Integration of Exponentials

The rule for integration with exponential functions follows straight from the derivative of exponential functions:

Let u be a differentiable function of x.

$$\frac{d}{dx}[e^u] = e^u \cdot u' \Longrightarrow \int e^u du = e^u + c$$

Examples:

a. $\int e^{3x+1}dx$

$u = 3x + 1$

$du = 3\,dx$

$\frac{1}{3}\int e^u\, du$

$\frac{1}{3} e^u + C$

$\boxed{\frac{1}{3} e^{3x+1} + C}$

b. $\int 5xe^{-x^2+1}dx$

$u = -x^2 + 1$

$du = -2x\,dx$

$-\frac{5}{2}\int e^u\, du$

$-\frac{5}{2} e^u + C$

$\boxed{-\frac{5}{2} e^{-x^2+1} + C}$

c. $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$

$\int x^{-2} \cdot e^{x^{-1}}\,dx$

$u = x^{-1}$

$du = -x^{-2}\,dx$

$-\int e^u\,du$

$-e^u + C$

$\boxed{-e^{1/x} + C}$

d. $\int e^{2\ln(x^2+1)}\,dx$

$\int \cancel{e^2 \cdot e^{\ln(x^2+1)}}\,dx$

$\int e^{\ln((x^2+1)^2)}\,dx$

$\int (x^2+1)^2\,dx$

$\int x^4 + 2x^2 + 1\,dx$

$\boxed{\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C}$

Evaluate the following definite integrals:

a. $\int_0^1 e^{-x}\,dx$

$u = -x$

$du = -dx$

$x = 0,\ u = 0$

$x = 1,\ u = -1$

$-\int_0^{-1} e^u\,du$

$\int_{-1}^{0} e^u\,du$

$\left[e^u\right]_{-1}^{0}$

$e^0 - e^{-1}$

$\boxed{1 - e^{-1}}$

$\boxed{1 - \frac{1}{e}}$

$\boxed{\frac{e-1}{e}}$

≈ 0.632

b. $\int_0^1 \frac{e^x}{1+e^x}\,dx$

$u = 1 + e^x$

$du = e^x dx$

$x = 0,\ u = 2$

$x = 1,\ u = 1+e$

$\int_2^{1+e} \frac{1}{u}\,du$

$\left[\ln|u|\right]_2^{1+e}$

$\boxed{\ln|1+e| - \ln|2| \approx 0.620}$

$\ln\left(\frac{1+e}{2}\right)$

c. $\int_{-1}^{0} e^x \cos(e^x)\,dx$

$u = e^x$

$du = e^x dx$

$\int \cos(u)\,du$

$\sin(u)$

$\left[\sin(e^x)\right]_{-1}^{0}$

$\sin(e^0) - \sin(e^{-1})$

$\boxed{\sin(1) - \sin\left(\frac{1}{e}\right) \approx 0.482}$

d. $\int_{-2}^{1} -2e^{2x-2}\,dx$

$u = 2x - 2$

$du = 2\,dx$

$x = -2,\ u = -6$

$x = 1,\ u = 0$

$-\int_{-6}^{0} e^u\,du$

$\left[-e^u\right]_{-6}^{0}$

$-e^0 + e^{-6}$

$e^{-6} - 1$

$\frac{1}{e^6} - 1$

$\frac{1-e^6}{e^6}$

≈ -0.998