2017 Question 2 (Calculator Active)

When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right)$$
 for $0 < t \le 12$,

where f(t) is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4 \ln(t^2 + 2t)$$
 for $3 < t \le 12$,

where g(t) is measured in pounds per hour and t is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find f'(7). Using correct units, explain the meaning of f'(7) in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time t = 5? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time t = 8?

(a)
$$\int_0^2 f(t) dt = 20.051175$$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

(b)
$$f'(7) = -8.120$$
 (or -8.119)

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

(c)
$$g(5) - f(5) = -2.263103 < 0$$

Because g(5) - f(5) < 0, the number of pounds of bananas on the display table is decreasing at time t = 5.

(d)
$$50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt = 23.347396$$

23.347 pounds of bananas are on the display table at time t = 8.

$$2: \begin{cases} 1 : integra \\ 1 : answer \end{cases}$$

$$2: \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$$

$$2: \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$$

$$3: \begin{cases} 2: \text{integrals} \\ 1: \text{answer} \end{cases}$$

Integration by Substitution

When integrating with composite functions, we can employ a technique called a *u-substitution*. The role of the substitution in integration is comparable to the role of the Chain Rule in differentiation. Recall:

$$\frac{d}{dx}\big[F\big(g(x)\big)\big] = F'(g(x)) \cdot g'(x) \qquad \longrightarrow \qquad \int F'(g(x)) \cdot g'(x) = F\big(g(x)\big) + c$$

Examples: Find the integral specified.

a.
$$\int \sqrt{2x-1} \, dx$$

$$\int (2x-1)^{1/2} \, dx \qquad dx = 2x-1$$

$$\int u^{1/2} \cdot \frac{1}{2} \, du \qquad du = 2 \, dx$$

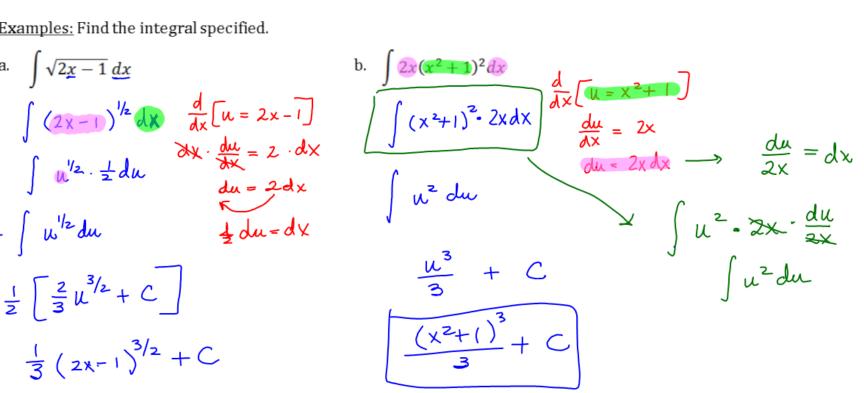
$$\int u^{1/2} \cdot \frac{1}{2} \, du \qquad du = 2 \, dx$$

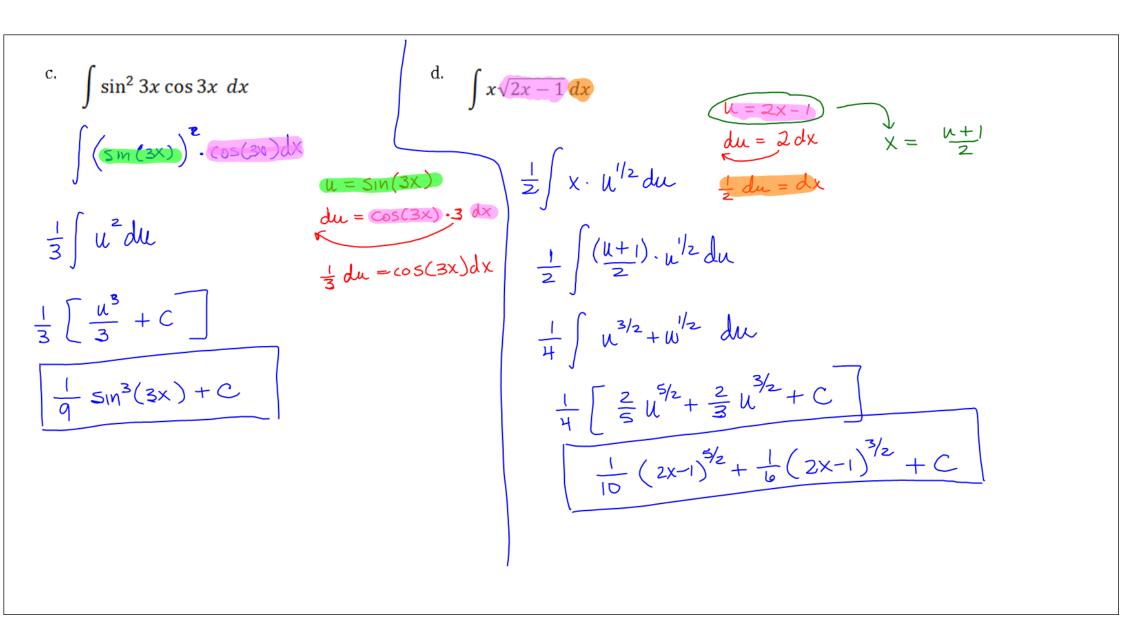
$$\int \frac{1}{2} \left[\frac{2}{3} \, u^{3/2} + C \right]$$

$$\frac{1}{3} \left(2x-1 \right)^{3/2} + C$$

$$\frac{1}{3} \left(2x-1 \right)^{1/2} \cdot \frac{1}{2}$$

$$\frac{1}{3} \left(2x-1 \right)^{1/2} \cdot \frac{1}{2}$$





Definite Integrals

When working with definite integrals, there are two ways to approach evaluating the integrals.

1) After integration, rewrite in terms of x and use the original limits of integration.

Examples:

a.
$$\int_{-1}^{1-x} \frac{x+1}{(x^2+2x+2)^3} dx$$

$$u = x^2 + 2x + 2$$

$$du = (2x+2) dx$$

$$du = 2(x+1) dx$$

$$\frac{1}{2} \int u^{-3} du$$

$$\frac{1}{2} \int u^{-3} du$$

$$\frac{1}{2} \left[\frac{u^{-2}}{-2} \right]$$

$$-\frac{1}{4(25)} - \frac{1}{4(1)^2} = \frac{6}{25}$$

$$-\frac{1}{4(1)^2} \int u^{-3} du$$

$$-\frac{1}{4(1)^2} = \frac{6}{25}$$

Examples:
a.
$$\int_{-1}^{1-x} \frac{x+1}{(x^2+2x+2)^3} dx$$
b. $\int_{0=x}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$

$$\lim_{x \to \infty} \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \int_{0=x}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$$

$$\lim_{x \to \infty} \frac{1}{\sqrt{x}} \int_{0=x}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$$

$$\lim$$

2) Redefine the limits of integration in terms of u.

Examples:

a.
$$\int_{0}^{1=x} x(x^{2}+1)^{3} dx$$

$$|x| = x^{2}+1$$

$$|x| = 2x dx$$

$$|x| = 0, |x| = 2$$

$$|x| = 1, |x| = 2$$

b.
$$\int_0^{\pi} \cos x \sqrt{\sin x} \, dx$$

$$\int_0^{\infty} \sqrt{u} \, du$$

$$u = \sin x$$

$$du = \cos x dx$$

$$x = 0, \quad u = \sin(0) = 0$$

$$x = \pi, \quad u = \sin(\pi) = 0$$