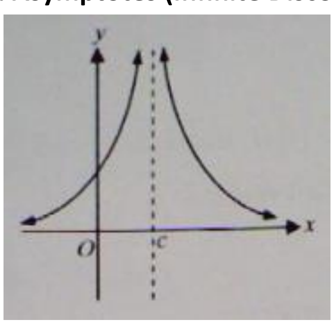
Non-Removable Discontinuities (Jumps and Asymptotes)

A. Jumps



$\lim_{x \to c} f(x) = DNE$ because $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

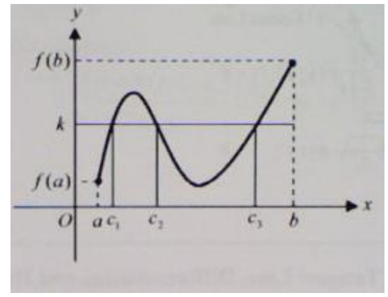
B. Asymptotes (Infinite Discontinuities)



$\lim_{x \to c} f(x) = \pm\infty$

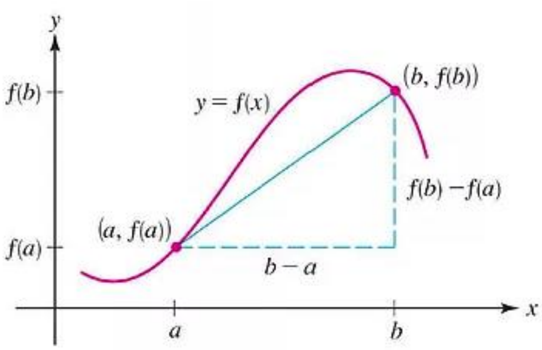
Intermediate Value Theorem

If *f* is a continuous function on the closed interval [a, b] and *k* is any number between *f(a)* and *f(b)*, then there exists at least one value of *c* on [a, b] such that *f(c) = k*. In other words, on a continuous function, if *f(a)< f(b)*, any y – value greater than *f(a)* and less than *f(b)* is guaranteed to exists on the function *f*.



Average Rate of Change

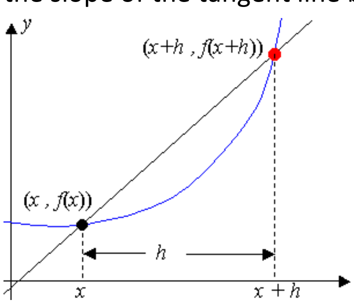
The average rate of change, *m*, of a function *f* on the interval [a, b] is given by the slope of the secant line.



$$m = \frac{f(b)-f(a)}{b-a}$$

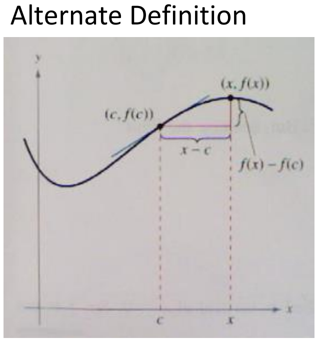
Definition of the Derivative

The derivative of the function *f*, or instantaneous rate of change, is given by converting the slope of the secant line to the slope of the tangent line by making the change is x, Δx or h, approach zero.



$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Alternate Definition



$$f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$$

Differentiability and Continuity Properties

A. If *f(x)* is differentiable at $x = c$, then *f(x)* is continuous at $x = c$.

B. If *f(x)* is not continuous at $x = c$, then *f(x)* is not differentiable at $x = c$.

C. The graph of *f* is continuous, but not differentiable at $x = c$ if:

I. The graph has a cusp or sharp point at $x = c$

II. The graph has a vertical tangent line at $x = c$

III. The graph has an endpoint at $x = c$

Basic Derivative Rules

Given *c* is a constant,

1. Constant Rule $\frac{d}{dx}[c] = 0$
2. Constant Multiple Rule $\frac{d}{dx}[cf(x)] = cf'(x)$
3. Sum Rule $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
4. Difference Rule $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
5. Product Rule $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
6. Quotient Rule $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
7. Chain Rule $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$ **2.** $\frac{d}{dx}[\cos x] = -\sin x$ **3.** $\frac{d}{dx}[\tan x] = \sec^2 x$

4. $\frac{d}{dx}[\sec x] = \sec x \tan x$ **5.** $\frac{d}{dx}[\csc x] = -\csc x \cot x$ **6.** $\frac{d}{dx}[\cot x] = -\csc^2 x$

Derivatives of Inverse Trig Functions

1. $\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$ **2.** $\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$ **3.** $\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$

4. $\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}}$ **5.** $\frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$ **6.** $\frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$

Derivatives of Exponential and Logarithmic Functions

1. $\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$
2. $\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$, $(a > 0,\ a \neq 1)$
3. $\frac{d}{dx}[\ln x] = \frac{1}{x}$
4. $\frac{d}{dx}[\ln|x|] = \frac{1}{x}$
5. $\frac{d}{dx}[\log_a |x|] = \frac{1}{x \ln a}$, $(a > 0,\ a \neq 1)$
6. $\frac{d}{dx}[e^x] = e^x$
7. $\frac{d}{dx}[a^x] = a^x \ln a$

Explicit and Implicit Differentiation

A. Explicit Functions: Function y is written only in terms of the variable x $(y = f(x))$. Apply derivatives rules normally.

B. Implicit Differentiation: An expression representing the graph of a curve in terms of both variables x and y.

I. Differentiate both sides of the equation with respect to x. (terms with x differentiate normally, terms with y are multiplied by $\frac{dy}{dx}$ per the chain rule)

II. Group all terms with $\frac{dy}{dx}$ on one side of the equation and all other terms on the other side of the equation.

III. Factor $\frac{dy}{dx}$ and express $\frac{dy}{dx}$ in terms of x and y.

Tangent Lines and Normal Lines

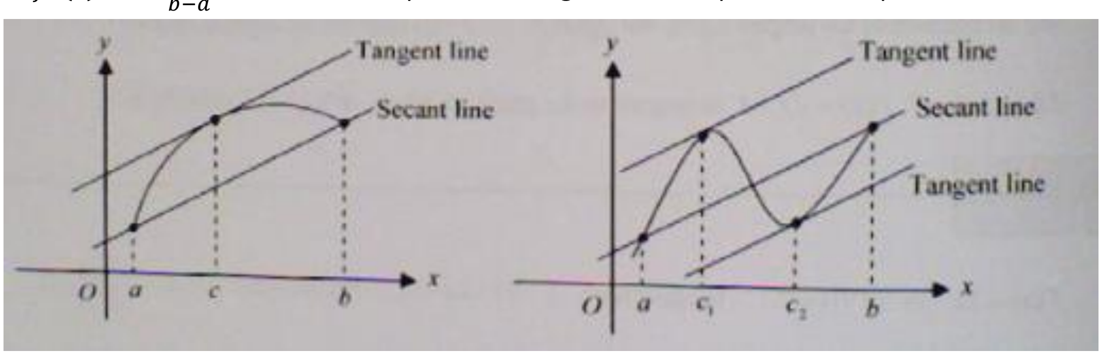
A. The equation of the tangent line at a point $(a, f(a))$: $y - f(a) = f'(a)(x - a)$

B. The equation of the normal line at a point $(a, f(a))$: $y - f(a) = -\frac{1}{f'(a)}(x - a)$

Mean Value Theorem for Derivatives

If the function *f* is continuous on the close interval [a, b] and differentiable on the open interval (a, b), then there exists at least one number *c* between *a* and *b* such that

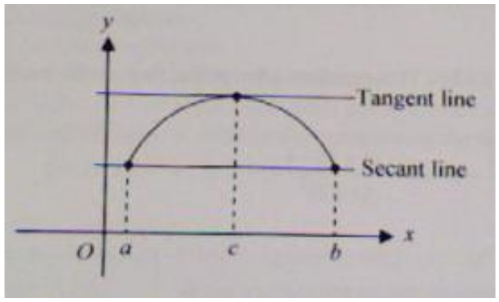
$f'(c) = \frac{f(b) - f(a)}{b - a}$ The slope of the tangent line is equal to the slope of the secant line.



Rolle's Theorem (Special Case of Mean Value Theorem)

If the function *f* is continuous on the close interval [a, b] and differentiable on the open interval (a, b), and *f(a) = f(b)*, then there exists at least one number *c* between *a* and *b* such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} = 0$$



Particle Motion

A velocity function is found by taking the derivative of position. An acceleration function is found by taking the derivative of a velocity function.

$x(t)$	Position	
$x'(t) = v(t)$	Velocity	* $\|v(t)\| = speed$
$x''(t) = v'(t) = a(t)$	Accleration	

Rules:

A. If velocity is positive, the particle is moving right or up. If velocity is negative, the particle is moving left or down.

B. If velocity and acceleration have the same sign, the particle speed is increasing. If velocity and acceleration have opposite signs, speed is decreasing.

C. If velocity is zero and the sign of velocity changes, the particle changes direction.

Related Rates

A. Identify the known variables, including their rates of change and the rate of change that is to be found. Construct an equation relating the quantities whose rates of change are known and the rate of change to be found.

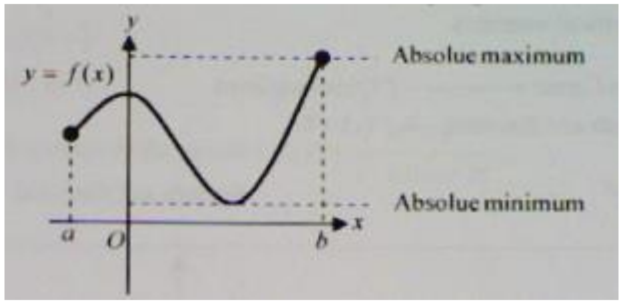
B. Implicitly differentiate both sides of the equation with respect to time. (Remember: DO NOT substitute the value of a variable that changes throughout the situation before you differentiate. If the value is constant, you can substitute it into the equation to simplify the derivative calculation).

C. Substitute the known rates of change and the known values of the variables into the equation. Then solve for the required rate of change.

*Keep in mind, the variables present can be related in different ways which often involves the use of similar geometric shapes, Pythagorean Theorem, etc.

Extrema of a Function

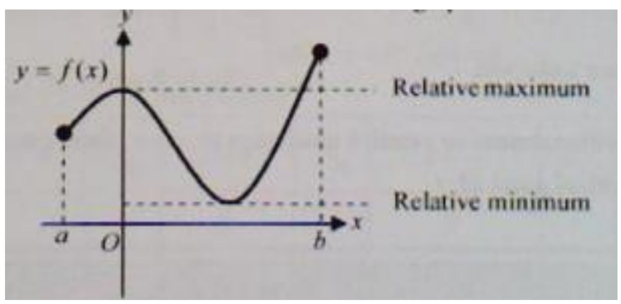
A. Absolute Extrema: An absolute maximum is the highest y – value of a function on a given interval or across the entire domain. An absolute minimum is the lowest y – value of a function on a given interval or across the entire domain.



B. Relative Extrema

I. Relative Maximum: The y-value of a function where the graph of the function changes from increasing to decreasing. Another way to define a relative maximum is the y-value where derivative of a function changes from positive to negative.

II. Relative Minimum: The y-value of a function where the graph of the function changes from decreasing to increasing. Another way to define a relative maximum is the y-value where derivative of a function changes from negative to positive.



Critical Value

When *f(c)* is defined, if *f ′ (c) = 0* or *f ′* is undefined at *x = c*, the values of the x – coordinate at those points are called critical values.

*If *f(x)* has a relative extrema at *x = c*, then *c* is a critical value of *f*.

Extreme Value Theorem

If the function *f* continuous on the closed interval [a, b], then the absolute extrema of the function *f* on the closed interval will occur at the endpoints or critical values of *f*.

*After identifying critical values, create a table with endpoints and critical values. Calculate the y – value at each of these x values to identify the extrema.

Increasing and Decreasing Functions

For a differentiable function *f*

A. If $f'(x) > 0$ in (a, b), then *f* is increasing on (a, b) → Tangent line has a positive slope

B. If $f'(x) < 0$ in (a, b), then *f* is decreasing on (a, b) → Tangent line has a negative slope

C. If $f'(x) = 0$ in (a, b), then *f* is constant on (a, b) → Tangent line has a zero slope (horizontal)

First Derivative Test

After calculating any discontinuities of a function *f* and calculating the critical values of a function *f*, create a sign chart for *f '*, reflecting the domain, discontinuities, and critical values of a function *f*.

A. If $f'(x)$ changes sign from negative to positive at $x = c$, then $f(c)$ is a relative minimum of *f.*

B. If $f'(x)$ changes sign from positive to negative at $x = c$, then $f(c)$ is a relative maximum of *f.*

*If there is no sign change of $f'(x)$, there exists a shelf point

Concavity

For a differentiable function *f(x)*,

A. If $f''(x) > 0$, the graph of $f(x)$ is concave up

→ This means $f'(x)$ is increasing

B. If $f''(x) < 0$, the graph of $f(x)$ is concave down

→ This means $f'(x)$ is decreasing

Second Derivative Test

For a function *f(x)* that is continuous at *x = c*

A. If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a relative minimum.

B. If $f'(c) = 0$ and $f''^{(c)} < 0$, then $f(c)$ is a relative maximum.

* If $f'(c) = 0$ and $f''(c) = 0$, you must use the first derivative test to determine extrema

Point of Inflection

Let *f* be a functions whose second derivative exists on any interval. If *f* is continuous at $x = c$, $f''(c) = 0$ or $f''(c)$ is undefined, and $f''(x)$ changes sign at $x = c$, then the point $(c, f(c))$ is a point of inflection.

Optimization

Finding the largest or smallest value of a function subject to some kind of constraints.

A. Define the primary equation for the quantity to be maximized or minimized. Define a feasible domain for the variables present in the equation.

B. If necessary, define a secondary equation that relates the variables present in the primary equation. Solve this equation for one of the variables and substitute into the primary equation.

C. Once the primary equation is represented in a single variable, take the derivative of the primary equation.

D. Find the critical values using the derivative calculated.

E. The optimal solution will more than likely be found at a critical value from **D.** Keep in mind, if the critical values do not represent a minimum or a maximum, the optimal solution may be found at an endpoint of the feasible domain.

Derivative of an Inverse

If *f* and its inverse *g* are differentiable, and the point (c, f(c)) exists on the function *f* meaning the point (f(c), c) exists on the function *g*¸ then

$$\frac{d}{dx}[g(x)] = \frac{1}{f'\left(f^{-1}(x)\right)} = \frac{1}{f'\left(f(c)\right)}$$

Antiderivatives

If $F'(x) = f(x)$ for all *x*, $F(x)$ is an antiderivative of *f*.

$$\int f(x) = F(x) + C$$

* The antiderivative is also called the Indefinite Integral

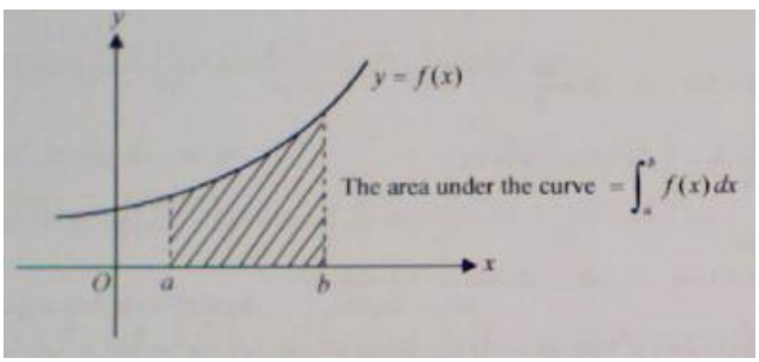
Basic Integration Rules

Let *k* be a constant.

$\int 0\, dx = C$	$\int k\, dx = kx + C$
$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$	$\int \cos x\, dx = \sin x + C$
$\int \sin x\, dx = -\cos x + C$	$\int \sec^2 x\, dx = \tan x + C$
$\int \sec x \tan x\, dx = \sec x + C$	$\int \csc^2 x\, dx = -\cot x + C$
$\int \csc x \cot x\, dx = -\csc x + C$	

Definite Integrals (The Fundamental Theorem of Calculus)

A definite integral is an integral with upper and lower limits, *a* and *b*, respectively, that define a specific interval on the graph. A definite integral is used to find the area bounded by the curve and an axis on the specified interval (a, b).



If $F(x)$ is the antiderivative of a continuous function $f(x)$, the evaluation of the definite integral to calculate the area on the specified interval (a, b) is the First Fundamental Theorem of Calculus:

$$\int_a^b f(x)dx = F(b) - F(a)$$

Integration Rules for Definite Integrals

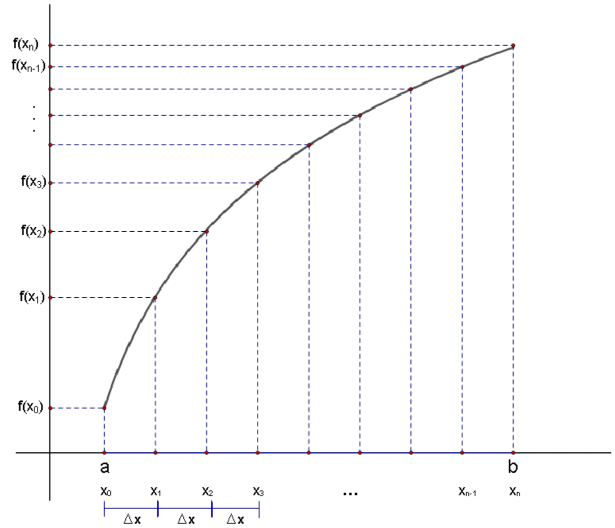
1. $\int_a^a f(x) = 0$
2. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for $c \in (a,b)$ → *This means that *c* is a value of *x*, lying between *a* and *b*
3. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
4. $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$, where f is an **even function**.
5. $\int_{-a}^a f(x)\,dx = 0$, where f is an **odd function.**

Riemann Sum (Approximations)

A Riemann Sum is the use of geometric shapes (rectangles and trapezoids) to approximate the area under a curve, therefore approximating the value of a definite integral.

If the interval [a, b] is partitioned into n subintervals, then each subinterval, Δx, has a width: $\Delta x = \frac{b-a}{n}$.

Therefore, you find the sum of the geometric shapes, which approximates the area by the following formulas:



A. Right Riemann Sum

$$Area \approx \Delta x\,[f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1})]$$

B. Left Riemann Sum

$$Area \approx \Delta x\,[f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_n)]$$

C. Midpoint Riemann Sum

$$Area \approx \Delta x\left[f\left(x_{1/2}\right) + f\left(x_{3/2}\right) + f\left(x_{5/2}\right) + \cdots + f\left(x_{(2n-1)/2}\right)\right]$$

D. Trapezoidal Sum

$$Area \approx \frac{1}{2}\,\Delta x\,[f(x_0) + 2\,f(x_1) + 2\,f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Properties of Riemann Sums

A. The area under the curve is under approximated when

I.A Left Riemann sum is used on an increasing function.

II. A Right Riemann sum is used on a decreasing function.

III. A Trapezoidal sum is used on a concave down function.

B. The area under the curve is over approximated when

I.A Left Riemann sum is used on a decreasing function.

II. A Right Riemann sum is used on an increasing function.

III. A Trapezoidal sum is used on a concave up function.

Riemann Sum (Limit Definition of Area)

Let f be a continuous function on the interval [a, b]. The area of the region bounded by the graph of the function f and the x – axis (i.e. the value of the definite integral) can be found using

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

Where c_i is either the left endpoint ($c_i = a + (i-1)\Delta x$) or right endpoint ($c_i = a + i\Delta x$) and $\Delta x = (b-a)/n$.

Average Value of a Function

If a function f is continuous on the interval [a, b], the average value of that function f is given by

$$\frac{1}{b-a}\int_a^b f(x)dx$$

Second Fundamental Theorem of Calculus

If a function f is continuous on the interval [a, b], let u represent a function of x, then

A. $\dfrac{d}{dx}\left[\int_a^x f(t)dt\right] = f(x)$

B. $\dfrac{d}{dx}\left[\int_x^b f(t)dt\right] = -f(x)$

C. $\dfrac{d}{dx}\left[\int_a^{u(x)} f(t)dt\right] = f\big(u(x)\big)\cdot u'(x)$

Integration of Exponential and Logarithmic Formulas

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{u'}{u}\,du = \ln|u| + C$, where u is a differentiable function of x.
3. $\int \frac{1}{x-a}\,dx = \ln|x-a| + C$, where a = constant.
4. $\int a^x\,dx = \frac{a^x}{\ln a} + C$
5. $\int e^x\,dx = \frac{e^x}{\ln e} + C = e^x + C$
6. $\int a^{u(x)}dx = \frac{a^{u(x)}}{(\ln a)u'} + C$
7. $\int e^{u(x)}dx = \frac{e^{u(x)}}{u'} + C$

Integration of Trig and Inverse Trig

1. $\int \cos x \, dx = \sin x + c$

2. $\int \sin x \, dx = -\cos x + C$

3. $\int \sec^2 x \, dx = \tan x + C$

4. $\int \csc^2 x \, dx = -\cot x + C$

5. $\int \sec x \tan x \, dx = \sec x + C$

6. $\int \csc x \cot x \, dx = -\csc x + C$

7. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$

8. $\int \frac{-1}{\sqrt{1-x^2}} dx = \arccos x + C$

9. $\int \frac{1}{1+x^2} dx = \arctan x + C$

10. $\int \frac{-1}{1+x^2} dx = \operatorname{arccot} x + C$

11. $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \operatorname{arcsec} x + C$

12. $\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \operatorname{arccsc} x + C$

If u is a differentiable function of x, then

1. $\int \frac{u'}{u} du = \ln|u| + C$

2. $\int \frac{u'}{\sqrt{1-u^2}} du = \arcsin u + C$

3. $\int \frac{u'}{1+u^2} du = \arctan u + C$.

Exponential Growth and Decay

When the rate of change of a variable *y* is directly proportional to the value of *y*, the function *y = f(x)* is said to grow/decay exponentially.

A. Differential Equation for rate of change: $\frac{dy}{dt} = ky$

B. General Solution: $y = Ce^{kt}$

I. If $k > 0$, then exponential growth occurs.

II. If $k < 0$, then exponential decay occurs.

Area Between Two Curves

A. Let $y = f(x)$ and $y = g(x)$represent two functions such that $f(x) \geq g(x)$(meaning the function *f* is always above the function *g* on the graph) for every *x* on the interval [a, b].

$$\text{Area Between Curves} = \int_a^b [f(x) - g(x)]\,dx$$

B. Let $x = f(y)$ and $x = g(y)$represent two functions such that $f(y) \geq g(y)$(meaning the function *f* is always to the right of the function *g* on the graph) for every *y* on the interval [a, b].

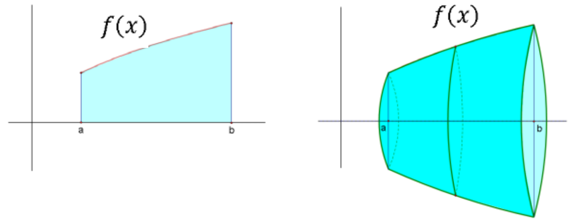
$$\text{Area Between Curves} = \int_a^b [f(y) - g(y)]\,dy$$

Volumes of a Solid of Revolution: Disk Method

If a defined region, bounded by a differentiable function *f*, on a graph is rotated about a line, the resulting solid is called a solid of revolution and the line is called the axis of revolution. The disk method is used when the defined region boarders the axis of revolution over the entire interval [a, b]

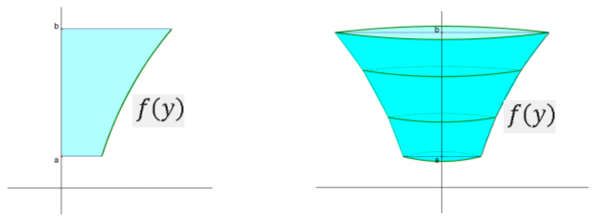
A. Revolving around the x – axis

$$\text{Volume} = \pi \int_a^b \left(f(x)\right)^2 dx$$



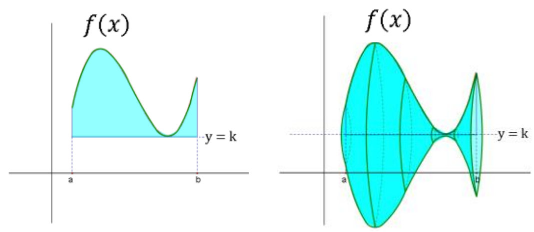
B. Revolving around the y – axis

$$\text{Volume} = \pi \int_a^b \left(f(y)\right)^2 dy$$



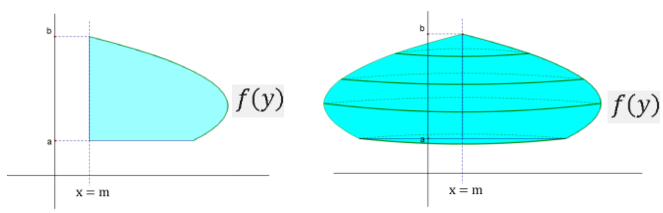
C. Revolving around a horizontal line *y* = *k*

$$\text{Volume} = \pi \int_a^b (f(x) - k)^2 dx$$



D. Revolving around a vertical line *x* = *m*

$$\text{Volume} = \pi \int_a^b (f(y) - m)^2 dy$$

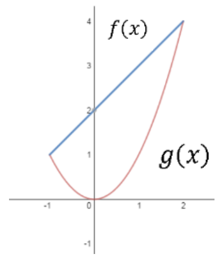


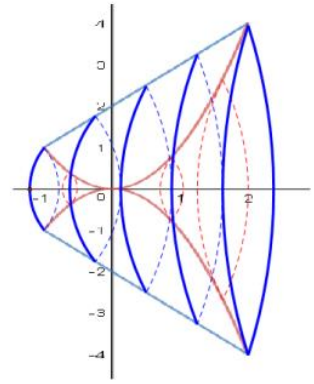
Volumes of a Solid of Revolution: Washer Method

If a defined region, bounded by a differentiable function *f*, on a graph is rotated about a line, the resulting solid is called a solid of revolution and the line is called the axis of revolution. The washer method is used when the defined region has space between the axis of revolution on the interval [a, b]

A. Revolving around the x – axis, where $f(x) \geq g(x)$(meaning the function *f* is always above the function *g* on the graph) for every *x* on the interval [a, b].

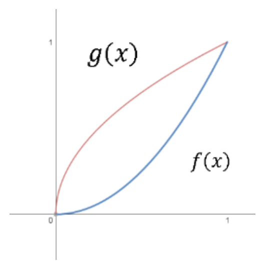
$$\text{Volume} = \pi \int_a^b ([f(x)]^2 - [g(x)]^2)dx$$

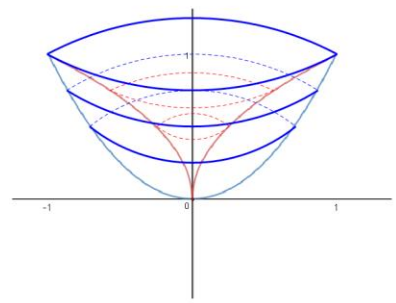




B. Revolving around the y – axis, where $f(y) \geq g(y)$(meaning the function *f* is always to the right of the function *g* on the graph)

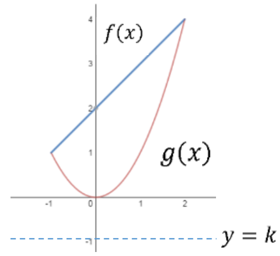
$$\text{Volume} = \pi \int_a^b ([f(y)]^2 - [g(y)]^2)dy$$

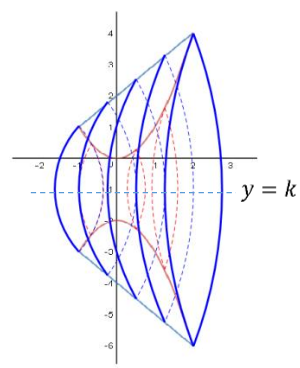




C. Revolving around a horizontal line *y* = *k*, where $f(x) \geq g(x)$(meaning the function *f* is always above the function *g* on the graph) for every *x* on the interval [a, b].

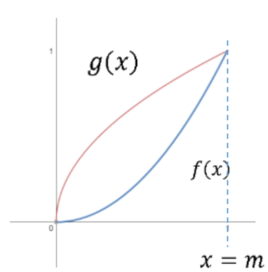
$$\text{Volume} = \pi \int_a^b ([f(x) - k]^2 - [g(x) - k]^2)dx$$

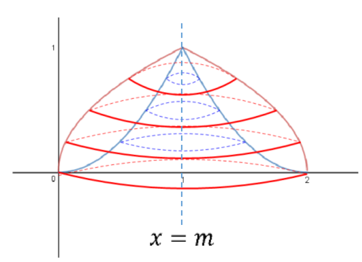




D. Revolving around a vertical line *x* = *m*, where $f(y) \geq g(y)$(meaning the function *f* is always to the right of the function *g* on the graph)

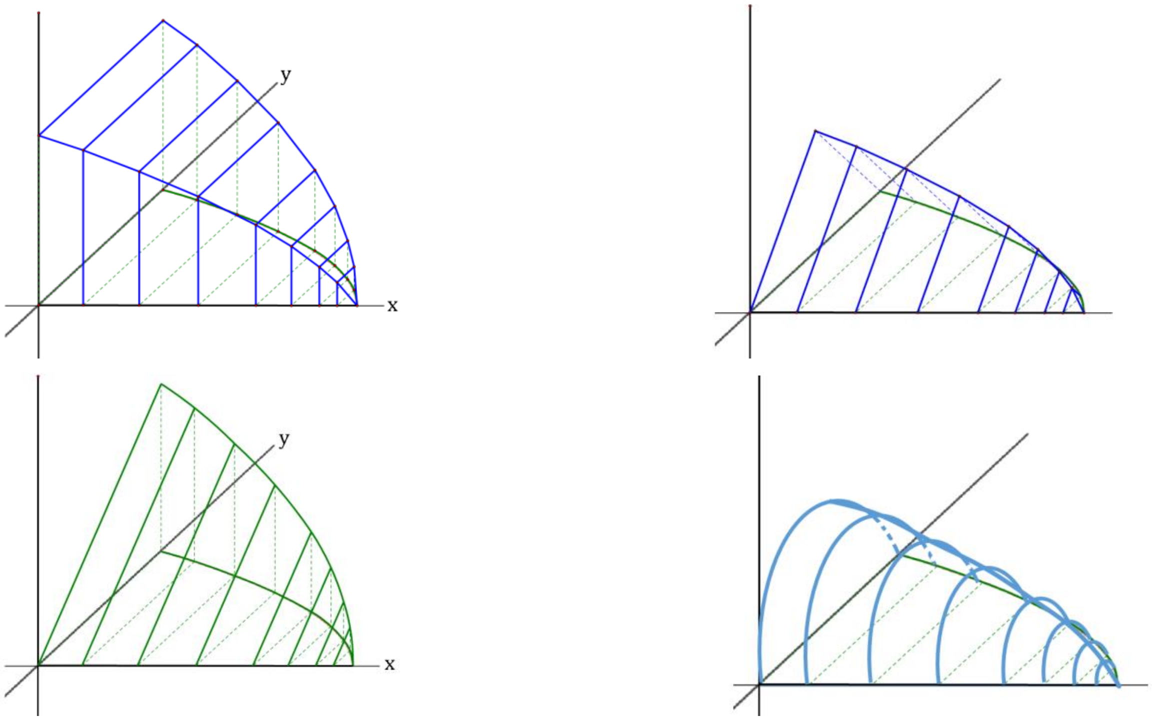
$$\text{Volume} = \pi \int_a^b ([f(y) - m]^2 - [g(y) - m]^2)dy$$





Volumes of Known Cross Sections

If a defined region, bounded by a differentiable function *f*, is used at the base of a solid, then the volume of the solid can be found by integrated using known area formulas.



For the cross sections perpendicular to the x – axis and a region bounded by a function *f* , on the interval [a, b], and the axis.

I. Cross sections are squares

$$\text{Volume} = \int_a^b [f(x)]^2 dx$$

II. Cross sections are equilateral triangles

$$\text{Volume} = \frac{\sqrt{3}}{4}\int_a^b [f(x)]^2 dx$$

III. Cross sections are isosceles right triangles with a leg in the base

$$\text{Volume} = \frac{1}{2}\int_a^b [f(x)]^2 dx$$

IV. Cross sections are isosceles right triangles with the hypotenuse in the base

$$\text{Volume} = \frac{1}{4}\int_a^b [f(x)]^2 dx$$

V. Cross sections are semicircles (with diameter in base)

$$\text{Volume} = \frac{\pi}{8}\int_a^b [f(x)]^2 dx$$

VI. Cross sections are semicircles (with radius in base)

$$\text{Volume} = \frac{\pi}{2}\int_a^b [f(x)]^2 dx$$

Differential Equations

A differential equation is an equation involving an unknown function and one or more of its derivatives

$\frac{dy}{dx} = f(x, y)$ → Usually expressed as a derivative equal to an expression in terms of *x* and/or *y*.

To solve differential equations, use the technique of separation of variables.

Given the differential equation $\frac{dy}{dx} = \frac{xy}{(x^2+1)}$

Step 1: Separate the variables, putting all *y's* on one side, with *dy* in the numerator, and all *x's* on the other side, with *dx* in the numerator.

$$\frac{1}{y}dy = \frac{x}{(x^2+1)}dx$$

Step 2: Integrate both sides of the equation.

$$\ln|y| = \frac{1}{2}\ln\sqrt{x^2+1} + C$$

Step 3: Solve the equation for y.

$$y = C\sqrt{x^2+1}$$

Given the differential equation $\frac{dy}{dx} = 2x^2$ with the initial condition $y(3) = 10$.

A. The general solution to a differential equation is left with the constant of integration, *C*, undefined.

$$dy = 2x^2\,dx \quad \to \quad \int dy = \int 2x^2\,dx \quad \to \quad y = \frac{2}{3}x^3 + C$$

B. The particular solution uses the given initial condition to calculate the value of *C*.

$$10 = \frac{2}{3}(3)^3 + C \quad \to \quad C = -8 \quad \to \quad y = \frac{2}{3}x^3 - 8$$

Slope Field

The derivative of a function gives the value of the slope of the function at each point *(x, y).* A slope field is a graphical representation of all of the possible solutions to a given differential equation. The slope field is generated by plugging in the coordinates of every point *(x, y)* into the differential equation and drawing a small segment of the tangent line at each point.

Given the differential equation $\frac{dy}{dx} = \frac{x}{y}$

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{0}{0}\ undefined$$

$$\left.\frac{dy}{dx}\right|_{(0,\pm1)} = 0$$

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{1}{2}$$

*These are only three example points. You would do this for every point in the given region of the graph.